MECHANISMS FOR PARTICLE ACCELERATION: POSSIBILITIES AND CONSTRAINTS

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Summary

An attempt is made to classify schemes that are used, or have been proposed, for accelerating particles. The basis for classification is the fundamental nature of the field-particle interaction; the photon viewpoint is introduced into the classical description normally used by accelerator designers.

1. Introduction

Very many ways of accelerating particles have been suggested over the past 50 years. Some of these have been tried, but only a few have led to useful devices. In the last few years efforts have been made to find new concepts for achieving high energies or high current economically, and a number of ideas have been proposed. There have been other periods of intense interest in novel ideas; the last wave some 15-20 years ago was concerned with various collective and coherent schemes following an earlier burst of interest in the mid-'fifties, stimulated particularly by Veksler, Budker and their colleagues in the Soviet Union. Although much interesting physics has been learned, none of these collective ideas has yet led to any devices in general use.

Recent lists of 'new approaches', making use of wake-fields, beat-waves, etc can, and have been, compiled. It seems desirable to try to structure such lists somewhat more than has been done so far, and furthermore, to enquire what basic classes of accelerator can be envisaged and to discern their limitations. It is difficult to generate a clear-cut and universal classification. The essential features of any scheme are to some extent a matter of judgement, and can appear different if one takes a physical or a technical point of view. Nevertheless, it is helpful to explore how the various schemes are related, and look at them from different points of view. In this way it is possible to grasp more easily the implications of new proposals, and to judge to what extent they can be considered as radically new, and to what extent they represent variations of existing ideas.

Accelerators form part of a more general class of system. Together with microwave tubes and lasers they are devices in which ensembles of charged particles, constrained in trajectories or bound in atoms or solids, experience stimulated absorption or emission of photons in coherent electromagnetic fields. The cyclotron maser is the perfect bridge, its manifestation as the gyrotron describable equally aptly in the languages of accelerators or conventional lasers. Here again distinctions are fuzzy, and it can be argued that it is more correct, for example, to call a tapered inverse free electron laser a particle decelerator. This broader problem, and the discussion of what is meant by laser action will not, however, be considered here, and the discussion will be confined to particle accelerators. The methodology will be synthetic.

2. A plane wave and a single particle, Cherenkov interaction

In any region of free space the electromagnetic fields can be expressed in terms of expansions into suitable sets orthogonal functions. Here we consider a manifold of plane waves, noting that in the presence of matter, evanescent as well as propagating waves need to be considered. The basic requirements for the most widely used accelerators, namely linacs and synchrotons, can be deduced from a discussion of the interaction of a particle with a single evanescent plane wave. It is well known that a particle cannot gain energy indefinitely from a single propagating plane wave. The wave always travels faster than the particle; for relativistic velocities, where the particles are moving almost at the light velocity, the electric field is perpendicular to the velocity, and furthermore is cancelled to order $1 - \beta$ by the $v \times B$ force. Non-relativistic particles proceed with uniform velocity, modulated by a 'figure of eight' motion as sketched in Fig 1.

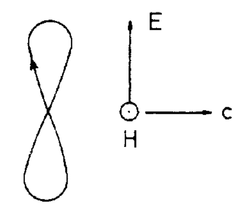

Fig. 1.

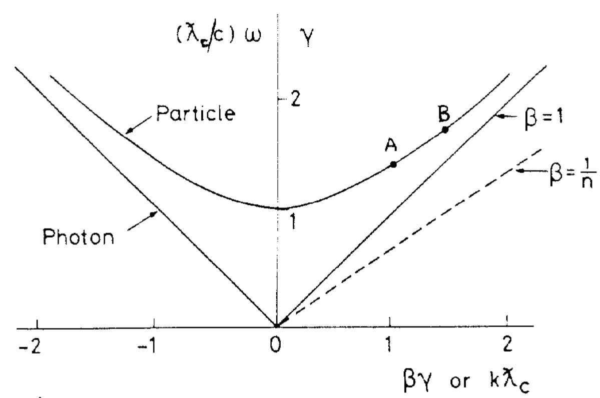

Fig. 2.

We now consider this fact in terms of the conditions required for photon absorption. Fig 2 shows a plot of energy versus momentum both for a particle with mass and a photon. The curves are also dispersion curves for the corresponding de Broglie waves, with the correspondence $E=\hbar\omega$, $p=\hbar k$. The particle velocity v is given either by $c d\gamma/d\beta\gamma$ or by the group velocity of the waves $d\omega/dk$. Clearly a single photon cannot be emitted or absorbed, because to get from A to B in one jump requires too great a ratio of momentum to energy. The introduction of some bulk matter, however, enables the extra momentum to be supplied or absorbed. The use of a dielectric shortens the wavelength and this increases the momentum of the photon in the medium. For a dielectric constant n, such a photon is represented by a line with slope $\beta=1/n$, shown dotted. At energies sufficiently high that the slope of the hyperbola is greater than this, a photon can be emitted or absorbed by the charge. The critical velocity is $\beta=1/n$. When the normalized particle velocity β is larger than this conditions can be matched if the photon moves at an angle θ, with forward component reduced by the factor $\cos\theta$ where

$$\cos\theta = 1/\beta n \qquad (1)$$

This is the well known Cherenkov condition.

In linacs and synchrotrons the particles do not pass through a dielectric block, but interact with an

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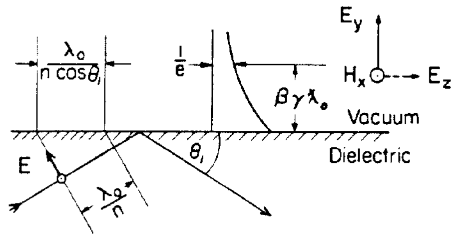

Fig. 3.

external evanescent field; this is illustrated in Fig. 3. The particle can be considered then as absorbing 'virtual spacelike photons' with negative (mass)^2. Fig. 3 shows the wave on both sides of the dielectric boundary. In free space, $\cos\theta = 1/\beta_w$, and $\sin\theta = i/\beta_w\gamma_w$, where the subscript w denotes 'wave'. Such a wave is synchronous with a particle of energy $\gamma_w m_o c^2$. The decay length $\beta\gamma\lambda\!\!\!^-$ (omitting henceforth subscript w) increases with γ, which becomes infinite at the 'critical angle' where the wave becomes a propagating plane wave travelling parallel to the surface. Unfortunately $E_z/E_y = 1/\gamma$, so that by the time the particle has become relativistic the desired accelerating field E_z has vanished! Before tackling this difficulty we note that since the outward impedance $Z_o\sin\theta$ in an evanescent wave is purely capacitive, it is possible to support the wave on an

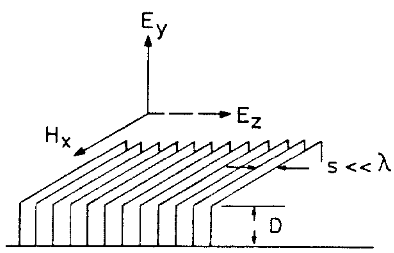

Fig. 4.

inductive surface. The simplest such surface is the 'Cutler surface' shown in Fig 4; this consists of shorted strip transmission lines of impedance $iZ_o\tan\omega D/c$. Setting this equal to $-Z_o\sin\theta$ yields the matching condition $\cos\theta = \sec(\omega Dc)$, for which there are solutions for discrete values of ω. Such a surface is more convenient than a dielectric for an accelerator, but there is the problem that at relativistic velocities, $E_z/E_y \to 0$. This can be overcome by opposing two surfaces such that E_y and H_x cancel on the mid-plane, but the E_z fields add. (This fact was noted by Lohmann[1] in his unpublished paper on laser acceleration written just 25 years ago.) Another solution is to retain a single-sided surface, but to arrange that there are two waves each with $\beta_w<1$ and finite $|\sin\theta|$ moving at angles $\pi/2 \pm \alpha$ to the grating lines. For a phase velocity equal to c,

$$\cos\theta \cos\alpha = 1 \tag{2}$$

as noted by Palmer[2]. This scheme is employed by Pickup[3] in his study of a grating accelerator. In both cases the particle travels within $\lambda\!\!\!^-$ of the surface.

Instead of two opposed surfaces, it is convenient to use a cylindrical manifold of plane waves with wave-normals at $\cos\theta$ to the z-axis to give the conventional disc loaded guide as shown in Fig 5a. This can also be turned inside out to form a set of discs on a rod, as shown in Fig 5b [4]; this figure can also be viewed as an array of dipoles, in a directive 'Yagi' antenna used for TV reception. Also shown in the figure is a solid dielectric rod, which can be used in tapered form as an antenna. These simple examples

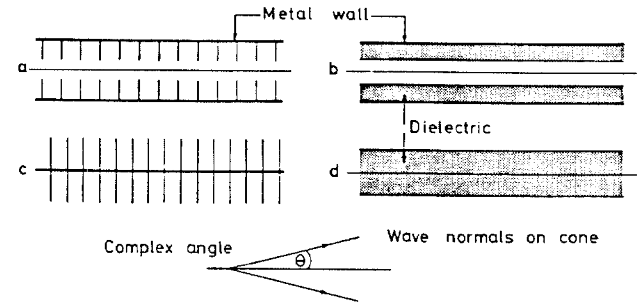

Fig. 5.

can be augmented by a wide range of periodic systems, for instance Alvarez structures, or synchrotron cavities. Periodic cavity arrays have a range of discrete values of k for given ω (space harmonics).

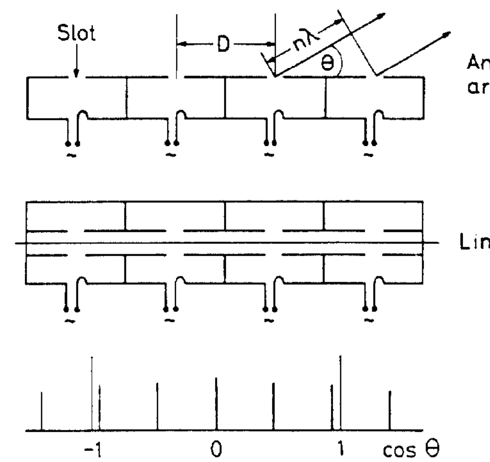

Fig. 6.

Fig 6 shows a further connection between antennas, diffraction gratings and accelerators. An infinite array of slots (equivalent, by Babinet's principle, to dipoles with the opposite polarization) fed as shown radiates a line spectrum in directions given by

$$\cos\theta = n\lambda/D \tag{3}$$

For $n\lambda<D$ these are radiated beams (or 'orders' in a diffraction grating), for $n\lambda>D$ they are evanescent space harmonics travelling along the surface. Excitation of the system by resonance with one space-harmonic in general excites the others. Evanescent and propagating modes are coupled, for example, in the Smith-Purcell effect[5]. Large n corresponds to low velocity and small $\beta\gamma$, so the exponentials drop away very fast, a fact normally explained in terms of a poor 'gap coupling factor' away from the surface. Rolling the system up produces a conventional cavity linac.

For the present we regard synchrotrons merely as similar in principle to a linac, with a slight curvature, and finite circumference which quantizes the azimuthal mode number. For the limiting case of a cyclotron, operating on a low harmonic, however, this is not a satisfactory approximation, and a new starting point has to be made to link up with the viewpoint here presented.

The class of interaction discussed so far can be described by the term 'Cherenkov'; it applies to conventional linacs and (almost) to large synchrotrons. These both have symmetry which is axial about the beam, or almost so. There is an interesting class of geometries without such symmetry. Examples are shown in Fig 7. The particles move in straight lines, but

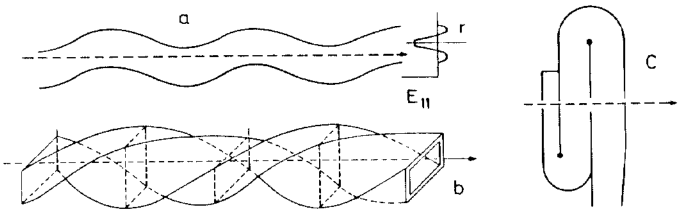

Fig. 7.

the structures, which carry waves with phase velocity greater then c, are curved in such a way that as the particle overtakes the wave, it moves into another position with respect to the wave where the phase is different. In the first two examples[6,7] a transverse phase jump associated with a higher order transverse mode is used, in the third the beam passes through holes in the waveguide wall[8].

3. Two plane waves and a single particle. Stimulated Compton interaction

The Cherenkov interaction studied in the previous section requires one plane wave (or a manifold of waves at the same frequency) and a nearby medium. An alternative is to consider interaction with two waves of different frequency; for simplicity we consider these in opposite directions, one 'forward', in the direction of the particle, and one 'backward'. Referring again to the dispersion diagram, the jump from A to B can be made in two steps, as illustrated in Fig 8. The forward photon is absorbed, and the backward one emitted, the energies therefore have to be subtracted, but since the momentum of the absorbed photon is negative, the momenta are numerically added.

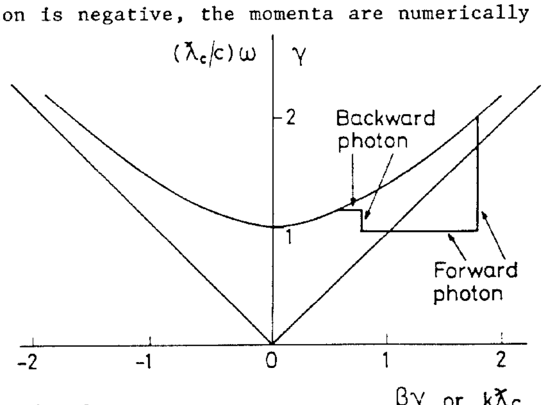

Fig. 8.

The resonance condition can be found from the geometry of Fig 8. If γ is large, so that $\beta \cong 1-1/2\gamma^2$, then an elementary geometrical construction shows that the ratio of forward to backscattered photon energies is $4\gamma^2$. The backward photon can be replaced by a 'static wiggler' of wavelength Λ, absorbing virtual photons with 'crystal momentum' h/Λ but no energy; the wavelength ratio is then $2\gamma^2$.

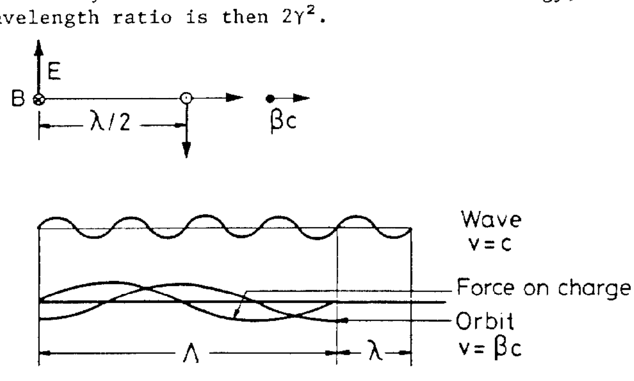

Fig. 9.

The familiar argument that resonance occurs when the particle overtakes the radiation by one wavelength in travelling through one wiggler peroid, illustrated in Fig 9, of course gives the same result

$$\Lambda/\lambda = \frac{1}{1-\beta} \cong 2\gamma^2 \qquad (4)$$

This is the 'two-wave' interaction used in the free electron laser. It is less effective than the Cherenkov interaction, since the transverse electric field acts on a second order component of transverse velocity. It does, however, dispense with nearby materials, giving more space and removing breakdown constraints. The same idea is used in the ubitron microwave tube, where coupling is to a wave in a waveguide [9]. The significance of the $2\gamma^2$ factor in producing ultra short wavelengths in a wiggler of practical dimensions was first appreciated by Motz[10]. Superficially, the two wave interaction with static wiggler may be regarded as similar to the 'wiggled' and twisted guide schemes illustrated in Fig. 7. In one case the particle trajectory wiggles whereas the wave goes straight, whereas in the other the wave wiggles and the particle goes straight. This would appear to have been the viewpoint of Gorn, who included both in his remarkable patent application filed just 40 years ago[7].

4. Another viewpoint; periodic potential and band structure

A different viewpoint on the two-wave interaction is to regard the wiggler as an external periodic perturbation of vector potential. This gives a band structure to the particle energies, and allows transitions between the bands. This is analogous to the effect of a periodic scalar potential, which produces the conduction bands in a metal. The metal crystal is analogous to the wiggler in the FEL; it has momentum but no energy. A microwave tube using this principle, the velocity jump amplifier, was invented by Field, Tien and Watkins in 1951[11].

5. Particle in a very intense wave

The motion of a particle in a single plane-wave was discussed in section 2, where it was shown that there is no net acceleration. If, however, the field is extremely strong, or the frequency low, a particle initially at rest can achieve a high energy before the sign of the field changes and it begins to decelerate. This can be important on an astrophysical scale, particularly in the rotating fields around pulsars. If the cyclotron frequency in the magnetic field of the wave exceeds its frequency the particle is accelerated to relativistic energies before the change in E and B are effective. It can be shown that in a plane wave the particle is accelerated into a direction following the wave, and it attains an energy of order $\gamma = (\omega_c/\omega)^2$ where ω_c is the cyclotron frequency of the particle in the magnetic field of the wave. In a rotating dipole, however, where the fields decrease with radius, and the wave velocity increases the peak energy is $(\omega_c/\omega)^{2/3}$.[12] In the Crab pulsar electrons and protons attain energies of about 100 and 1000 TeV respectively.

6. A single particle, one plane wave, and a spontaneously emitted photon. Radiation pressure

An acceleration mechanism of some importance in astrophysics is radiation pressure. Though hardly useful for practical accelerators it has been considered as an example of a second order coherent mechanism, where synchronism between the wave and the accelerated particle is not required. It is a second order effect, in which acceleration is proportional to the square of the product of field and charge. In section 2, where interaction between a single wave and charge was considered, it was stated that although the energy of the charge fluctuates under the action of the electric field, the net time averaged acceleration is zero. If, however, the charge radiates, this is no longer the case. Considered classically, there is a radiation reaction force proportional to $\dddot{x}$. Without this term, the net force can be shown to be proportional to $\langle \cos\omega t \cdot \sin\omega t \rangle$, which is equal to $\frac{1}{2} \langle \sin 2\omega t \rangle = 0$. An elementary small amplitude analysis[14] shows that the extra term introduces a phase shift ϕ in the $\sin\omega t$ term, which modifies the bracket from zero to $\langle \sin^2\phi \rangle$. Using the standard expression for the radiation reaction force gives a force on a bunch of n particles of charge q

$$F = \frac{1}{3} n^2 q^2 r_e E_o^2 \qquad (5)$$

where E_o is the field and r_e the classical radius of the charge.[13]

The following macroscopic analogy, but with resistive dissipation, illustrates the effect. A long bar magnet moves with uniform velocity through a conducting ring. If this is superconducting, the first pole pushes it forward, and the second stops it. If there is finite restivity, the second impulse does not quite cancel the first, since the current will have decayed, and energy is communicated to the ring. Obviously there is an optimum resistivity, if it is zero there is again no forward pressure. The mechanism is formally the same as that which drives an induction motor, where the rotating wave is not synchronous with the rotor.

7. Travelling pulse in vacuum

In the systems considered so far, the particles are accelerated in vacuum, and the accelerating fields are harmonic. Although in some cases the fields are provided by a series of independent cavities, this can be fourier analysed into travelling waves, even though the group velocity of the system may be zero. The manifolds of waves assumed so far have had a broad distribution in k-space, but a δ-function in ω-space. Some schemes, involving wakefields or triggered gaps, have nonharmonic accelerating fields. This involves broadening the δ-function in ω-space. An ideal pulse for acceleration has been sketched by Tigner[14], Fig 10. This has the desirable property that the phase and group velocities are nearly equal, and also the spectral bandwidth of the pulse is large. Such a pulse, with $E_z \nparallel E_\perp$ cannot be propagated remote from its source, since it rapidly spreads from dispersion. It would be interesting in this connection to find some criteria relating the relevant parameters, so that some optimum configurations can be found.

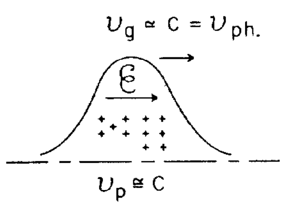

Fig. 10.

If the charges generating the pulse are nearby, or if the structure consists of a series of independently fed gaps, then dispersion does not occur, but an efficient system may be difficult to devise. The wake field accelerator of Voss and Weiland[15], and triggered gap accelerator of Willis[16] are in this class. It is interesting to regard the original 'Donkeytron' proposed Alfvén and Wernholm in 1952 as an early example of a similar idea, though strictly the field is not perhaps a wake field[17]. In this device an intense electron beam with a cross-over, near which is an intense 1/r electrostatic field, moves in a direction perpendicular to its length. A bunch of protons is dragged along behind the cross-over. At first sight, the compact pulse in these schemes seems to offer an advantage over the spread out fields in conventional devices. When factors of efficiency and tolerances are properly taken into account, however, this advantage may turn out to be somewhat illusory.

8. Betatron and induction linac

It is difficult to give a useful description of betatron interactions in terms of photon exchange; a plane wave expansion is not appropriate, and, as with a cyclotron operating at its fundamental frequency it is preferable to start from considerations of angular momentum. Coupling between the sources of the fields and the particles to be accelerated is very tight, which implies that the photons exchanged are remote from the energy shell. A spectrum of very low frequencies is involved, so that the device is related to those of the Cherenkov group. Similar remarks apply to the induction linac, though if one expands about the beam axis it is linear rather than angular momentum which is important, as with conventional linacs.

9. Acceleration in a plasma medium

A number of schemes have been suggested in which the particles to be accelerated are embedded in plasma. Conceptually the simplest of these starts from a stationary neutral plasma, and we begin by noting some properties of this medium. Propagation of transverse electromagnetic waves is dispersive, $\omega^2 - \omega_{p\perp}^2 = c^2k^2$, where ω_p is the plasma frequency $(nq^2/\varepsilon_0 m)^{\frac{1}{2}}$. The plasmons have rest mass $\hbar\omega$ which for $n = 10^{16}\,cm^{-3}$ is about 3×10^{-3}eV. For longitudinal (or 'Langmuir') waves, on the other hand, which are purely electrostatic in nature, the plasma frequency (for a cold plasma) is independent of wavelength, and the group velocity $d\omega/dk$ is zero. Waves below the plasma frequency do not propagate, and the dielectric constant $1 - \omega_p^2/\omega^2$ is negative. A 'rod' of such plasma, however, is capable of supporting a mode with $\beta_w < 1$. In planar geometry the corresponding mode is a surface wave that decays both into the plasma, with skin-depth of order c/ω, and into free space with decay length $\beta\gamma\lambda\!\!\!^-$. Probably the first suggestion to use plasma in accelerators was that of Fainberg in the early 'fifties[18]; he proposed a plasma rod of this form, and pointed out that the fields are such that simultaneous phase and radial stability is possible. Accelerators of this type, never actually realized in useful form, belong to the Cherenkov class.

A newer idea is the beat-wave accelerator of Tajima and Dawson[19]. Since, in a plasma, a Langmuir wave of given frequency can take any value of k, a transverse electromagnetic wave can couple directly to a Langmuir plasmon. This process is particularly efficacious if two transverse waves with frequency difference equal to ω_p are present. If $\omega \gg \omega_p$, then the phase velocity of the Langmuir wave is just $d\omega/dk$, the group velocity of the transverse plasmon. Extremely high fields can be produced in this way, and there is no breakdown constraint. Langmuir waves may be excited by other means, in particular by the injection into the plasma of charged bunches. These produce a harmonic wake with wavelength $2\pi v/\omega_p$.

Many other acceleration schemes involving plasma have been proposed and tried. Some of these make use of waves on beams; in this case the plasma is moving, and furthermore, not in general neutral. For non-neutral systems the net space-charge fields of the beam impose limitations on current and charge density, which restrict the accelerating gradient, and generally make the generation of a well-controlled beam difficult. The various plasma schemes are generally referred to as 'collective'; the accelerating fields are produced by free charges rather than charges confined in metals. There are many sub-groups, some of which are complicated. For example Olson's 'ionization front' accelerator and Briggs' recent collective accelerator involve a controlled variation of plasma properties[21,22]. The electron ring accelerator involves a very special non-Maxwellian plasma that moves bodily at the same velocity as the particles to be accelerated[23]. As with microwave tubes, where there was great activity some 25 - 30 years ago with the aim of making constructive use of the beam-plasma interaction, the problems of proper control of plasma constitute at least one reason why plasma accelerators have not come into widespread use.

10. Classification schemes

Accelerators have been classified in many ways in the past. Some of these classifications are straightforward and convenient, such as electrostatic, inductive and r.f., with the latter divided into cyclic or linear machines. The appearance of Veksler's enigmatic paper[24] at the 1956 accelerator conference introduced the term 'coherent', and the word 'collective' was certainly in use some ten years later. The use of radiation pressure, discussed in section 6, was suggested in Veksler's paper. An attempt to define the terms coherent and collective in a systematic manner is made in ref 13.

Despite the identification of physical mechanisms attempted in this paper, it is difficult to draw up an interesting an illuminating classification scheme. The boundaries are not sharp, and divisions which look satisfactory for some devices separate others that seem in other ways to be similar. Furthermore, the approach used here is not strictly correct. Except in the extreme relativistic regime, either k or ω varies during the acceleration. Nevertheless, an attempt at classification has been made, and is exhibited in the table. Not included in the table is the class of 'stochastic' accelerators. This cuts across the classification as shown. It is of considerable importance in astrophysics, in association with various forms of the Fermi mechanism[25], and was for a time advocated as a practical laboratory scheme, particularly applied to the cyclotron. A good account of this work, and early references, (again from the USSR) are given by Livingood[26].

Mechanism	Medium / ω-spectrum	Free space	Stationary plasma	Moving plasma
Cherenkov and plasmon ('Near field' and 'media')	δ-function	Conventional linacs Inside-out linacs (gratings, droplets) Synchrotron (approx) [Cyclotron]	Plasma waveguide Beat-wave	Beam-wave (e.g. auto-resonant) Plasma wake field Ionization front Electron ring
	broad	Wake field Triggered multi-gap Induction linac Betatron		
Stimulated Compton ('Far field')	δ-function	Two waves Tapered wiggler + 1 wave	Ditto + plasma background	
Spontaneous Compton	δ-function	Radiation pressure		[Induction motor, strictly Raman scattering]

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