COLLECTIVE ACCELERATION OF PROTONS BY THE PLASMA WAVES IN A COUNTERSTREAMING ELECTRON BEAM

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Abstract

A novel advanced accelerator is proposed. The counterstreaming electron beam accelerator relies on the same physical mechanism as that of the plasma accelerator but replaces the stationary plasma in the plasma accelerator with a magnetized relativistic electron beam, drifting antiparallel to the driving source and the driven particles, as the wave supporting medium. The plasma wave in a counterstreaming electron beam can be excited either by a density-ramped driving electron beam or by properly beating two laser beams. The fundamental advantages of the counterstreaming electron beam accelerator over the plasma accelerator are a longer and tunable plasma wavelength, a longer pump depletion length or a larger transformer ratio, and easier pulse shaping for the driving source and the driven beam. Therefore, the energy gain of the driven particles can be greatly enhanced whereas the trapping threshold can be dramatically reduced, thus admitting the possibility for proton acceleration.

Introduction

Using plasma waves to accelerate electrons have attracted extensive attention ever since the first proposal of the plasma beat-wave accelerator (PBWA) by Tajima and dawson in 1979.[1] By properly tuning two lasers they demonstrated that a plasma wave with phase velocity near the speed of light c can be generated in the plasma. A bunch of electrons can then ride the wave electric field and accelerate to high energy. Another promising scheme that can also generate relativistic plasma waves, the plasma wake-field accelerator (PWFA), has also been proposed recently.[2] The PWFA differs from the PBWA in that it replaces the two laser beams in the PBWA with a relativistic electron beam as the driving source. The basic advantage of using the plasma waves to accelerate charged particles is that there is no electrical breakdown since the plasma is already fully ionized. Thus the acceleration gradient is only limited by the plasma wave breaking field which can be an order of a Gev/m at present.[3]

Although both the PBWA and the PWFA are very attractive, there are some fundamental shortcomings associated with them, namely the inherent short plasma wavelength, the inherent short pump depletion length, and the difficulty for pulse shaping of the driving source and the driven beam. Aiming at improving these drawbacks in the plasma accelerators, the author proposes using a counterstreaming electron beam as the relativistic plasma wave supporting medium for charged particle acceleration.

Similar to the plasma wake-field accelerator, the plasma wave in a counterstreaming electron beam can be excited by a shaped driving electron beam. This is called the counterstreaming electron beam wake-field accelerator (CWFA). Similar to the plasma beat-wave accelerator, the plasma wave in a counterstreaming electron beam can also be excited by properly beating two laser beams. This is called the counterstreaming electron beam beat-wave accelerator (CBWA). Exciting the plasma wave in a counterstreaming electron beam has several fundamental advantages over the plasma accelerators: (1) a longer plasma wavelength, (2) a longer pump depletion length or a larger transformer ratio, and (3) easier pulse shaping. With a longer plasma wavelength, beam loading onto the plasma wave can be made easier; phase slippage between the driven particles and the plasma wave is reduced; energy spread among the accelerated particles is made smaller; and the particle trapping threshold is lower. With a longer pump depletion length or a larger transformer ratio, the driven beam can be accelerated for a longer distance in a single acceleration stage; thus a larger energy gain per acceleration stage can be achieved. Easier pulse shaping particularly means that the sharp tail cutoff of the driving source can be made easier because of the Lorentz contraction effects. The most interesting characteristic of the counterstreaming electron beam accelerators is that they (both the CWFA and the CBWA) admit the possibility of trapping and accelerating moderately relativistic protons, a step impracticable with the plasma accelerators.

Basic Feature of the Plasma Waves

Consider a counterstreaming electron beam of rest frame density n_o and relativistic velocity $\underset{\sim}{v}_d = -\beta_d c\hat{e}_x$ as the plasma wave supporting medium. For the CWFA, let us assume that the driving source is an ideal "doorstep" linearly risisng density and sharp tail cutoff electron beam of length L_b, peak density n_b, and with a relativistic velocity $\underset{\sim}{v}_b = \beta_b c\hat{e}_x$ - as that has been considered in the plasma wake-field accelerator.[4,5] For the CBWA, let us assume that the driving source is a pulse of light consisting of two laser beams with respective frequencies ω_1, ω_2 and fields E_1, E_2 moving in the x-direction and beating at the frequency $\Delta\omega = \omega_1 - \omega_2 = \omega_p/f$. Here $\omega_p = (4\pi n_o e^2/m)^{1/2}$ is the proper (rest frame) plasma frequency of the counterstreaming electron beam; m is the rest mass of an electron; and f is denfined as the counterstreaming factor and is given by

$$f = \gamma_d\,(1 + \beta_b\beta_d) \sim 2\gamma_d, \qquad \text{(in the CWFA)}$$

or

$$f = \gamma_d\,(1 + \beta_d) \sim 2\gamma_d, \qquad \text{(in the CBWA)}$$

One way to examine the counterstreaming electron beam accelerator is to analyze this accelerator in the rest frame of the counterstreaming electron beam since, in this frame, the basic physical mechanism of the PWFA or the PBWA is recovered so one can directly apply those results obtained for the plasma wake-field accelerator,[4,5] or for the plasma beat-wave accelerator.[1,6,7] Lorentz transforming those results obtained in the counterstreaming electron frame to the laboratory frame, one then obtains the following:[8,9]

Phase velocity: The phase velocity of the plasma wave in the counterstreaming electron beam is given by

$$\underset{\sim}{v}_p = \underset{\sim}{v}_b, \qquad \text{(in the CWFA)}$$

or

$$\underset{\sim}{v}_p = (1 - \omega_p^2/\omega_1^2)^{1/2} c, \qquad \text{(in the CBWA)} \tag{1}$$

where $\omega_1 \geq \omega_2 \gg \omega_p$ has been implicitly assumed.

Amplitude: The electrostatic amplitude of the plasma wave in the counterstreaming electron beam is given by

$$E_+ = \varepsilon E_m, \tag{2}$$

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where $E_m \approx mc\omega_p/e$ is the plasma wavebreaking field in the counterstreaming electron beam; and ε is given by

$$\varepsilon = fn_b/n_o, \quad \text{(in the CWFA)}$$
or
$$\varepsilon = (\tfrac{16}{3}\, a_1 a_2)^{1/3}, \quad \text{(in the CBWA)} \tag{3}$$

where $\varepsilon < 1$, - that is, $fn_b < n_o$, or $a_1 a_2 < 3/16$, - has been implicitly assumed; and $a_{1,2} = eE_{1,2}/mc\omega_{1,2}$ is a measure of the laser pump intensity.

Wavelength: The plasma wavelength in the counter-streaming electron beam is given by (for both of the CWFA and the CBWA)

$$\lambda_p \approx f\lambda_p' \approx f(2\pi c/\omega_p), \tag{4}$$

where λ_p' is the plasma wavelength referred to the rest frame of the counterstreaming electron beam.

Since the acceleration gradient is basically limited by the plasma wave-breaking field E_m, for purposes of comparison it is reasonable to assume that ε is a chosen constant. Therefore, for a given rest frame density n_o of the counterstreaming electron beam, from Eqs. (1)-(4), one finds that the plasma wavelength is proportional to f whereas the amplitude and the phase velocity of the plasma wave are independent of f. Note that f = 1 ($\beta_d = 0$) stands for the plasma accelerators.

Pump Depletion

In general, any accelerator suffers pump depletion as the accelerating field loses energy to the accelerated particles. This is usually referred to as the beam loading energy loss. Using plasma wave to accelerate particles, however, there is another significant channel of energy loss, that is, the plasma waves extract energy from the driving source sc that eventually the energy contained in the driving source will be depleted. The pump depletion length is thus defined as the length the driving source travels before its energy gets depleted. this is the maximum distance the driven particles can be possibly accelerated in a single acceleration stage.

From energy conservation, the pump depletion length can be roughly given by[8,9]

$$L_p \approx R(\gamma_b - 1)mc^2/e\varepsilon E_m, \quad \text{(in the CWFA)}$$
or
$$L_p \approx 4f(c/\omega_p)(\omega_1{}^2/\omega_p{}^2)(a_1 a_2/4)^{-1/3}, \quad \text{(in the CBWA)} \tag{5}$$

where R is the transformer ratio in the CWFA and is given by[8,10]

$$R = E_+/E_- = 2\pi L_b/\lambda_p \approx 0.03f\Psi\gamma_b/\varepsilon, \quad \text{with } \Psi \leq 1. \tag{6}$$

Here E_+ and E_- are respectively the maximal accelerating electric field behind the driving electron beam and the maximal decelerating electric field within the driving electron beam. Neglecting phase slippage effects, the transformer ratio actually represents the ratio of the maximum energy gain ΔW of an accelerated particle with charge $|q|=e$ to the initial energy of a driving electron, $(\gamma_b - 1)mc^2$. Note that Ψ is the length control factor of the driving electron beam. The transformer ratio R is limited ($\Psi \leq 1$) because the driving electron beam length can not be to long; otherwise the two-stream instability will be strong enough to degrade the driving electron beam. The reason why the transformer ratio R is proportional to f is because the Lorentz effects resulting from the relativistic streaming of the counterstreaming electron beam can help suppress the harmful two-stream instability.[8] Similarly, the reason why the pump depletion length in the CBWA is proportional to f is because the Lorentz contraction effects can accumulate more laser energy which is spreaded in a longer distance in the laboratory frame into a useful shorter distance in the counterstreaming electron frame.[9]

Energy Gain

In the CWFA, since the pump depletion length given by Eq. (5) is not long enough to have appreciable phase slippage effects, the energy gain of a driven particle with charge q and mass m_p can be given approximately by[8]

$$\Delta W \approx qE_+L_p \approx 0.03f\Psi(q/e)\gamma_b{}^2mc^2/\varepsilon. \tag{7}$$

On the other hand, in the CBWA the pump depletion length given by Eq. (5) is long enough that the energy gain is basically limited by the phase slippage and is given approximately by[8]

$$\Delta W \approx 2\gamma_p\xi\beta_p(1 - \xi^{-2})^{1/2}m_pc^2. \tag{8a}$$

Here $\xi = 1 + \Gamma f(q/e)(m/m_p)\varepsilon\beta_p\gamma_p$, with $0 < \Gamma \leq 2$, depending on the loading phase and the allowable phase slippage (normally, $\Gamma = 1$ because the allowable phase slippage is usually one fourth of the plasma wavelength); $\beta_p = v_p/c$; and $\gamma_p = (1 - \beta_p^2)^{-1/2} \approx \omega_1/\omega_p$ is the relatvistic factor of the plasma wave. For $\xi >> 1$ (always true for electron acceleration), Eq. (8a) can be simplified as ($\Gamma = 1$, q/e=1)

$$\Delta W \approx 2f\gamma_p^2\varepsilon mc^2. \tag{8b}$$

Trapping Threshold

If a driven particle of charge q and mass m_p, with a velocity lower than the phase velocity of the plasma wave at the beginning of acceleration, can catch up the phase velocity of the plasma wave in a short period during which the relativistic factor of the plasma wave γ_p changes little, The trapping threshold is given approximately by[8]

$$W_{th} \approx \left\{ \gamma_p\xi \left[1 - \beta_p(1 - \xi^{-2})^{1/2} \right] - 1 \right\} m_pc^2. \tag{9}$$

This is generally true for electron acceleration in both the CWFA and the CBWA but is only true in the CBWA for proton acceleration provided that $\omega_1 \geq \omega_2 >> \omega_p$.[9] As for proton acceleration in the CWFA, if the protons are initially loaded onto the plasma wave in phase with the acceleration gradient near E_+ and the allowed phase slippage distance $\alpha\lambda_p$ for the plasma wave to outrun the protons is much smaller than λ_p, the trapping threshold can be calculated to be[8]

$$W_{th} \approx \left\{ \frac{h\left[1+\frac{2}{h}(1+\frac{M}{Rm}) \right]^{1/2} - h - 1}{2hM/Rm - 1}\gamma_b - 1 \right\} Mc^2, \tag{10}$$

where $h = 2\pi f\alpha\varepsilon\gamma_b(m/M)$ and M is the proton rest mass.

Proton Acceleration

Since the energy gain of the driven particles is basically proportional to the counterstreaming factor f as shown by Eqs. (7) and (8) and the trapping threshold can be appreciably reduced with a large f as can be numerically calculated from Eqs. (9) and (10),[8] it would be interesting to consider the possibility for proton acceleration. Indeed, based on recent advances in the electron beam technology, one might consider a counterstreaming electron beam of energy 50MeV ($\gamma_d \approx 100$, $f \approx 200$) with rest frame density $n_o = 10^{12}cm^{-3}$ ($E_m \approx 100MV/m$). Taking CWFA for example, one might also consider a shaped driving electron beam of energy 50MeV ($\gamma_b \approx 100$), with peak density $n_b = n_o\varepsilon/f \approx 2 \times 10^9 cm^{-3}$ if one chooses ε to be 0.4. The maximum proton energy gain per acceleration stage can then be calculated from Eq. (7) to be $\Delta W \approx 42\Psi$GeV. Furthermore, if the length control factor of the driving electron beam, Ψ, is chosen to be the maximum allowable value (Ψ = 1) and a reasonable value of α = 0.18 is used, from Eq. (10) the proton trapping threshold can be calculated to be $W_{th} \approx$ 6 GeV. The trapping efficiency would then be

$$\eta = \Delta W/(\Delta W + W_{th}) \approx 42\text{GeV}/(42\text{GeV} + 6\text{ GeV}) \approx 88\%.$$

However, this requires a long driving electron beam and a long counterstreaming electron beam (about 1000 meters)[8] that may not be achievable with current technical ability. Therefore, for a practical experiment to be performed now, one may have to choose a small length control factor of the driving electron beam, Ψ. For example, if $\Psi = 0.05$, one has $\Delta W \approx$ 2.1GeV, $W_{th} \approx$ 2.4, and $\eta \approx$ 47%, and that both the driving electron beam and the counterstreaming electron beam are within 100 meters.[8] One can even choose a lower value of Ψ than 0.05 to further reduce the required length of the driving electron beam and the counterstreaming electron beam. Although this will further reduce the energy gain ΔW per acceleration stage, the required initial energy W_{th} is also reduced so that the trapping efficieny η is essentially not much affected.[8] On the other hand, let us consider the case of using a 50MeV driving electron beam but replacing the counterstreaming electron beam with a stationary plasma (PWFA), that is f =1. Then even if $\Psi = 1$, the energy gain is only $\Delta W \approx$ 0.21GeV whereas the trapping threshold becomes $W_{th} \approx$ 17GeV; thus the trapping efficiency becomes extremely low ($\eta \approx$ 1.2%). These numbers clearly show why the counterstreaming electron beam accelerator might be considered as a collective proton accelerator whereas the plasma accelerator is indeed impracticable for accelerating protons.

Shortcomings

One shortcoming of using a relativistic counter-streaming electron beam as the charged particle acceleration medium is that the kinetic energy of the counterstreaming electron beam is not transferrable to the accelerated particles. However, as has been done for free-electron lasers, there may be ways of retrieving this energy, or it might be used for other stages of particle acceleration. This remains further investigation. Another shortcoming is the present inability of generating a high-energy, high-density, and long electron beam that can have a rest frame density equal to that can be achieved for a plasma at rest. Thus the achievable acceleration gradient in the counterstreaming electron beam accelerator is actually lower than that can be attained using a plasma accelerator. Nonetheless the attainable acceleration gradient ($E_m \approx$ 100MV/m) in a counterstreaming electron beam is still comparable to or larger than those can be obtained via conventional accelerator means.

Conclusions

In conclusion, the author has proposed a collective plasma wave accelertor using a counterstreaming electron beam as the wave supporting medium. The plasma wave in a counterstreaming electron beam can either be excited by a shaped driving electron beam, just as the plasma wake-field accelerator does, or be excited by beating two laser beams, just as the plasma beat-wave accelerator does. Despite the above-mentioned shortcomings, the fundamental advantages of this newly proposed accelerator over the plasma accelerators, - e.g. a longer plasma wavelength, a longer pump depletion length or a larger transformer ratio, and easier pulse shaping, particularly the ability for accelerating protons, - make this an interesting scheme worth pursuing further. **Finally,** The author would like to emphasize that the effective sharpness of the driving electron beam tail cutoff (counted in the counterstreaming electron beam frame) in the CWFA is enhanced by a facotr of f^2 because of the Lorentz constraction effects.[8] This will greatly improve the performance of the wake-field accelerator.

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