THE *ULTRALAC*: A COLLECTIVE WAVE ACCELERATOR FOR ULTRARELATIVISTIC PARTICLES

J. A. Nation and A. P. Anselmo
Laboratory of Plasma Studies and School of Electrical Engineering
Cornell University, Ithaca, NY 14853-5401

Summary

Collective wave accelerator schemes utilizing slow waves on an electron beam have an upper energy limit on the load particles set by the drive beam drift velocity. Typical intense electron beams operating near limiting current have drift velocities between 0.7c and 0.9c, setting an energy limit for protons of about 1 GeV. In order to collectively accelerate electrons or ions above this limit, the drive wave must have a phase velocity greater than electron beam drift velocity. The *Ultralac* concept utilizes the fast upper hybrid wave for particle acceleration. It has a phase velocity that is bounded only in the lower limit by the electron drift velocity. Calculations are presented showing the operating parameters of such a device including beam and wave parameters and achievable field gradients, load particle current limits and particle focusing.

Introduction

Up to now, attempts to build a collective particle accelerator using intense electron beam technology has emphasized the use of slow waves. The first concept suggesting the use of beam slow waves for acceleration of particles was the autoresonant accelerator (ARA) described by Sloan and Drummond[1] (1974). The ARA concept utilized the slow upper hybrid mode on an unneutralized relativistic electron beam for ion acceleration. The second collective wave concept forwarded was the slow space wave accelerator proposed by Sprangle, Manheimer and Drobot[2] (1976). Since both techniques use slow waves, a velocity barrier will exist on the load particles, resulting in a final energy limit equal to the mass ratio times the kinetic energy of the electrons. In actual practice, the ultimate velocity may be limited to a lower useful value by the available field gradient. Obviously slow waves on an electron beam are of no use for high energy applications.

The Ultralac Concept

Up to now, little or no work has been done on excitation of waves on electron beams in an effort to collectively accelerate particles to ultra-relativistic energies. In a magnetized bounded electron beam, many different linear waves exist in the fluid model. For the lowest radial and azimuthally symmetric modes, eight different branches of the dispersion relationship can be plotted on a Brillouin diagram[3]. The electromagnetic waves have four branches, the modified TM_{01} and TE_{01} waves, each with a positive and negative phase velocity branch. Four other eigenmodes of a electromechanical nature can exist in this system due to the collective aspects of the electron beam. These electrokinetic modes are the fast and slow space charge waves and the fast and slow upper hybrid waves. Of all these waves, only the fast upper hybrid mode satisfies all of the accelerator criteria for ultra-relativistic particles. The upper hybrid modes have also been referred to as Trivelpiece-Gould[4] modes, vortex waves[5], and cyclotron modes[3]. For waves whose upper hybrid frequency is less than the empty waveguide cutoff for the TM_{01} electromagnetic wave, a quasi-static approximation can be used where the electric field is derivable from the gradient of a scalar potential.

Ideally, if the fast upper hybrid mode can be excited precisely at the point which has phase velocity equal to c, this mode would then become extremely useful for the collective acceleration of ultra-relativistic particles. This is the *Ultralac* concept. Since the fast upper hybrid mode is a positive energy mode in a non-neutral particle beam, energy must be added in order to excite it. We propose using a three wave method employing a wiggler to couple energy from the negative energy modes to the fast wave. The success of using wigglers to excite the positive energy electromagnetic modes in beam-guide systems is well understood both theoretically and experimentally[7]. In particular, the free electron laser has been successfully developed, using the parametric excitation of fast waves. In our case we want to excite a fast electrokinetic and not an electromagnetic mode. This condition can be ensured if the wave is excited at a frequency below the lowest guide electromagnetic cutoff frequency.

Using a linear, electrostatic fluid theory for a confined flow beam in a finite magnetic field, we can derive a relativistic dispersion relationship. If a relativistic rigid rotor equilibrium is used, we recover a dispersion relationship similar to that of Potzl[5] (1960). The use of this dispersion relation allows us to calculate the trapping electric field, the field structure and the power flow associated with the wave.

Basic Theory

For cases when the hybrid wave frequency is lower than the TM_{01} cutoff frequency, the theoretical analysis can be greatly simplified by employing a quasi-static approximation. This requires $\nabla \times \mathbf{E} \approx 0$. The analysis yields the electrokinetic modes that exist in the beam. For cases where the upper hybrid frequency is equal to or greater than the TM_{01} cutoff frequency, a fully electromagnetic model is necessary. This analysis, both linear and non-linear, is presently being undertaken by Seyler[6]. The quasi-static analysis employed here uses the continuity equation, the momentum conservation equation, and Poissons' equation to yield a characteristic dispersion relation.

The linear dispersion relation is derived from matching the jump conditions for beam-vacuum boundary and setting the wave potential at the wall equal to zero. For all values of azimuthal mode number, n, this characteristic equation is

$$\frac{I_n'(ka)K_n(kb) - I_n(kb)K_n'(ka)}{I_n(ka)K_n(kb) - I_n(kb)K_n(ka)} = A(\omega,k)\frac{J_n'(k_\perp a)}{J_n(k_\perp a)} + B(\omega,k) \tag{1}$$

where

$$A(\omega,k) = \left(\frac{(\omega_v^2 + \gamma^2\omega_p^2 - \omega_b^2)(\omega_p^2 - \omega_b^2)}{(\omega_v^2 - \omega_b^2)\omega_b^2}\right)^{1/2}$$

and

$$B(\omega,k) = \frac{n\gamma^2\omega_p^2\omega_v}{ka(\omega_v^2 - \omega_b^2)\omega_b}$$

In addition to these definitions, the perpendicular wavenumber, $k_\perp$, is defined as

$$k_\perp^2 = \frac{k_z^2(\omega_p^2 - \omega_b^2)(\omega_v^2 - \omega_b^2)}{(\gamma^2\omega_p^2 + \omega_v^2 - \omega_b^2)\omega_b^2} \tag{2}$$

To simplify the equations, the Doppler shifted eigenfrequency of the modes is defined as $\omega_b = \omega - k\mathbf{v}_d - n\dot{\Theta}_0$. Using the

CH2387-9/87/0000-0145 $1.00 © IEEE

PAC 1987

rigid rotor equilibria as an approximate model, the bulk rotation frequency in the lab frame for a completely unneutralized, relativistic beam is given by

$$\dot{\Theta}_0 = \frac{\Omega_c}{2}\left(1 \pm (1 - \frac{2\omega_p^2}{\Omega_c^2})^{1/2}\right) \tag{3}$$

The other characteristic frequencies of the system are the vortex frequency,

$$\omega_v^2 = \Omega_c^2 - 2\omega_p^2 = (\Omega_c - 2\dot{\Theta}_0)^2$$

where the cyclotron frequency and the plasma frequency for the system are,

$$\Omega_c = \frac{eB_0}{\gamma m} \quad \text{and} \quad \omega_p^2 = \frac{e^2 n_0}{\gamma^3 m \epsilon_0}$$

Using the characteristic equation, known values of injection voltage, beam current, guide field and geometry, we can calculate the dispersion relationship numerically (Figure 1). Since the dispersion relationship produces essentially a straight line at the Doppler shifted cyclotron frequency, we may use the approximate relation for electric field scaling.

$$\omega \simeq k v_d \pm \Omega_c \tag{4}$$

This is valid for regimes where the vortex frequency is greater than the plasma frequency or $\Omega_c^2/\omega_p^2 \geq 3$, and also ensures beam stability exists. Using the simple dispersion formula, and requiring the interaction to occur such that the fast hybrid wave has a phase velocity equal to c, we arrive at a simple formula for the operating frequency

$$\omega_o = \frac{eB_0}{m}\left(\frac{1+\beta}{1-\beta}\right)^{1/2} \tag{5}$$

Since the relativistic cyclotron frequency, Ω_c, depends directly on magnetic field and drift energy, the operating frequency is strongly dependent on guide field strength and beam drift velocity. The beam drift velocity is determined by the prescribed beam equilibria.

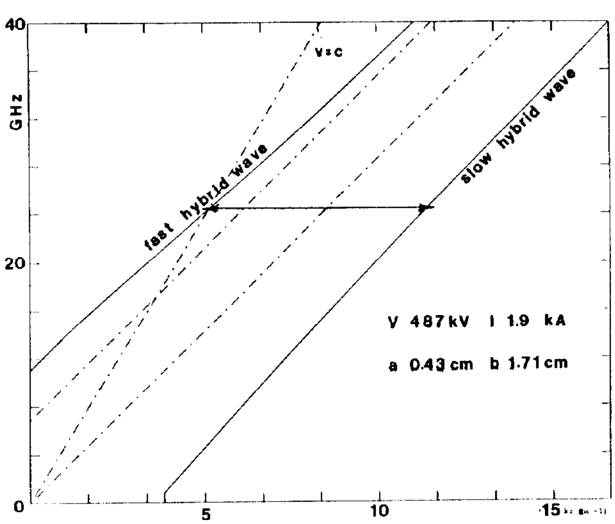

Fig. 1 Dispersion relation calculated for a 487 keV beam at 1.9 kA in a 3.4 cm diameter guide showing the hybrid modes.

Accelerating Gradients

To determine the effectiveness of the hybrid wave accelerator, a estimate of the achievable electric field gradient using the simple dispersion relationship can be made. At the operating frequency, the Manley-Rowe relations for the conservation of wave energy and conservation of wave momentum are

$$\omega_o = \omega_{\text{fast}} = \omega_{\text{slow}} \quad \text{and} \quad k_{\|\text{slow}} = k_{\|\text{fast}} + \frac{2\Omega_c}{v_d}$$

The slow mode should self trap at lower axial electric field since it has smaller relativistic factor in the beam frame than the fast mode. The condition for self-trapping is the given by

$$E_{\text{trap}} = \frac{k_z mc^2}{e\gamma_\phi}(\gamma_r - 1) \quad \text{where} \quad \gamma_r = \gamma_\phi \gamma (1 - \beta\beta_\phi)$$

Using the trapping relation and the operating frequency condition, we can determine the self-trapping field for the slow hybrid mode. In the collective (Raman) regime the self-trapping field of this mode sets a limit on the achievable accelerating gradient. The resulting trapping formula is derived using these basic arguments and is given by

$$E_{\text{trap}} = \alpha c B_0 \left(\frac{(2+\beta) - 2(1+\beta)^{1/2}}{\beta}\right) \tag{6}$$

The coefficient α is of order 5. For slow space charge waves, it has been experimentally measured at ~ 5.

We can now determine the most effective use of beam energy and guide geometry by finding the greatest electric field gradient possible for a given injection energy. This is determined by maximizing accelerator action per unit length,

$$J_{\text{accel}}(\beta) = \frac{eE_{\text{trap}}}{\omega_o} = \alpha mc(1-\beta)^{1/2}\left(\frac{(2+\beta) - 2(1+\beta)^{1/2}}{\beta(1+\beta)^{1/2}}\right) \tag{7}$$

The accelerator action per unit length is maximized for $\beta \approx 0.555$. If we insert this into the self-trapping condition we have the linear limit given by

$$E_{\text{trap}} = (0.110)\alpha c B_0$$

The numerical values for a possible 30 GHz device can now be calculated using the simple relations (Table 1).

Wave Power Flow

If we use the Poynting theorem to calculate the wave power flux we can determine the relative magnitude of axial electric field for the parametrically grown waves. Substituting for the various terms in the power equation we obtain the energy conservation equation for the electrostatic modes

A Thirty Gigahertz Accelerator			
Drift Velocity (c)	0.5	0.6	0.7
Diode Voltage (keV)	276	487	892
Magnetic Field (kG)	6.19	5.36	4.50
Trapping Field (MV/m)	93.8	94.1	89.1
Limiting Current (kA)	0.88	1.91	4.26
Cyclotron Frequency (GHz)	17.3	15.0	12.6
Plasma Frequency (GHz)	9.18	9.64	10.6
Slow Wavelength (cm)	0.33	0.43	0.54
Wiggler Period (cm)	0.50	0.75	1.16
Maximum Field (MV/m)	123	225	432
Beam Power (GW)	0.241	0.931	3.80

Table 1 Calculation of parameters for a 30 GHz accelerator using the simple model.

$$\frac{\partial}{\partial t}\left(\frac{1}{2}nm|\mathbf{v}|^2+\frac{1}{2}\epsilon_0|\mathbf{E}|^2\right)+\nabla\cdot\left(\frac{1}{2}mn\mathbf{v}|\mathbf{v}|^2\right)=0 \qquad (8)$$

Inserting for the values of the density, velocity, and electric field from the linear theory, and integrating over the volume of the beam, we derive the net power flow for the waves.

$$P_{\text{total}} = C(k_\perp a)k_z v_d\left(\frac{k_z\omega_p^2}{\omega_b^2}+\frac{(k_\perp^2+k_z^2)v_d}{\omega_b}\right) \qquad (9)$$

$$\text{where} \quad C(k_\perp a) = \pi a^2\epsilon_0\varphi_0^2(J_1^2(k_\perp a)+J_0^2(k_\perp a))$$

From the linear dispersion relationship, we calculate the perpendicular wavenumber and a normalized effective velocity β_{eff} (Figure 2)

$$\beta_{\text{eff}} = \frac{P_{\text{total}}}{\pi a^2\epsilon_0 c(k_z\varphi_0)^2} \qquad (10)$$

For a coupled set of waves excited at the same frequency in the lab frame, the root of the ratio of effective velocities measures the ratio of the axial electric field. Examination of Figure 2 shows that the electric field in each mode is roughly equal when the fast wave is excited near the light line. This ensures that the amplitude limited slow hybrid wave does not severly restrict the gradient associated with fast wave. Although this may be a manifestation of the quasi-static analysis, if the waves are excited below cutoff this approach is probably valid.

Conclusion

We have proposed an approach to use a weakly relativistic electron beam as a means of collectively accelerating particles to ultra-relativistic energies. Using this technique, it seems possible to achieve fields in excess of 100 MV/m in a simple linear model. The fast hybrid wave seems to be the eigenmode capable of accelerating particles to velocities arbitrarily close to speed of light. Given a 700 kV, 3 kA particle beam as the driver, a 5 meter module could be used to accelerate 1.0 amp of load particles through 1 GeV if the device could transfer 50 percent of its energy to load particles. Since the accelerator is a wave driven device, the stage to stage phasing of the waves should not present a difficulty. In particular, the intrinsic electrostatic focusing of a collective accelerator will permit high currents of accelerated protons. Furthermore, the use of a smooth wall device with a confining electron beam eliminates the problems of wake fields associated with coupled cavity linacs. If the accelerating wave potentials can approach the beam-wall potential drop of the drive electron beam, axial gradients of 200 MV/m may be achieved. As a device for accelerating high energy electrons, the inherent stability of the high current driving electron beam in each stage ensures net stability. Alternating the guide magnetic field stage to stage may also provide an overall focusing.

With the recent developments in induction linacs and high current cathodes, the technology required to produce the high quality particle beam necessary for such a device exists. An r.f. accelerator transforms power from a low energy electron beam in a klystron and places it in a high energy beam. These two beams are separated both in time and space. The *Ultralac* concept uses a low energy drive beam and a high energy load beam in the same region of space at the same time. Development of these types of devices provides an interesting option for future accelerator design.

This work was supported by the U.S. D.O.E

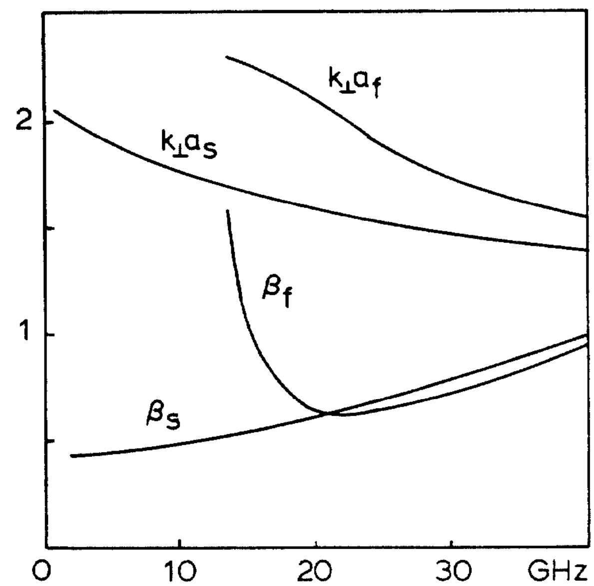

Fig. 2 Calculation of the normalized effective velocity and the perpendicular wavenumber for the example in Fig. 1

References

[1] M. L. Sloan and W. E. Drummond, Phys. Rev. Lett. **31**, p. 1234, (1974).

[2] P. Sprangle, A. Drobot and W. Manheimer, Phys. Rev. Lett. **36**, p. 1180, (1976).

[3] V. Bevc and T. E. Everhart, Journal of Electron Control **13**, p. 185, (1962).

[4] A. W. Trivelpiece and R. W. Gould, J. Appl. Phys. **30**, p. 1784, (1959).

[5] H. Potzl, Proc. of 4th International Congress on Microwaves Tubes, p. 649.

[6] C. E. Seyler, private communication.

[7] P. Sprangle, R. A. Smith, and V. L. Granatstein, Infared and Submillimeter Waves, K.J. Button ed., Vol **I**, p. 279, Academic Press, New York, (1979).