DESIGN STUDY OF THE THIRD ORDER RESONANCE EXTRACTION SYSTEM AT TARN II

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ABSTRACT

TARN II is designed to accelerate ions with charge to mass ratio of 1/2 up to 450 MeV/u. The slow extraction system utilizes a third order resonance (ν_h=5/3). The system is designed according to the approximate theory which deals with dependence of the separatrix size on the closed orbit displacement. The circulating beam with large emittance (up to 48πmm×mrad) and fractional momentum spread of ±0.2% can be extracted with small emittance (1.1πmm×mrad) and with high extraction efficiency (beyond 90%).

Introduction

A beam was first resonantly extracted by Hereward with CERN PS.[1] An analytical treatment of the resonant extraction was given by Kobayashi based on an approximate Hamiltonian, and was applied to integral, half-integral and third resonant extraction.[2] The third resonant extraction has been performed at synchrotrons. SATURNE II has been so designed that the momentum spread of the extracted beam becomes small ($\Delta p/p$=3×10^{-4}).[3] At LEAR a beam has been successfully extracted with small emittance (0.8πmm×mrad).[4] Chromaticity adjustment has been important in the both cases.

TARN II lattice is designed to be able to operate with two different excitation modes.[5] One of them, Synchrotron Mode serves the slow extraction which is carried out by using third order resonance (νres=5/3). The transverse emittance is not so much shrunk by acceleration as at high energy accelerators. The extraction system is designed so that a circulating beam has as large separatrix area as possible at the time of extraction and so that an extracted beam has as small emittance as possible.

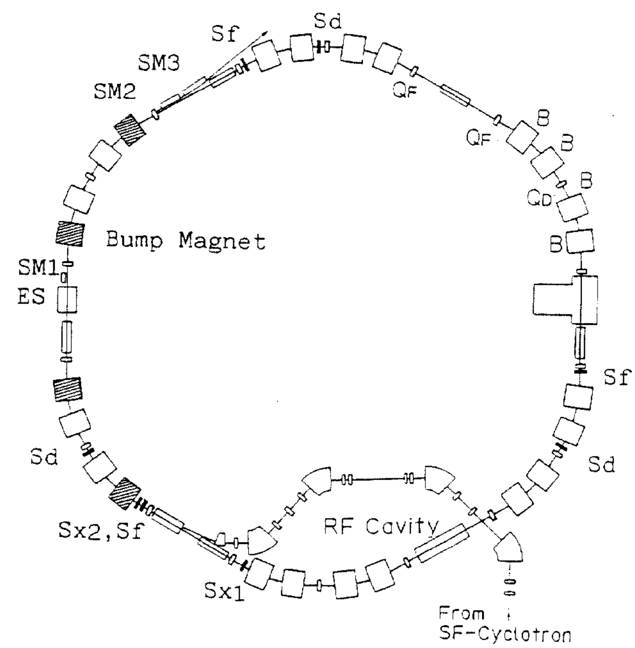

Fig. 1 Extraction System
Sx1 and Sx2 are sextupole magnets for resonance excitation, and Sf and Sd for chromaticity adjustment. ES is an electrostatic septum and SM's are septum magnets.

Beam Extraction System

The beam extraction system consists of four elements as shown in Fig. 1:

i) four bump magnets which distort the closed orbit

ii) six sextupole magnets(Sf and Sd) as a chromaticity adjuster and two magnets(Sx) as a resonance exciter

iii) main dipole and quadrupole magnets as the closed orbit shifter

iv) an electrostatic septum(ES) and three septum magnets(SM) as an extractor.

As a circulating beam at the time of multi-turn injection must be given a large open horizontal space, the nearest thing to the beam, the anode plane of ES is set at x=84mm as far from the central orbit as possible. The closed orbit is distorted between the bending section just before ES and the next bending section by four bump magnets

i) for the reason that an extracted beam should go into ES parallel to the anode plane on the anode side,

ii) in consideration of turn separation and of the horizontal space for a circulating beam.

Four main dipole magnets among 24 have the function of bump magnets with backleg winding coils.

Description of sextupole magnets is given in later Sections.

The beam extraction follows the following procedure:

i) closed orbit distortion with bump magnets, and applying high voltage to ES and excitation of SM's

ii) excitation of the exciter, and chromaticity adjustment

iii) ν-value shift from the injection value of 1.75 to a given value near the resonance of 5/3 with main quadrupole magnets

iv) approach to the resonance point by gradual decrement of main dipole magnet field with tracking of the quadrupole magnet field.

During the approach, as the separatrix is shrunk, the beam out of it has the rapidly increasing amplitude of the betatron oscillation. When the beam jumps over the septum of ES, it is extracted with ES, SM-1 at a straight section and two SM's at the next straight section. The phase advance between ES and SM-2 is about π/2. The acceptance at ES is 20mm×2mrad. Parameters of the ES and SM's are listed in Table 1.

Table 1
Parameters of the ES and SM's

type	septum thickness (mm)	field (kV/cm),(kG)	length (m)	kick (mrad)
ES	0.1	60	1.0	2.3
SM-1	1	1	0.5	7.3
SM-2	9	5	1.0	76.4
SM-3	35	16	1.3	302.9

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Dependecne of the Separatrix Size on the Closed Orbit Displacement

Theoretical study about the separatrix size in the third order resonance is described which explicitly deals with the closed orbit displacement at the sextupole magnet. A particle with ν-value near the resonance($\nu=\nu res+\Delta\nu$) is considered to receive a kick from a sextupole magnet, whose strength is $\Delta X'=SX^2$, every revolution, S being $B''l/2B\rho\times\sqrt{\beta_S}^3/\beta_n$ (β_S:the beta function at the sextupole magnet, β_n:the normalized beta function). The closed orbit of the particle at the sextupole magnet is positioned $Xd=\eta_S\sqrt{\beta_n/\beta_S}\Delta p/p_0$ from the central orbit, η_S being the dispersion function at the sextupole magnet, and Δp momentum displacement from the on-central-orbit momentum p_0. Consideration is simplified by using normalized units[6] and the complex notation $Zb=X-Xd+i(X'-Xd')$. After three revolutions the phase space point(X,X') of the particle at the azimuthal of the magnet is shifted by

$$\Delta Zb=3i[(-2\pi\Delta\nu+SXd)Zb+SZb^{*2}/4] \ . \tag{1}$$

When the phase space point(X_j,X_j') at the phase advance Ψ_j is expressed by using $Zbe^{-i\Psi_j}=X_j-Xd_j+i(X_j'-Xd_j')$, the shift by a sextupole magnet S_j at the phase advance Ψ_j is

$$\Delta(Zbe^{-i\Psi_j})=3i[(-2\pi\Delta\nu+S_jXd_j)Zbe^{-i\Psi_j}+S_jZb^{*2}e^{2i\Psi_j}/4] \ . \tag{2}$$

The shift becomes at the phase advance of 0

$$3i[(-2\pi\Delta\nu+S_jXd_j)Zb+S_je^{3i\Psi_j}Zb^{*2}/4] \ . \tag{3}$$

Therefore, after three revolutions the phase space point at the phase advance of 0 is shifted under all sextupole magnets by

$$\Delta Zb=3i[(-2\pi\Delta\nu+\Sigma S_jXd_j)Zb+\Sigma S_je^{3i\Psi_j}Zb^{*2}/4] \ . \tag{4}$$

An effective sextupole magnet with S can be considered to be located at the phase advance Ψ instead of the sextupole magnets:

$$Se^{3i\Psi}=\Sigma S_je^{3i\Psi_j} \ . \tag{5}$$

Equation (5) holds at all phases= Ψ(mod $2\pi/3$).
The term including Xd_j in Eq. (4) has a relation with the horizontal chromaticity:

$$\Sigma S_jXd_j=\Sigma B_j''l_j\beta_j\eta_j/2B\rho\times\Delta p/p_0=-2\pi\xi_X\nu_0\times\Delta p/p_0 \ . \tag{6}$$

When we consider new coordinates $Z=Zbe^{-i\Psi}$ and follow Barton:[7]

$$\begin{aligned}&\Delta Re(Z)=dRe(Z)/dt= dH/dIm(Z)\\ &\Delta Im(Z)=dIm(Z)/dt=-dH/dRe(Z)\\ &\epsilon=6\pi(\Delta\nu+\xi_X\nu_0\times\Delta p/p_0) \ ,\end{aligned} \tag{7}$$

the separatrix forming a regular triangle is described by

$$[SRe(Z)/4+\epsilon/6][3Im(Z)+Re(Z)-4\epsilon/3S]\times[3Im(Z)-Re(Z)+4\epsilon/3S]=0 \ . \tag{8}$$

The distance between the stable fixed point and the unstable fixed point is

$$r=4\epsilon/3S=8\pi[\nu_0+\nu_0(\xi_n+\xi_X)\Delta p/p_0-\nu res]/S \ , \tag{9}$$

ξ_n being natural chromaticity.
The separatrix at the phase advance of 0 is shown in Fig. 2. The particle on the outgoing trajectory

$$Zb=re^{i\Psi}+le^{i(\Psi+\pi/6)} \tag{10}$$

has turn separation $\Delta l\times\cos(\Psi+\pi/6)$, Δl being given by using Eqs. (4), (9) and (10):

$$\Delta l=3S(\sqrt{3}rl+l^2)/4 \ . \tag{11}$$

Overlap of the Outgoing Trajectories

At the entrance of ES, a beam must have no slope toward the anode plane of ES. The circulating beam on the region A in Fig. 2 declines outwards. It hits the anode and the septum of the following magnet SM-1, and is lost. The extracted beam on the region B going inwards also hits them and is lost. The boundary line C has a slope depending mainly on length of ES and SM-1 and their arrangement. To make beam loss as small as possible, it is important to overlap outgoing trajectories of the beams with different momenta at the azimuthal of ES, and to inject the beam parallel to the anode plane.[8]

In Fig. 2, separatrices and outgoing trajectories at ES are shown on condition that the trajectories are overlapped each other. The phase advance at ES is defined to be 0. Stable fixed pionts are always on the dispersion line. This condition is made by

i) defining the intersecting point of the trajectory and the dispersion line

ii) having a zero emittance separatrix at the intersecting point.

The trajectory belonging to the beam with $\Delta p=0$ is given by

$$X'=\frac{\sin(\Psi+\pi/6)-r_0\sin(\pi/6)}{\cos(\Psi+\pi/6)} \ , \tag{12}$$

where r_0 is r for $\Delta p=0$. The X coordinate of the intersecting point is given by

$$X=\frac{r_0\sin(\pi/6)}{\sin(\Psi+\pi/6)-(\alpha+\beta_n\eta_e'/\eta_e)\cos(\Psi+\pi/6)} \ . \tag{13}$$

When the X or a zero emittance point is defined, Ψ is seeked. The momentum displacement Δp of the zero emittance beam must have the relation

$$\eta_e\sqrt{\beta_n/\beta_e}\Delta p/p_0=-[\text{right hand side of Eq. (13)}] \ , \tag{14}$$

where η_e and β_e are dispersion and beta functions at ES, respectively.

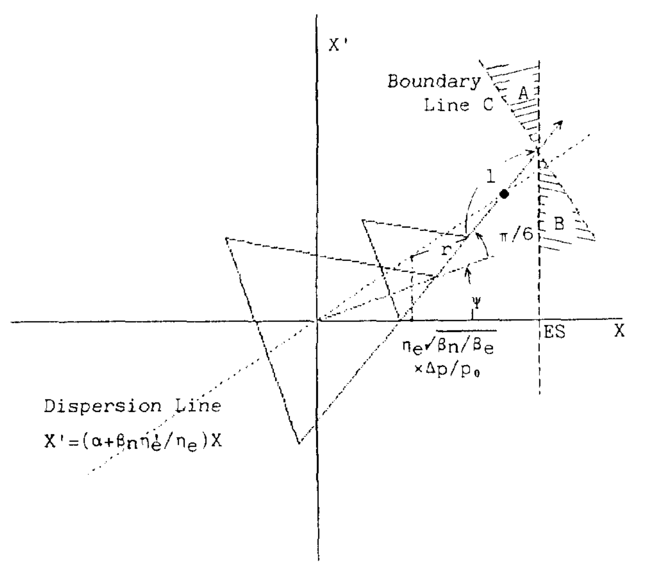

Fig. 2 Overlap of outgoing trajectories on the normalized phase space at ES

Thus, the overlap is realized when the chromaticity is

$$\xi_h=\xi_n+\xi_x=S\eta_e\sqrt{\beta_n/\beta_e}/4\pi\nu_0 \times[\sin(\Psi+\pi/6)-(\alpha+\beta_n\eta_e'/\eta_e)\cos(\Psi+\pi/6)] \quad . \quad (15)$$

Sextupole Magnets Arrangement

Four parameters (S, Ψ, ξ_h and ξ_v) are tuned with eight sextupole magnets. Two of them are used as a resonance exciter to have an effective strength S at the phase advance Ψ. These are so installed that their phase difference Φ satisfies $|\sin(3\Phi)|\approx 1$, as the required strengths of the magnets are inversely proportional to $\sin(3\Phi)$. Two families (Sf and Sd) consisting of the other three magnets respectively are a chromaticity adjuster. Each family is arranged three-hold symmetrically so that it has no contribution to S in Eq. (5).

These magnets are same ones. The maximum strength of B"l is $1000 kG/m^2 \times 0.1 m$.

The arrangement reduces superperiodicity of the lattice from 6 to 1. A sector resonance induced near the working point is $\nu_h+2\nu_v=5$. The bandwidth is calculated according to Guignard.[9] The bandwidth is smaller than that of $3\nu_h=5$.

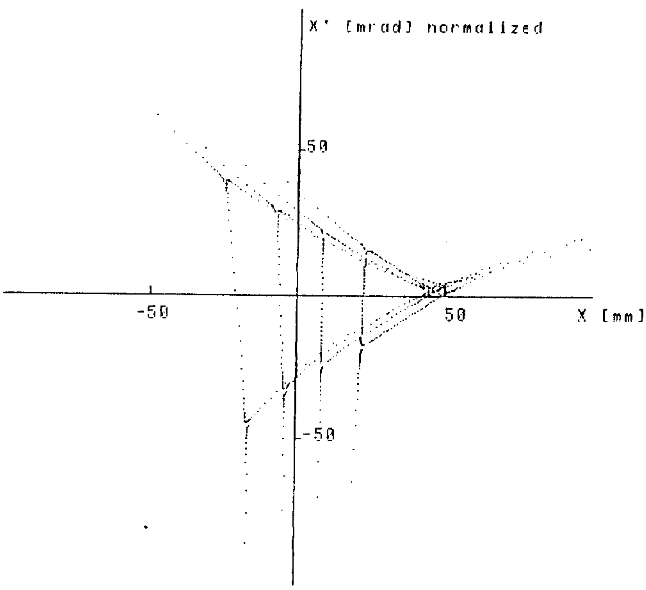

Fig. 3 The separatrices and outgoing trajectories of beams with $\Delta p/p_0$=0.0, 0.2, 0.4 and 0.6%, respectively, at ES(α=0)

Beam Trace with Computer Simulation

A beam is expected to have the horizontal emittance of 400πmm×mrad at most after multi-turn injection. After acceleration, the momentum spread is $\Delta p/p=\pm 0.2\%$. Then the emittance is dependent on ion species, charge to mass ratio and energy. We have searched the case where the separatrix area for the circulating beam can be considerably large at the maximum magnetic rigidity. Following requirement has been imposed upon the beam at the azimuthal of ES:

i) turn separation of 7mm at X=66mm for the beam with $\Delta p/p_0$=0.2%

ii) ξ intersecting point (50mm, 0mrad)

iii) the phase advance of the effective magnet Ψ=0.

The eight sexupole magnets have been treated as thin lenses. A beam has been traced in the normalized unit system.

Results of the simulation show that the separatrix for the beam with $\Delta p/p_0$=0 is a little larger than prediction by the theory. We have made ν_0 small to get the separatrix with r=50mm. Outgoing trajectories can not be overlapped each other, as the strength S is not so weak but that separatrices and outgoing trajectories are curved. The phase Ψ has been tuned in order to reduce wideness of the path of the trajectories. Dependence of the separatrices and trajectories on the momentum displacement Δp are shown in Fig. 4, where ES is positioned at X=66mm and the slope of the line C is -5mrad/mm. Comparison is shown in Table 2 between the results of computer simulation and theoretical predictions. Turn separation has become larger than the prediction.

During the extraction the fields of the main dipole and quadrupole magnets are gradually decreased by about 1%, tune value is kept constant, and $\Delta\nu/\nu res$ is 0.53%, where $\Delta\nu$ is tune value distance from νres. Thus, stability of the power supply is required to be better than 10^{-4}.

Table 2
Comparison between computer simulation and theory

	Simulation		Theory	
S (/m)	6.42		6.42	
tune value ν_0	1.6756		1.6794	
phase Ψ (rad)	0.063		0.	
chroma. (h,v)	(-0.55, 0.)		(-0.75, 0.)	
natural(h,v)	(-0.72,-0.60)			
	Sx1=-0.67,Sx2=6.43			
	Sf=-1.18,Sd=-0.88			
$\Delta p/p_0$ (%)	emittance (mm×mrad)	separation (mm)	emittance (mm×mrad)	separation (mm)
0.0	79π	5.3	105π	8.1
0.2	48π	8.8	67π	6.8
0.4	26π	9.5	38π	5.4
0.6	14π	9.1	17π	4.1
0.8	3π	8.9	4π	2.8
1.0	0	8.3	0	1.4

Conclusions

The slow extraction system with the third order resonance at TARN II has been designed. The circulating beam with the emittance upto 48πmm×mrad can be extracted under the condition that the fractional momentum spread is ±0.2%. The emittance of the extracted beam is about 1.1πmm×mrad. The extraction efficiency exceeds 90%.

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