Mixing, Staging and Phasing for a Proton-Driven Wake Field Accelerator

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Summary

In this paper we expand on few important details of the Wakeatron concept. This is a device where electrons can be accelerated by the wake field of short intense proton bunches travelling along the axis of an rf structure. Specifically, we have examined the consequences of the longitudinal dynamics of both the electron and the proton bunches. Included were "mixing" in the proton bunches (crucial to the overall concept) and phase shifts (electron bunches relative to proton bunches) in the acceleration process. Because of the deterioration of the proton bunches, due to the "mixing" process, it is required that the Wakeatron is indeed staged in a number of consecutive sections.

The Wakeatron Concept

The Wakeatron has already been discussed in other papers /1,2,3/ to which we refer the reader. It consists of a sequence of cylindrical rf cavities or cells. Each cell has the shape of a pill-box with a length of 2 mm, an outer radius of 4 mm, and an inner diameter of 1 mm. The parameters of the geometry have been adjusted so that the lowest resonating frequency mode corresponds to a wavelength of 1 cm, or 30 GHz.

The Wakeatron is meant to accelerate electrons to very large energies, possibly in the TeV range, over a relative short distance. Two of these accelerators could be configured to provide an electron-positron linear collider with luminosity of 10^{33} cm^{-2} sec^{-1}.

The Wakeatron concept proposes to power the 30 GHz linac with relatively short intense proton bunches with an initial energy of 110 GeV. The proton bunch, which we shall call the "driver", is assumed to have a gaussian distribution with an rms bunch length of 3 mm. The driver, travelling along the axis of the rf structure, leaves a wake field. A test particle, moving in the same direction, would therefore be accelerated by an amount which depends on the distance from the driver. If we assume 3×10^{11} protons in a bunch, a maximum accelerating gradient of 80 MeV/m has been estimated, which is of reasonable interest. Moreover, the so called transformer ratio for the geometry described is 10, that is, with a 110 GeV proton bunch one should, in principle, be able to accelerate electrons to 1 TeV in one single stage until the protons have exhausted almost their energy, for instance down to 10 GeV, at which point they are disposed of because their velocity is no longer relativistic.

This large transformer ratio can be obtained by using a driver bunch of a length comparable to the wavelength to be excited in the rf structure and by employing an averaging process on the amount of energy loss per particle in the driver. If the particles in the driver are heavy (protons) they move as they lose (or gain) energy with respect to each other and they "mix". We have performed a computer simulation of this process and found that indeed "mixing" occurs and provides the expected transformer ratio. Unfortunately, the shape of the proton bunches will deteriorate by lenghtening to a point where no useful accelerating gradient is provided to a trailing particle. This will require disposing of the proton bunches before they have lost most of their energy, and to build the linear accelerator in stages energized by different fresh proton bunches.

Electrons move practically at the speed of light, whereas protons have somewhat less velocity. This will cause the electrons to advance their motion with respect to protons and to slip out of the optimum phase of the accelerating field. To control the slippage, it is required to stop the acceleration at some point, to apply some mechanism to the two beams so that they are brought again at the right rf phase distance. We call "staging" the process of dividing the length of the accelerator in sections to minimize the slippage problem; and "phasing" the mechanism applied to both beams in between sections to compensate for the rf slippage. We have investigated the "staging" and came up with a criterion for the division of the accelerator length. This should be compared with the requirement due to "mixing". We have also investigated methods for "phasing" which would be useful for a one-stage design. One is forced to stage the accelerator because of the "mixing" problem. By using a fresh proton bunch for each stage, the requirement on "phasing" is softened.

Mixing

For a proton driven wake field accelerator, the energy gain or loss of a particle in the driver beam depends on the particles position within the bunch. Particles will acquire different momenta and, because of their heavy mass, will exchange position with respect to each other, a process called "mixing".

The equations of motion of the i-th particle in the proton bunch with N particles are

$$\frac{dz_i}{dt} = c/\sqrt{1 + (E_o/p_i c)^2} \qquad (1)$$

$$\frac{dp_i}{dt} = -\sum_{j \neq i} w_o \cos k(z_j - z_i) - \frac{w_o}{2} \qquad (2)$$

Here z_i, p_i are the position and momentum of the particle with rest energy E_o. Our model assumes that only one mode, the fundamental, is excited because of the length of the driver bunch. w_o and k, the wake field amplitude and number, are determined by the structure of the Wakeatron, as it has been explained above.

Equations (1) and (2) describe a system consisting of charged particles moving due to applied fields of other particles and forces of their own. The physics comes from two parts; the wake field produced by the particles will cause a change in the particle momentum (Eq. (2)); this in turn will affect the speed of motion (Eq. (1)).

We have used several methods to integrate numerically Eqs. (1) and (2). For instance, we have written codes where we solve the corresponding Vlasov equation, or many-particle simulations. We found the following method more convenient and time saving.

We performed a numerical simulation with a smaller number Np of (macro-) particles, each representing a sample of many real particles. We used the PIC (Particle In Cell) method /4/. The method divides the phase space (z,p) into Ng cells each of length Δ, with Ng << Np. The motion of a macro-particle of charge q is approximated by the nearest grid nods of coordinates X_j and X_{j+1} with the weight

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factors:

$$\rho(X_j) = \frac{q}{\Delta} \sum_{i=1}^{N} (X_{j+1} - z_i) \qquad (3a)$$

$$\rho(X_{j+1}) = \frac{q}{\Delta} \sum_{i=1}^{N} (z_i - X_j) \qquad (3b)$$

Then we transform Eq. (2) to

$$\frac{dp_i}{dt} = - \sum_j \rho(X_j) w_o \cos k(X_j - z_i) - \frac{w_o}{2} \qquad (4)$$

By using the PIC method, the CPU time is considerably less than that required by those methods which directly integrate (1) and (2).

We have calculated (1) and (4) for up to 3000 macro-particles, and adjusted parameters to make them correspond to 3×10^{11} protons. The results are shown in Fig. 1. Here we use an initial Gaussian distribution with momentum spread of ± 0.1 GeV and rms bunch length 3 mm. Other parameters are Ng = 256, N_{pW_o} = 0.4 GeV/m, which would correspond to a maximum accelerating gradient of 80 MeV/m for a transformer ratio of 10, and an initial proton momentum of 110 GeV.

We see that the average momentum of the proton beam decreases during the first 1000 m. After that, the proton bunch does not lose more energy. Because of a large increase in length, almost all the energy lost by the front of the bunch is recovered by the trailing particles. The energy gain of trailing electrons increases rapidly during the first 1000 m, and then diminishes (Fig. 2). The net energy gain for electrons is 120 GeV, compared to the average energy loss of 20 GeV for the proton driver bunch. We have an overall transformer ratio R = 6 and acceleration gradient G = 120 MeV/m. This is somewhat larger than originally estimated under the assumption of constant bunch length. The difference we believe is caused by an initial reduction of the bunch length due to early process of mixing. After about one half of the mixing period the proton bunch lengthens. After about 1000 m, the proton driver bunch is not useful. This proton bunch would be removed from the Wakeatron and a new proton driver bunch injected to accelerate the same electron bunch. The used proton beam could be taken to the proton accelerator for "recycling" and used again as a new proton driver. In this way, we can obtain 1 TeV electron beams with about 10 proton driver bunches as shown in Fig. 3.

Staging

One serious problem for a proton driven wake field accelerator has to do with the difference in velocity of the proton and the trailing electron bunches. The assumed 1 cm wavelength of the wake field could allow, for instance, perhaps ± 1 mm slippage and maintain a high acceleration gradient. At the beginning the electrons could lag behind about 1 mm from the peak of the field. After electrons have advanced 2 mm with respect to the protons, one stops accelerating and re-adjust the distance between the two beams. We call this procedure "staging" and "phasing".

The length L of each stage is strongly energy dependent. Figure 4 shows the relation between proton energy and stage length based on 2 mm slippage requirement. For 110 GeV proton bunches, L = 50 m. But at 90 GeV the stage is only 30 m long. We will require that a new proton bunch is used only after the previous one has lost 20 GeV, from 110 down to 90 GeV, or equivalently.

Phasing

As we have discussed in the "mixing" section, one needs a fresh proton driver bunch for every ~ 1000 m. But the stage length varies between 50 and 30 meter, so that as many as 25 stages are needed. The "phasing" provides the correction for the distance between electrons and protons, at the end of each stage, so the electrons can be brought back on top of the wake field for maximum acceleration. We have considered two methods:

1. Use of a wiggler magnet to delay the motion of one beam with respect to the other. Since the proton and electron beams have different momenta, the distance travelled in the magnet would also be different. By adjusting the strength of the magnetic field and the length of magnets, one can cause enough delay to "phase" the electrons with respect to the peak of the wake field. Because of the considerations discussed in the "staging" section, a 2 mm path difference is required at each "phase" step.

   Synchrotron radiation has to be taken into account because of the high energy of the electrons. We find that a considerable space is needed to contain these magnets. For example, it would require 22.8 m for a 20 kG wiggler to phase 100 GeV protons and 50 GeV electrons, but the staging length is only 40 m, and the energy loss due to synchrotron radiation is 2.5 GeV which almost cancels the energy gain in that stage.

2. Another possible solution is to adjust the wavelength of the wake field. This depends on the geometry structure of the cavity. This can be done by simply varying the physical dimension of the structure in the different stages. For example, at the end of i-th stage, the distance d between the electrons and the protons is given by $n_i\lambda_i - s/2$, here n_i is an integer, λ is the wavelength and s is the maximum allowed slippage in one stage. Thus by changing the wake field wavelength of the next stage such that $d = n_{i+1}\lambda_{i+1} + s/2$, the electron will be again in phase at the beginning of (i+1)th stage.

One can also consider the possibility of continuously adjusting the wavelength of the field so that the distance between the proton and electron bunches always be a multiple integer of the wake field wavelength. Since in this case there is no slippage, staging is necessary in order to recover the value of $\lambda(z)$ to its initial value $\lambda(0)$. Comparing this method with the mixing, one could use one stage for every new proton bunch. This method may be very impractical, nevertheless, because it requires precise mechanical tolerances in the construction of the cavities.

Acknowledgements

We are very grateful to J. Rosenzweig for his help in solving some of the "phasing" problems.

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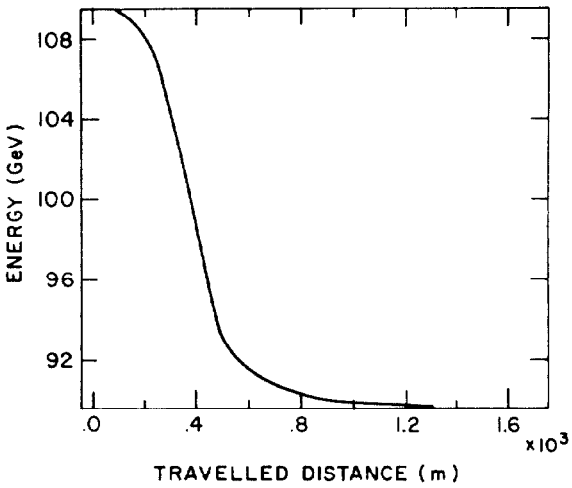

Fig. 1. Average Momentum of a proton driver bunch in the Wakeatron.

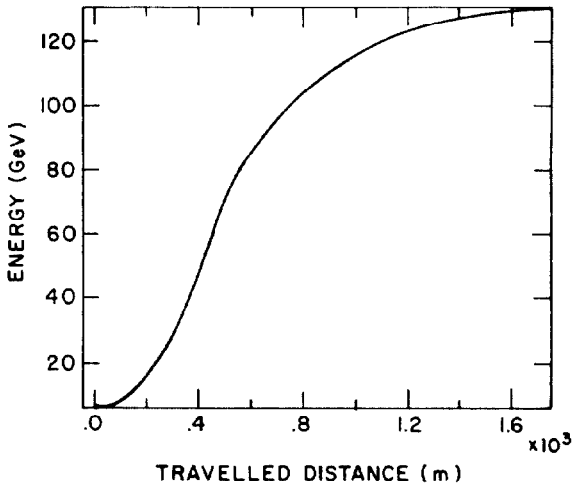

Fig. 2. Energy gain of electrons in the Wakeatron.

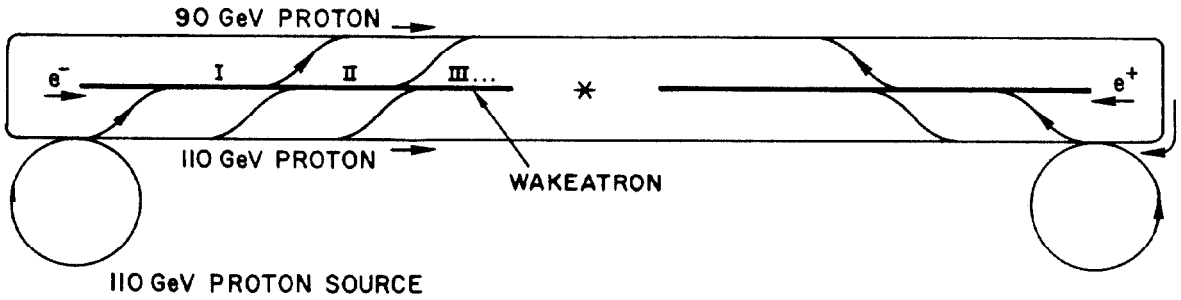

Fig. 3. Schematic design of a multi-stage Wakeatron.

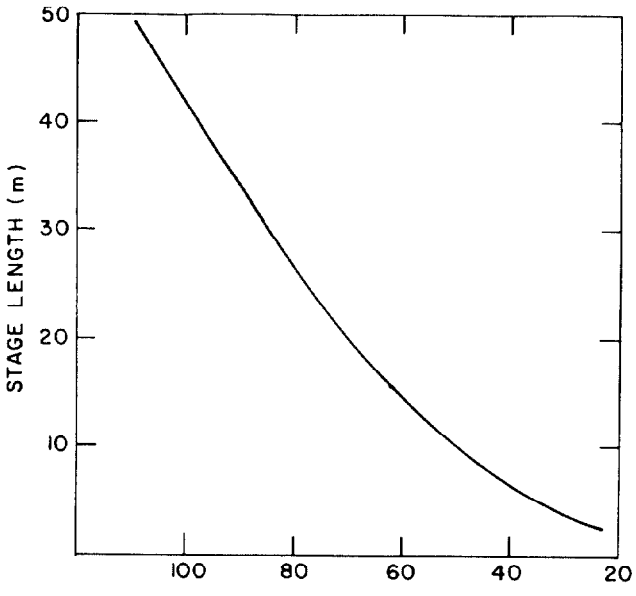

Fig. 4. Proton energy and stage length based on 2 mm slippage requirement.