# ASYMMETRIC DEE-VOLTAGE COMPENSATION OF BEAM OFF-CENTERING IN THE MILAN SUPERCONDUCTING CYCLOTRON

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Abstract: An analysis of the effects of orbit off-centering on the beam extraction in the Milan superconducting cyclotron is made, and the sensitivity of axial beam loss and radial phase space distortions to beam off-centering determined for various acceleration conditions. We conclude that the first field harmonic compensation of beam off-centering is ineffective in the region of the operating diagram where the Walkinshaw resonance precedes the  $\nu_{\rm T}=1$  resonance. Asymmetric dee-voltage compensation is considered in these cases, and the domain of validity of the method determined. A semi-empirical relation for dee-voltage distribution is deduced, and the extraction efficiency discussed.

#### Introduction

One of the general features of superconducting cyclotron magnets is the large field drop in the fringing region, which substantially influences the beam dynamics and extraction in these machines. Because of tight constraints on the position of extraction elements, one is forced to accelerate the beam far below the  $\nu_r = 1$  resonance before passing through the focusing elements. Consequently, in dealing with beam extraction in superconducting cyclotrons, a multitude of resonances of different orders are encountered, whose sequence is of paramount importance in obtaining high quality beams.

The extraction system of the Milan superconducting cyclotron follows closely the MSU K500 design, |1|. The basic feature of this system is that the  $\nu_{T}{=}1$  resonance at extraction is driven by a first harmonic of the magnetic field whose amplitude and phase are controlled by adjusting the currents of the last triming coil set. It was, however, noted in the first runs of the MSU K500 that the beam could be extracted only in a very limited range of first harmonic amplitudes and phases, which were considerably shifted in relation to ideal field calculations |2|. This unexpected behaviour was related to the acceleration of off-centered beams, which was shown to be difficult to avoid in cyclotrons with spiraled dees, |3|.

In order to establish conditions under which an off-centered beam can survive various resonances during acceleration in the Milan cyclotron, we have investigated the beam dynamics on a set of nine ions representative of the entire operating range, both in terms of charge-to-mass ratio and the magnetic field level. We found that in a number of cases the radial phase space distortions due to beam off-centering can not be compensated by first field harmonic at extraction. Asymmetric dee-voltage compensation at inner radii was hence studied. Results pertaining to these investigations are presented below.

### Equilibrium orbit properties

The equilibrium orbit properties relevant for extraction studies, the average orbit radius and energy per nucleon at which the ions cross the most important resonances, are given in table 1. From these data and corresponding  $(\nu_{\text{T}},\nu_{\text{Z}})$  plots, one may observe that the position of the resonances moves inwards as the field level is lowered, so that the crossing of the Walkinshaw resonance may occur before or after the  $\nu_{\text{T}}=1$ 

resonance. Furthermore, in a certain part of the operating diagram, multiple crossings of the Walkinshaw resonance are possible (one says that the beam "sits" on the Walkinshaw resonance). In all cases considered, the effective limit of beam acceleration is the  $\nu_r + 2\nu_Z = 3$  resonance, which always occurs after the  $\nu_r = 1$  resonance, and in certain cases is beyond the extraction region. Due to the coupling nature of the Walkinshaw resonance, the possibilities of extraction of an off-centered beam depend crucially on the relative position of this resonance.

Table 1

Z/A	B <sub>O</sub> (kG)	vr=2vz		ν <sub>r</sub> =1		νr+2νz=3	
		R <sub>e0</sub> (cm)	T/A <sub>e0</sub> (MeV/n)	R <sub>e0</sub> (cm)	T/A <sub>e0</sub> (MeV/n)	R <sub>e0</sub> (cm)	T/A <sub>e0</sub> (MeV/n)
0.5	31.1	83.7	95.7	83.9	96.0	87.0	100.8
0.5	26.0	81.7	60.0	83.0	61.7	85.7	64.7
0.5	24.8	70.4	39.7	82.7	55.5	85.2	58.0
0.5	22.0	56.6	19.5	82.0	42.2	84.3	43.8
0.3	34.1	82.8	36.9	82.8	36.9	87.1	39.4
0.3	22.0	50.1	5.4	81.7	14.5	83.7	15.0
0.15	47.0	83.4	17.2	82.2	16.8	**	**
0.15	30.5	74.6	5.7	82.0	6.9	86.0	7.3
0.15	25.0	58.6	2.4	81.2	4.6	85.0	4.8

Multiple crossings of the Walkinshaw resonance beginning at given radius

# Beam Acceleration Studies

All studies of the accelerated orbits reported here were performed with the Spiral Gap orbit program. Each of the nine ions has been accelerated in a 120 deg. symmetric field map with an initial energy,  $T/A_{\rm i}$ , corresponding to the  $R_{\rm EO}$  between 15 and 17 cm. The initial conditions for the central ray, i.e. the radius  $R_{\rm i}$ , the radial momentum  $p_{\rm ri}$ , and the phase  $\phi_{\rm i}$  with respect to the RF were given by equilibrium orbit data. The radial phase space is represented by eight particles whose initial conditions were determined by the beam emittance for the injection energy. For each of these radial phase points, two particles lying on the axial eigenellipse were considered, serving for axial and radial emittance calculations.

The static phase plot of an ion with Z/A=0.15,  $B_0{=}25~kG$  and T/A=0.21 MeV/n is shown in fig.1. Here, the beam off-centering is defined in relation to the accelerated equilibrium orbit, and is determined by its amplitude and phase,  $\rho_0$  and  $\psi_0$ , in relation to an arbitrary axis, which we have chosen to connect the static and accelerated equilibrium orbits. The precessional motion around  $(x_c,p_{Xc})$  may be represented by three types of cycloids, depending on the ratio  $\Lambda{=}2\pi\rho_0\,(\nu_T{-}1)/\Delta r_e$ , where  $\Delta r_e$  is the turn spacing. Since  $|\nu_T{-}1|$  is

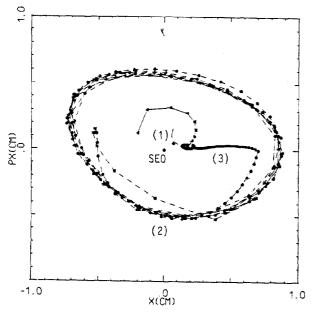


Fig.3 Dynamical phase plot of a particle Z/A=0.5 and B =25 kG. The central ray path is given by (1). The trajectories of off-centered beams in a symmetric dee system (2), and an asymmetric dee-voltage system (3) are also shown.

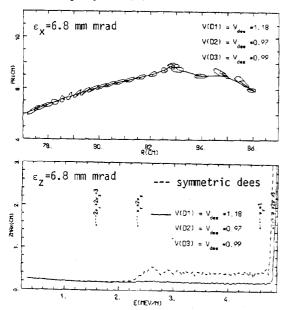


Fig.4 The radial phase space and axial envelope of the beam Z/A=0.15 and  $B_{\rm O}$ =25 kG, initially off-centered by  $\rho_{\rm O}$ =7 mm, and recentered by asymmetric dee-voltage compensation.

(N=3). A satisfactory approximation for the amplitude of perturbation  $\Delta V_{\Omega}/V$  is given by:

$$\Delta V_{O}/V = (\rho_0 / \Delta r_e)(R_{eo}^{0.61}/B_O)$$
 ,  $\alpha_0 = \psi_O$ 

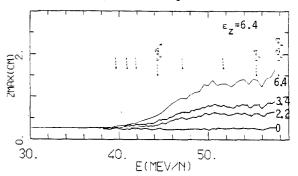
where  $\rm B_{\rm O}$  is expressed in kG, and all other values in cm. In table 2, values of  $\Delta V_{\rm O}/V$  given by this relation are compared for a number of beams with empirically determined values. The agreement is seen to be quite satisfactory. On the basis of these results we conclude that values of asymmetric dee-voltages for recentering of beams for which the Walskinshaw resonance occurs at internal radii can always be found.

Table 2

	Z/A	,B <sub>O</sub> (kG)	R <sub>eo</sub>	Рo (сm)	ψο (deg)	△V/V	h <sup>α₀</sup> (deg)	ΔV/V <sub>e</sub>
_	0.15	25.0	18.05	.228	0.	.048	-10.9	.053
				.462	87.8	.097	100.2	.098
				.568	180.0	.119	180.0	.115
				.433	-90.8	.091	-90.8	.092
	0.5	26.0	9.33	.496	89.2	.110	75.9	.121
			21.25	.481	103.1	.379	99.35	.352
	0.5	22.0	16.40	.284	99.2	.171	101.3	.177
	0.15	47.0	17.31	.670	87.0	.90	77.26	.079

### Multiple Crossings of the Walkinshaw resonance

A particularly interesting case occurs when the extracted beam crosses the Walkinshaw resonance more than once. Cumulative effects in the axial phase space may be expected which may drastically reduce the extraction efficiency. This has been found to be the case, as is illustrated in fig.5. Studying the phase space behaviour for various beams, we conclude that multiple crossings of the Walkinshaw resonance can be survived only if the beam size is significantly reduced. This limitation may be understood as an effective constraint on the operating diagram, since achievable intensities are of no physical significance.



 $\frac{\text{fig.5}}{24.8}$  kG in case of acceleration with various radial emittances (given in mm mrad). Dashed lines indicate the positions of the Walkinshaw resonance

#### Conclusions

In this paper we report some of the results of extensive studies of beam acceleration in the Milan superconducting cyclotron, especially those pertaining to analysis of off-centered beam dynamics. We conclude that recentering is a very important aspect of beam dynamics in this cyclotron. The conventional technique of recentering with magnetic bump at extraction is seen to be applicable only in a limited part of the operating range. Due to internal position of the Walkinshaw resonance at low fields, which is typical for superconducting magnets, asymmetric dee-compensation is shown to be neccessary for recentering of a number of beams.

# References

- |1| E. Fabrici, A. Salomone, "Acceleration studies for the Milan superconducting cyclotron", INFN/TC-83/9, 1983.
- [2] F. Marti, H. Blosser, M.M. Gordon, 10<sup>th</sup>Int. Conf. on Cycl. and Appl. p.44, 1984.
- |3| M.M. Gordon, IEEE Trans.Nucl.Sci. NS-30(1983)2439

close to zero, the beam precession evolves during acceleration according to the monotonic fall of  $\Delta r_e,$  and sharply changes in the vicinity of the  $\nu_r=1$  resonance. For multiple crossings of the Walkinshaw resonance, the relevant parameter for determining the character of precession is  $\Delta_{tot}=n_{\text{C}}\Delta,$  where  $n_{\text{C}}$  is the number of Walkinshaw resonance crossings.

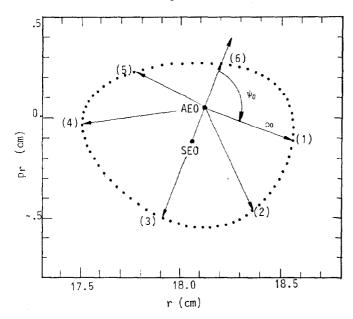


Fig.1 Static phase plot of Z/A=0.15,  $B_0$ =25 kG and  $\overline{1/A}$ =0.21 MeV/n particle. The amplitude and phase ( $\rho_0$ , $\psi_0$ ) of an off-centered beam are defined in relation to AEO and axis connecting SEO and AEO. The off-cenetered beam acceleration has been studied for initial phases  $\psi_1$ - $\psi_6$ .

A thorough study of the acceleration of an off-centered beam indicates that:

1) An off-centered beam that approaches the Walkinshaw resonance with  $\Lambda>1.2$  cannot survive the vertical blow-up effect during the crossing. In determining these conditions, we considered a factor of two as a limiting value of amplitude growth. Also, no significant effect of this resonance on the radial emittance was noted.

2) It is impossible for an off-centered beam to pass through the  $\nu_{r}{=}1$  resonance without significant distortions of the beam radial phase space.

Due to these consequences, it is clear that the centering of the beam is of paramount importance for beam extraction. Two methods of compensation of beam off-centering have been considered, and their effectiveness and domains of validity are as follows.

#### Magnetic Field Bump Compensation

This method of compensating the beam off-centering has been applied in those cases when the Walkinshaw resonance is in the vicinity of the  $\nu_{\Gamma}=1$  resonance. An illustration of the effectiveness of the method is shown in fig.2 in the case of the most relativistic particle Z/A=0.5 and B\_0=31.1 kG. In this case the position of the Walkinshaw resonance is very close to the  $\nu_{\Gamma}=1$  resonance, so that we could measure the effects of latter by observing the behaviour of an off-centered beam in the axial phase space.

In general, we found that in these cases the centering of the beam could be accomplished with acceptable values of the amplitudes and phases of the magnetic bump that are linearly correlated to the parameters of the off-centered beam  $\rho_0,\psi_0.$  Also, as the off-centered

ing is larger, the tolerances on the magnetic bump values were shown to be tighter. However, the radial phase space corresponding to a central ray, although symmetric prior to the  $\nu_{\rm p}=1$  resonance, was significantly distorted after acceleration through this resonance. Adequate behaviour of initially off-centered beam in passing both through the  $\nu_{\rm p}=1$  and  $\nu_{\rm p}=2\nu_{\rm Z}$  resonances could be obtained for a slightly off-centered beam.

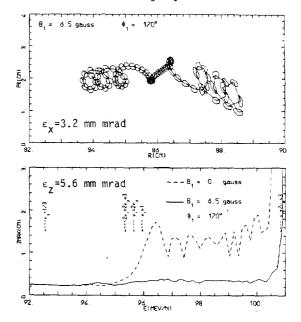


Fig.2 Axial envelope of an off-centered beam with Z/A=0.5 and  $B_0$ =31.3 kG in cases of no magnetic field bump and recentering with a first harmonic amplitude  $B_1$ =6.5 G and  $\phi$ =170 in which case the radial phase space is shown in a)

# Asymmetric Dee-voltage Compensation

Although quite efficient in a number of cases, the magnetic field bump is of no use in recentering of beams for which the Walkinshaw resonance occurs well before the  $v_r=1$  resonance. A similar method, centering of the beam by means of a magnetic bump in the central region, is inapplicable in the case of the Milan superconducting cyclotron, since the beam is injected from a Tandem at radii beyond the  $v_r=1$  resonance in the central region. Consequently, asymmetric dee-voltage compensation was examined as an alternative method, which is known to have a similar effect on an off-centered beam as the first harmonic of the magnetic field.

The beam dynamics in the case of asymmetric dee-voltage recentering may be appreciated by considering the dynamical phase plot, fig.3, where the  $(x, \mathsf{p}_\mathsf{X})$  coordinates are plotted for each turn of acceleration. Unlike the off-centered beam in symmetric accelerating field, which conserves the initial value of off-centering during acceleration, the off-centered beam in an asymmetric dee system evolves towards the AEO with the rate of encounter depending on the energy gain per turn. The trajectory of a beam centered in this manner is given by a cycloid which degenerates into a line converging towards the central ray. One observes from fig.4 that the beam is then well behaved both in the axial and radial phase space.

Assuming constant energy gain per turn, dee voltage perturbations neccessary for recentering of a radially injected beam are related by:

$$\Delta V_{k}/V = \Delta V_{o}/V \sin{\alpha_{o} + (k-1)\frac{2\pi}{N}}$$

where N is the periodicity of the accelerating system