SUPERGOBLIN - THE ACCELERATED ORBIT CODE IN USE FOR THE CHALK RIVER SUPERCONDUCTING CYCLOTRON

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Summary

SUPERGOBLIN is a general cyclotron orbit code of distinguished lineage having reached Chalk River via Oak Ridge, Michigan State and TRIUMF. Several new features have been added to aid in cyclotron design and analysis. Features described are a treatment of spiral dees, a second order transport matrix option, phase-space projection tracking, an automatic injection matching algorithm, a method of treating the extraction system and parameter search and fitting options.

Treatment of Spiral Dees

SUPERGOBLIN integrates particle motion (fourth order Runge-Kutta with Gill constants) through a magnetic field mapped at discrete azimuthal angles. The outwards spiral path of the beam will intersect the spiral dee gaps at angles between these azimuths. The location of the beam intersection with the gap has to be determined precisely if dee effects are to be modeled well.

This is achieved by performing a short iterative search initiated when a gap is about to be crossed in an integration step. By interpolation in the trajectory radius and azimuth coordinates, the location of the spiral gap intersection with the trajectory can be predicted. This prediction is then used to reintegrate over the last azimuthal step and a new trajectory to the gap found. Another interpolation is made in the trajectory and a new prediction of the gap location made and so on. In this way the gap intersection can be located to 1/1000 of a degree in a few iterations.

Once the dee gap is reached, the particle momentum is resolved into components parallel and perpendicular to the dee gap and the component perpendicular to the gap increased to match the potential drop (times the transit time factor) across the gap.

Second Order Beam Transfer Matrix

There is a need to be able to calculate the beam transfer matrix between two locations within the cyclotron. In particular, for the Chalk River cyclotron, the beam transport system between the tandem and the cyclotron has been designed[1] to match the calculated beam optical properties of the cyclotron injection trajectory and so optimize the transverse and longitudinal phase-space projections at the stripper foil.

Because this beam transport system has been designed to second order, initially using TRANSPORT[2] and more recently using TRANSOPTR[3], there has been a requirement to calculate the second order matrix representative of the injection trajectory.

This new option allows the user to request a second order matrix calculation at several specified turn numbers and azimuth angles. Each output matrix is the cumulative transfer matrix between the beam starting location and the specified output location.

The method used is to ray-trace 28 particle trajectories. The first ray is the reference trajectory, then one ray displaced in the positive direction along each of the six phase-space axes, then six rays displaced negatively along the same axes and finally the 15 possible rays resulting from a simultaneous positive displacement along each of two phase-space axes.

These 28 rays form a sufficient set to calculate the complete second order matrix including all square and cross-product terms.

If x and y are two phase-space coordinates and x/x_+, x/y_+ and x/x_+y_+ etc. represent the calculated displacement in x resulting from initial positive displacement x_+, y_+ etc. and (x/x), (x/y), (x/x^2) and (x/xy) etc. represent the values of the matrix elements, then the matrix elements are

$$(x/y) = \{x/y_+ + x/y_-\}/\{2\,y_+\}$$

$$(x/y^2) = \{x/y_+ - x/y_-\}/\{2\,y_+^2\}$$

$$(x/xy) = \{x/x_+y_+ - (x/x) * x_+ - (x/y) * y_+$$
$$-(x/y^2) * y_+^2 - (x/x^2) * x_+^2\}/\{x_+y_+\}$$

Although in principle the matrix element values calculated should not depend on the magnitude of the displacements used, (as long as they are small) the magnitudes are in fact definable by the user and should be appropriate to the beam dimensions to give a meaningful matrix.

The output matrix may be requested in a format suitable for direct input into TRANSPORT.

The cyclotron coordinates defining the start of a ray are the azimuth angle, radius, radial momentum, vertical displacement, vertical momentum, rf phase and energy. For rays displaced from the reference trajectory the starting coordinates have to be determined with respect to the reference trajectory by a simple geometrical transformation.

Similarly, at the output matrix location, the displaced ray must be examined to extract the appropriate values of the phase-space coordinates. This more difficult transformation is accomplished by storing a short history table of ray coordinates as a function of azimuth. When the matrix output azimuth is reached, an interpolation within the table is made to find where the plane normal to the reference ray is crossed. This is illustrated in Fig. 1. Once this intersection is found all the displacements from the reference ray can be calculated.
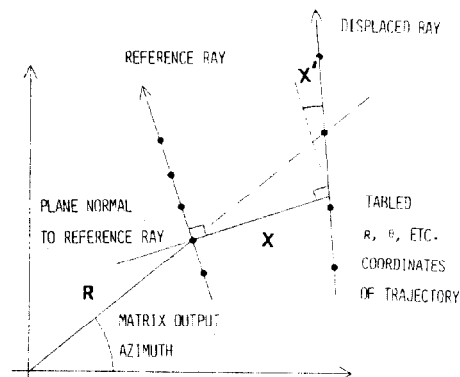

Fig. 1   At the output angle the phase-space projection coordinates x, x', etc. must be calculated by determining the intercept of the displaced ray with the normal to the reference trajectory.

Phase-Space Projection Tracking

To evaluate beam behaviour it is useful to define the beam phase-space projections at a given location and examine how they have become transformed when the beam reaches a second given location. Such an option is available and has been used to look at the phase-space evolution during injection, during acceleration and along the extraction channel elements.

Six parameters are available to specify an elliptical phase-space projection at the starting location. The first five of these are the selection of the two phase-space coordinate axes for the projection, (for example x and x'), their maximum extents x_{max} and x'_{max} and the emittance ε for this projection. (The convention used is that the area of the ellipse is $\pi\,\varepsilon$.) These parameters are sufficient to form the bounding ellipse defined by the equation:

$$\gamma\,x^2 - 2\,\alpha\,x\,x' + \beta\,x'^2 = \varepsilon \qquad (1)$$

where $\gamma = x'^2_{max}/\varepsilon$

$$\alpha^2 = \frac{x^2_{max} \cdot x'^2_{max}}{\varepsilon^2} - 1$$

$\beta = x^2_{max}/\varepsilon$ are the usual Twiss parameters.

A user defined number of particles are placed on the perimeter of the ellipse in a cosine distribution between $-x_{max}$ and x_{max}. The positive and negative roots for α result in two possible ellipses with their major axes in the first and second quadrants respectively and the sixth parameter (equal to -1 or +1) is necessary to select which is intended.

The output projections may be requested at several turn numbers and azimuth angles.

The projections are constructed by tracing individual rays for each of the starting coordinates on the ellipse perimeter. The starting projection coordinates x, x', etc. are translated, by relation to the central ray, into cyclotron coordinates r and r' etc. Similarly, at each output location, the cyclotron coordinates are translated back into projection coordinates. The same method described above for the second order matrix option is used for these translations.

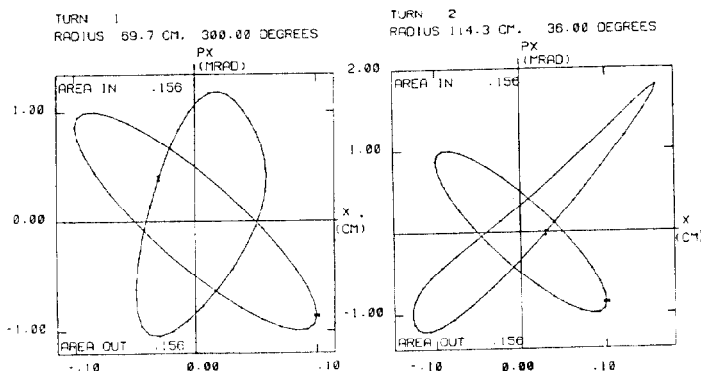Figure 2 shows example plots for a beam moving along the magnetic extraction channel.


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TURN   1                          TURN  2
RADIUS  69.7 CM. 300.00 DEGREES   RADIUS 114.3 CM.  36.00 DEGREES
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Fig. 2   Example of the radial plane phase-space projection plots.

Injection Orbit Matching

There are two steps required to match an injected beam into the cyclotron. The first step is to find the injection trajectory that results in the best centered accelerated turn pattern after stripping that also results in the largest energy gain per turn. The second step is to calculate the injection matrix for this trajectory and use it to set up the injection beam line to provide radial, axial and longitudinal phase-space projections at the stripper which minimize radial and axial incoherent (beam size) oscillations and minimize the beam bunch length. This second step has been described elsewhere[4]. This section will describe how to find the reference injection trajectory.

All injection paths from the external beam line are collinear until they reach the inflection magnet in the yoke wall. From this point each heavy ion species is deflected along a different path to the stripping foil. The problem is to optimize these paths.

The method used, while not a formal function minimization, results empirically in the minimization of a test function $F = F_1 + F_2$.

The first component F_1 of the test function is a measure of the "centeredness" of the beam and is defined as

$$F_1 = \sum_i (x_i - x_{av})^2 + \sum_i (p_{x_i} - p_{x\,av})^2$$

where x_i is the radial displacement of the beam orbit (after stripping) from the coordinates of an equilibrium orbit (E.O.) of the same energy. The coordinates of the E.O. are known at four locations 90° apart for every turn. Similarly p_{x_i} is the displacement in radial momentum from the E.O. The summation is over the number of E.O. locations and one turn with its four E.O. locations is usually sufficient. x_{av} and $p_{x\,av}$ are the average values of x_i and p_{x_i} at the E.O. locations. Thus F_1 is a minimum when x and p_x remain constant and so is a measure of the x,p_x oscillation amplitude.

The second component F_2 is given by

$$F_2 = \sum_i (\phi_i - \phi_{ref_i})^2$$

where ϕ_i are the rf phases at the E.O. angles and ϕ_{ref_i} is the calculated rf phase at that angle which would result if the phase at the dee center were 90° (i.e., voltage equal to zero). The two resonators of the Chalk River cyclotron can operate in zero or π mode and at three harmonic numbers h=2, 4 and 6[5]. For all of these conditions the design rf phase in the center of the dee is 90°. Thus F_2 is a test function which is a minimum when the energy gain per turn is a maximum.

The method used is an iterative Newton-Raphson method having three parameters as the function variables. These are the rf phase ϕ_0, the direction of the beam α_0 and the beam energy E_0, each defined at the inflection magnet.

The method is as follows: Initial values for α_0, ϕ_0 and E_0 are chosen and a ray is traced to the stripper and beyond for a few accelerated turns. The matrices M_i which relate the changes in x_i, p_{x_i} and ϕ_i at the E.O. locations to the changes in the input variables α_0, ϕ_0 and E_0 are then calculated. (Use is made of the matrix software described

earlier.) A mean correction vector can then be calculated by averaging the correction predicted at each of the N E.O. locations where the x_i etc. are calculated.

$$\begin{pmatrix} \Delta\alpha_0 \\ \Delta\phi_0 \\ \Delta E_0 \end{pmatrix} = \frac{1}{N} \sum_{i=1}^{N} M_i^{-1} \begin{pmatrix} \Delta x_i \\ \Delta p_{x_i} \\ \Delta\phi_i \end{pmatrix}$$

where the Δx_i, Δp_{x_i} and $\Delta\phi_i$ are the calculated deviations of x_i, p_{x_i} and ϕ_i from the ideal values x_{av}, $p_{x_{av}}$ and ϕ_{ref_i} at the E.O. locations. The input variables can then be corrected

$$\alpha_0' = \alpha_0 + \Delta\alpha_0$$
$$\phi_0' = \phi_0 + \Delta\phi_0$$
$$E_0' = E_0 + \Delta E_0$$

and the iteration continued until the values of both F_1 and F_2 are acceptably small. This takes three to five iterations and depends on the choice of the starting parameters.

Extraction System Handling

The Chalk River cyclotron extraction system uses a first harmonic field perturbation to increase the turn separation at the entrance to a single electrostatic deflector. This deflects the beam into a multi-component magnetic channel with active and passive elements providing both bias and gradient fields. SUPERGOBLIN has the capability to model this entire system.

The first harmonic bump is generated using the outermost trim rod set[6] and the required field perturbation from the rods is calculated and added to the magnetic field map when the bump radius, azimuth and magnitude are specified. The perturbation shape functions (calculated by TRIM[7] and verified by measurement) are stored within the code as a function of amplitude.

The deflector azimuthal position and electrostatic gradient may be defined. The gradient is converted to an equivalent magnetic field perturbation using the mass and energy of the entering particle and so the deflecting field is always normal to the beam trajectory.

The magnetic channel elements are modeled by the addition of up to five perturbation maps to the main field map. These are generally calculated using EXMAP[8] and may be defined over annular sectors between defined radii and azimuths.

A trajectory may also be specified which defines a reference path through the deflector and magnetic channel elements. The geometry of the hardware within the system may be specified with respect to this trajectory allowing system acceptances to be calculated.

An alternative analytical (as opposed to mapped) representation of the magnetic channel is also available. This allows up to five sectors (each with bias, gradient and second gradient field components) to be defined over specified azimuthal intervals. The field in the channel is calculated from these specified values using the reference trajectory as the origin for the gradient terms, which are perpendicular to the reference trajectory.

The analytical representation has been used for scoping studies of the extraction system and the mapped representation for the detailed design and evaluation stage.

Search and Fitting Options

The search option is a simple one-dimensional Newton-Raphson iterative minimization which allows variation of a single parameter to place the beam on a specified radius at a specified turn and azimuth. The variable may be any valid cyclotron parameter, e.g., dee voltage, first harmonic parameter, deflector gradient, magnetic channel field, etc.

The fitting option is a more elaborate multi-dimensional Newton-Raphson iterative minimization allowing simultaneous variation of several parameters within defined bounds to find a fit which best matches the beam to specified conditions at a large number of turns and azimuths. Again the variables may be any valid cyclotron parameter. The beam properties requested may be the radius, radial momentum, rf phase and energy and each may have an associated weight. The method is to numerically calculate the elements of a Jacobian matrix which is inverted and used to calculate the required damped corrections.

Both of these options have been used to analyze the performance of the extraction channel[9]. It has been found most useful to use the search option to find a first approximation to the cyclotron parameters one at a time and then use these values for a more elaborate fit to optimize several parameters simultaneously.

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