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CHARACTERISTICS AND APPLICATIONS OF RADIATION FROM CHANNELED PARTICLES

R. H. Pantell, H. Park, R. L. Swent, J. O. Kephart and R. K. Klein
Department of Electrical Engineering
Stanford University
Stanford, CA 94305

and

S. Datz
Oak Ridge National Laboratory
Oak Ridge, Tennessee 37830

and

B. L. Berman
Lawrence Livermore National Laboratory
University of California
Livermore, California 94550

Summary

Charged particles channeled in crystals traverse nearly periodic trajectories, which produce radiation that has spectral peaks at photon energies determined by the particle energy and the periodicity of the motion. This effect is analogous to the radiation enhancement that occurs when a magnetic undulator is placed in a storage ring. For γ=10 to 100, the period of the motion is several hundred to several thousand lattice sites in the crystal, so that the forward directed emission is in the soft-to-hard x-ray portion of the spectrum. Channeling radiation can be used to study the channeling phenomenon, to investigate the properties of crystals in which channeling occurs, and as a source of x-rays.

Introduction

Charged particles channeled in crystals, either along an axis or between planes, traverse periodic trajectories with a periodicity that is typically on the order of one hundred to one thousand lattice sites. This motion results in forward directed emission of electromagnetic waves, which, for MeV electrons, has several peaks in the x-ray portion of the spectrum. Channeled particles are trapped by the electrostatic field in a crystal and, from a quantum mechanical point of view, radiation results from spontaneous transitions between eigenstates associated with this potential. From a classical point of view, the acceleration of the particle caused by its periodic channeling motion produces emission, which is reinforced at particular photon energies because of the periodicity of the trajectory.

For the case of planar-channeled positrons, the potential is close to harmonic, resulting in a multiplicity of energy eigenvalues that are nearly equally spaced. The radiation is, therefore, basically at a single frequency. Electrons, on the other hand, are in a planar potential that is exponential in nature, which means that the eigenvalues are not equally spaced and so several radiation frequencies can be generated. The eigenstates for axial-channeled particles have features in common with the eigenstates of electrons around an atomic nucleus, again giving rise to a multiplicity of frequencies.

The energy radiated per unit frequency is up to an order of magnitude larger than the bremsstrahlung generated by the channeled particle, so that for electrons and positrons the effect is readily observable. Heavier particles, such as protons, produce secondary electrons that generate bremsstrahlung in excess of the channeling radiation from the primary particle, which would make observation difficult.

The crystalline potential giving rise to channeling radiation may be calculated from the Hartree-Fock model modified by a Debye-Waller factor to account for thermal vibrations of the atoms. The eigenvalues can be calculated using a many-beam analysis[1], in which Bloch wave eigenfunctions are substituted into the wave equation.

Characteristics of Radiation

Channeling radiation has a number of interesting features concerning its spectrum, polarization, directionality, time structure and intensity.

Energy.

The energy of the radiation emitted by the charged particle is directly related to its periodic motion in the crystal. If the distance of crystal traveled during one cycle of the particle motion in the laboratory frame is λ_1, the wavelength of the radiated electromagnetic wave is

$$\lambda = \lambda_1(1 - \beta\cos\theta) \approx \frac{\lambda_1}{2\gamma^2} \qquad \text{(forward direction)} \tag{1}$$

where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$ and θ is the angle between the particle direction and the observation point.

For example, planar channeled electrons of γ = 100 traverse periodic trajectories with a periodicity of ≈ 2000Å and the radiated wavelength is ≈ 0.1Å which is 124 keV of photon energy. The channeling radiation can be tunable from the soft x-ray to the γ-ray region by changing the energy of the channeled electrons or positrons.

Linewidth.

The linewidth of the radiation depends on various factors. Factors contributing to the linewidth are:

i) Limited Coherence Length Due to Atomic Thermal Vibrations, Crystal Defects, and Electronic Collisions. A radiating particle undergoes transitions to other bound states or to the continuum, thereby causing a discontinuity or termination of the emission. The width is given by

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$$\mathrm{FWHM} \approx \frac{\gamma^2 \lambda}{\pi \ell} \tag{2}$$

where ℓ is the e^{-1} coherence length.

For planar-channeled electrons of $\gamma = 100$ in Si, the coherence length is ≈ 0.5-µm and it contributes about 13% of the linewidth at 0.2Å of channeling radiation wavelength.

ii) Doppler Effect. Non-zero beam divergence and finite detector aperture result in a Doppler shift in the forward directed frequency. The divergence results from the intrinsic beam emittance and from multiple scattering in the crystal. If the multiple-scattering angle in the crystal is θ, the FWHM is

$$\mathrm{FWHM} \approx \gamma^2 (\Delta\theta)^2 \tag{3}$$

The scattering angle $\Delta\theta$ can be calculated from the multiple-Coulomb scattering formula for a nonchanneled particle.[2] For electrons of $\gamma = 100$ in 20-µm thick Si, $\Delta\theta = 2.65$ mrad and it will contribute $\approx 7\%$ of linewidth.

iii) Finite Crystal Thickness L. Because of the restricted length for coherent emission, the width is

$$\mathrm{FWHM} \approx 1.8\, \gamma^2 \cdot \frac{\lambda}{L} \tag{4}$$

If 50.6-MeV ($\gamma = 100$) electrons channel through a 10-µm thick crystal, then the crystal thickness only contributes 1.8% of the linewidth to channeling radiation at 0.1Å of wavelength.

iv) Energy Spread of the Particle Beam.

$$\mathrm{FWHM} \approx 1.5 \cdot \Delta E \tag{5}$$

where ΔE is the energy spread in percent.

v) Bloch-Wave Broadening.

vi) Potential Anharmonicity for Planar-Channeled Positrons.

These factors are not additive contributions, and the quadrature sum of these widths is a better estimate of total width than is the sum of all.

Directionality.

In a reference frame moving at the longitudinal particle velocity the radiation pattern is that of a dipole. In the laboratory frame the emission is highly forward-peaked, approximating a cone in the forward direction with a cone half-angle $\approx 1/\gamma$.

Polarization.

The emission is linearly polarized for radiation from planar-channeled particles. It may be linearly polarized for axial channeling also.

Time Structure.

The time structure of the radiation reproduces the time structure of the particle beam. As an example, for the linear accelerator at Lawrence Livermore National Laboratory (LLNL) this means that emission occurs in ≈ 10 picosecond bursts, 350 picoseconds apart, over 0.1 - 3 microsecond intervals, at a repetition rate of up to 1440/sec.

Intensity.

To determine the intensity of channeling radiation as a source of x-rays it is necessary to consider the maximum tolerable electron current that could be used to generate the x-rays. Three factors are important: the current limitations of an accelerator; heating of the crystal; and the rate of defect formation in the crystal. Based upon these considerations, a reasonable average current is 10-µA in a 5mm diameter beam. This beam can be produced by a linac. The temperature rise in an 18-µm thick Si crystal with radiation cooling is only ≈ 600°C, and there is ≈ 1 chance in 10^3 of a defect forming per hour at an atomic site. This latter condition means that the crystal would have to be rotated to a new position in a time scale of a second; but Si is self-annealing at this temperature, so that defect formation may not be a limitation.

An average current of 100-µA of 54-MeV electrons will give $\approx 10^{10}$ photons/sec in $\approx 15\%$ linewidth at 122 keV. This rate was calculated from the measured photons per electron per electron-volt at the low currents (10^{-10} - 10^{-11}A) used to obtain the present data. The above value is for a single transition, from the first excited state to the ground state for (110) electron planar channeling in 18-µm thick silicon.

Experimental Work

We have observed radiation from both positrons and electrons, for axial and planar channeling in Si[3], Ge[4], diamond[5] and LiF[6] crystals. Other workers have studied diamond[7] Au[8] and Ni[9]. Particle energies in experiments have ranged from 1-MeV to 10 GeV. To observe distinct peaks with reasonable linewidth, one has to use lower Z material. For electrons, $Z \lesssim 20$ is required. The strict requirement may not be needed for positrons, since there is basically one peak in the spectrum. The optimum particle energy is from $\approx$ 1-MeV to $\approx$ 100-MeV for both electrons and positrons.

The electron and positron beams for our experiment were produced at the Lawrence Livermore National Laboratory Electron-Positron Linear Accelerator. The electron energy ranges from 10-MeV to 170-MeV and the maximum average current is 700-µA. The emittance of the electron beam is generally $\lesssim 0.3$ mrad-cm. The detailed experimental set-up is described in Ref. 10.

Figure 1 shows the major interplanar potentials that we have calculated for LiF. The eigenfunctions and energy levels were computed with the many-beam approximation, which takes into account the periodicity of the crystal potential. The Bloch-wave nature of the eigenfunctions becomes apparent only near the tops of the potential wells where the energy levels broaden into bands. The measurements were carried out with a 54.5-MeV ($\gamma = 107.6$) positron beams incident upon LiF crystals having thickness of 25-µm. The channeling-radiation spectra are shown in Figure 2, together with the theoretical predictions based upon the potentials shown in Figure 1. The good agreement of the results of this calculation with the data, especially the more complex (111) planar case, demonstrates the essential correctness of our calculation.

Figure 3 shows the interplanar potentials and eigenvalues calculated for 54.5-MeV electrons channeled in diamond. Note the double well characteristic of the (111) direction, because of the unequally spaced planes, in contrast to the simple (100) and (110) directions. Figure 4 shows the experimental results for the three cases of Figure 3, together with the theoretical predictions of the energies and relative intensities of the spectral lines (in the forward direction), shown as the vertical lines along the abscissae. The calculated and measured peak energies are in good agreement (≲ 2%). Positron channeling radiation was also measured, and compared with the theoretical prediction. But here, the calculated energies are higher than the measured spectral peaks by 5 to 10%.

It is possible that the origin of these discrepancies lies in the calculation of the crystalline potential, which is obtained from a superposition of the potentials for free atoms; the alteration of the potential by the covalent bonding was not considered. For diamond, where four out of six electrons per atom take part in the bonding, this could be a severe effect. However, after making a correction to take into account the non-spherical distribution of electrons in the bonds between the carbon atoms, the calculated peaks are still too high by 4 to 5%.

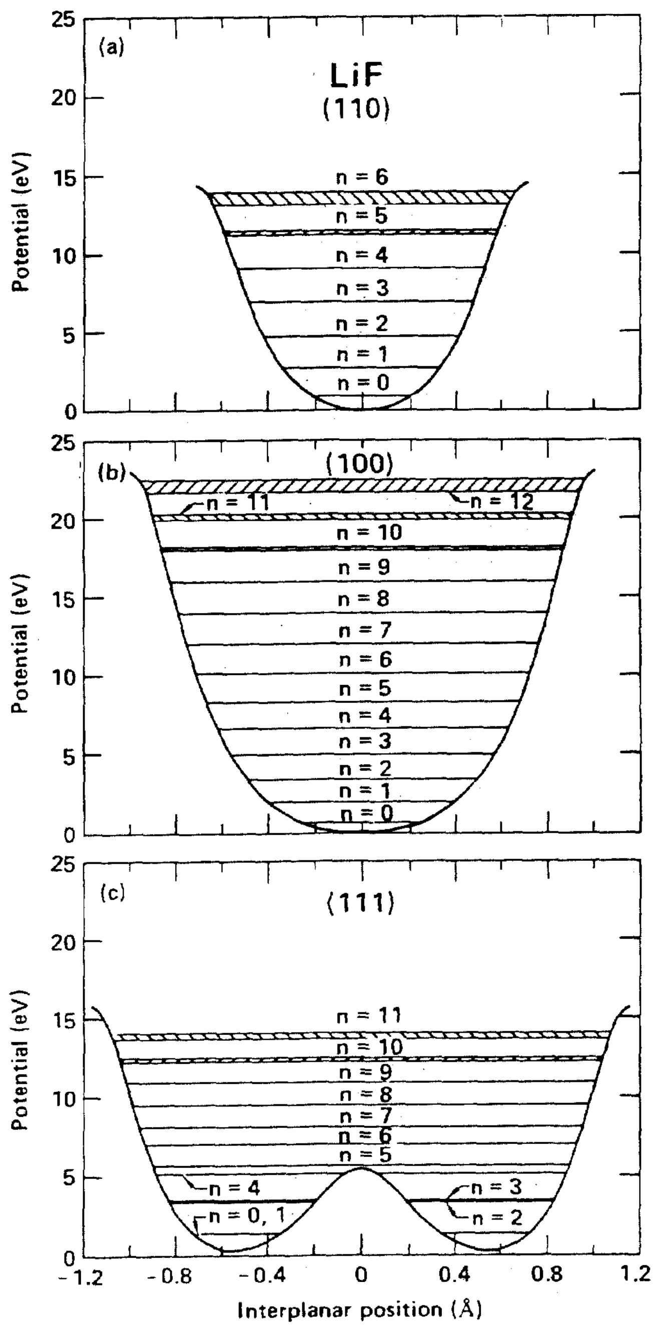

Figure 1. Calculated interplanar potentials for LiF: (a) for the (110) plane, (b) for the (100) plane, (c) for the (111) plane.

This discrepancy may lie in the different regions of the potential being probed by positrons and electrons. Positrons are confined between planes, rather than close to planes as is the case with electrons. Thus, their eigenstates should be more sensitive to the details of the potential in the midplane region. The indication from the lower energy of the positron radiation is that the potential at the midplane is somewhat shallower than anticipated on the basis of the model used. Hence we must conclude that, although the agreement is quite close, a more sophistiated theoretical treatment is needed, at least for the case of positrons.

Applications of Channeling Radiation

Channeling emission can be used as a source of radiation; to investigate various aspects of channeling

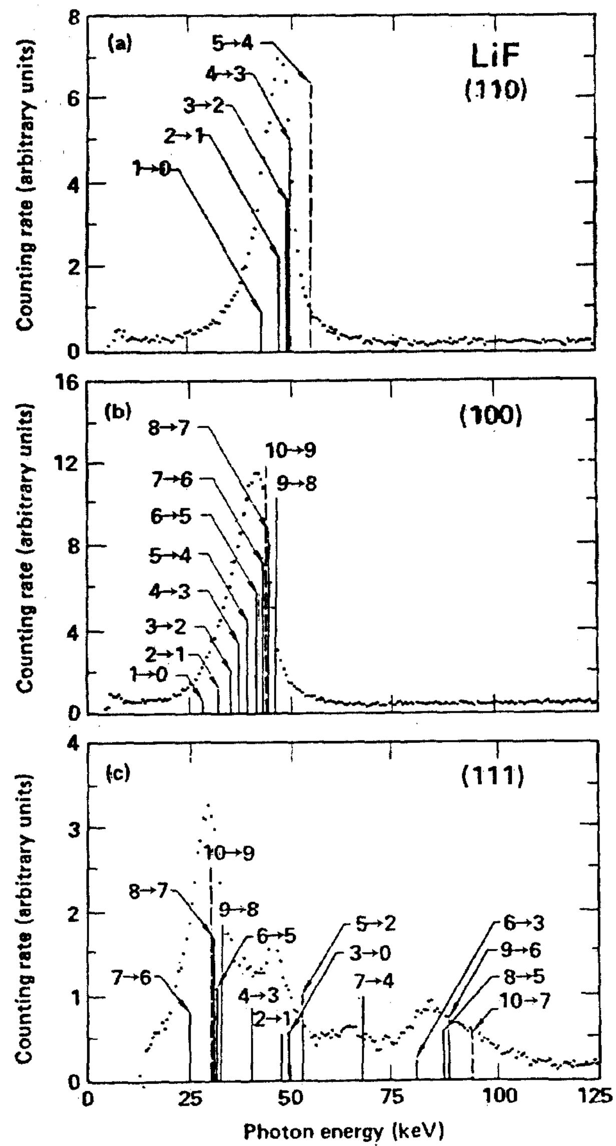

Figure 2. Measured background-corrected radiation spectra from 54-MeV positrons channeled in LiF: (a) along the (110) plane, (b) along the (100) plane, (c) along the (111) plane. The relative intensities of the spectral lines calculated with the potentials of Figure 1, assuming equal initial level populations, are shown as vertical lines. The dashed lines are somewhat uncertain because of the proximity of the initial level to the top of the well and its consequent band broadening.

phenomenon; and to study the properties of crystals in which the particle is channeled.

As an x-ray source, channeling radiation has a number of interesting properties: it is intense, easily tunable, highly directional, 100% plane polarized, and can have picosecond time structure.

Channeling radiation has been used to measure coherence length.[3] Another feature that is being investigated is the occupation length of the various bound states. Occupation length refers to the penetration distance in the crystal for which the occupation probability of the state falls to e^{-1} of its initial value (which is not the same as the coherence length). The distinction arises because the electron or positron oscillates between the bound states such that its mean lifetime in a state can be much longer than the mean coherence time of a state.

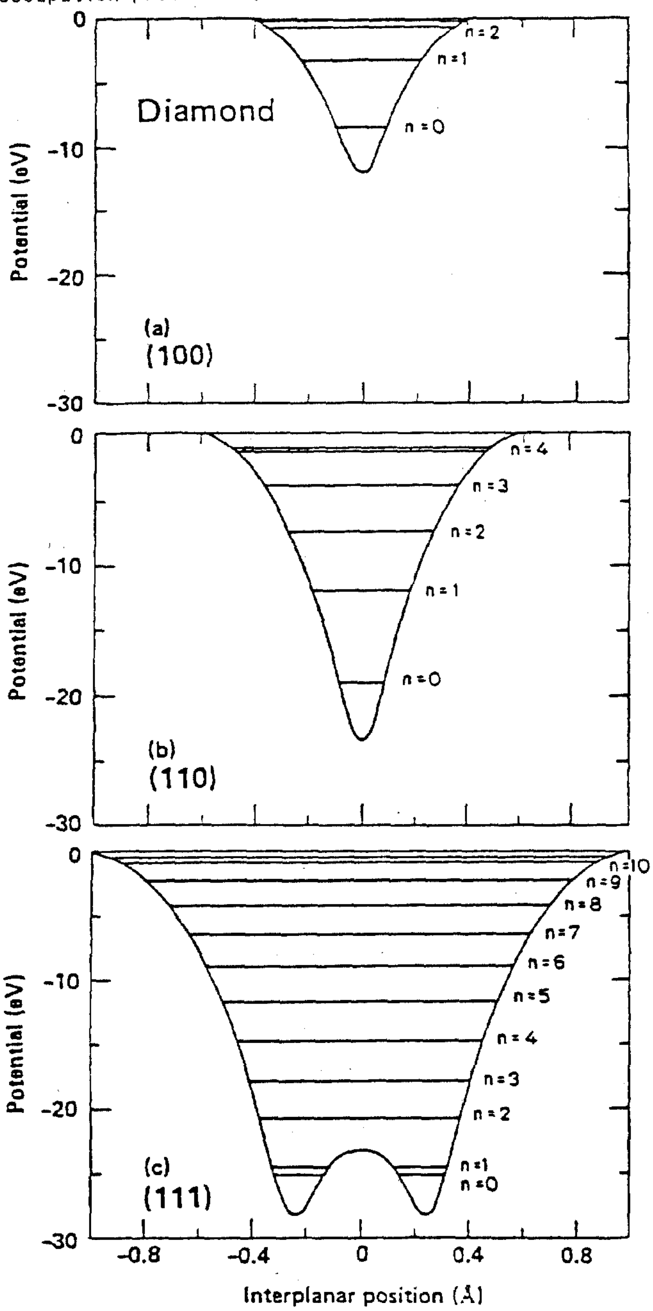

Figure 3. Calculated interplanar potentials and eigenstates for 54.5-MeV electrons channeled along the (a) (100), (b) (110), and (c) (111) planes in diamond. The interplanar spacings are 0.89 Å for (100), 1.26 Å for (110), and 1.54 for the wide (111) and 0.51 Å for the narrow (111). The interplanar position for parts (a) and (b) is measured from the planes and for part (c) from the midpoint between the narrow planes.

A potential application of channeling radiation is to study defects and impurities in crystals. Recently we reported the first observation of the differences in channeling radiation in diamond with and without the presence of platelets.[11] Platelets are clusters of nitrogen impurities present in many natural diamonds, which are in the form of thin disks. These platelets are precipitated on the (100) planes and displace the normal (100) crystal planes by a distance a/3, where a = 3.567Å is the diamond lattice constant. A schematic diagram of such platelet is shown in Figure 5.[12] Channeling radiation is particularly sensitive to this type of defect because the effects are different depending upon the crystal orientation.

Table I summarizes the results for positron planar

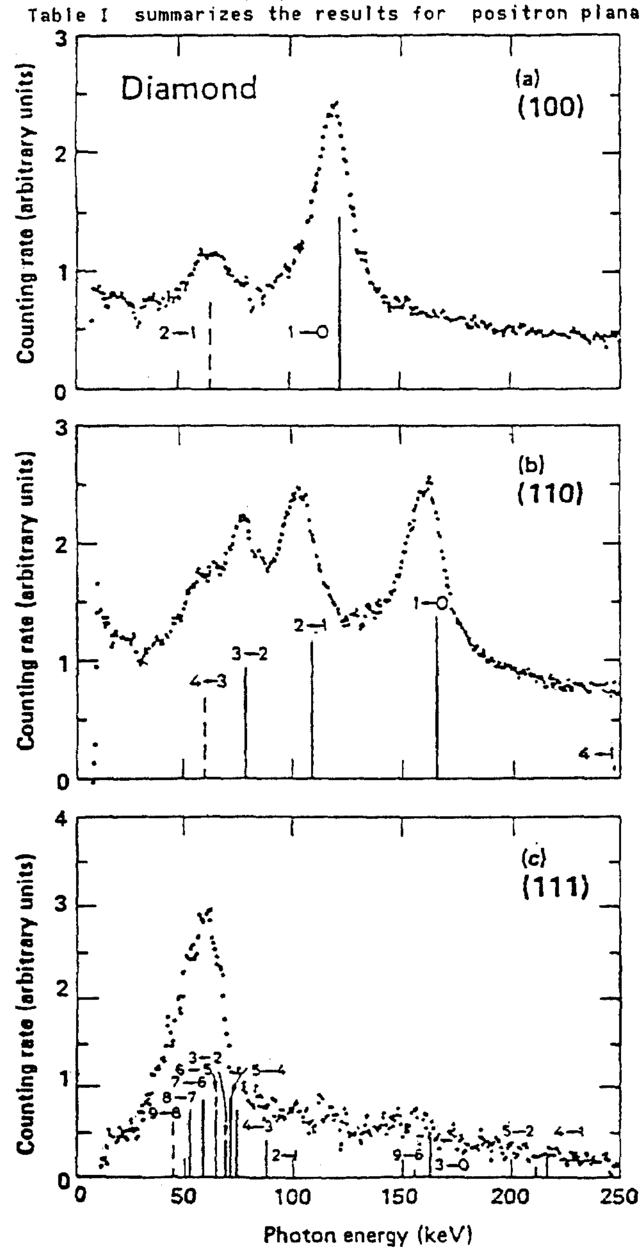

Figure 4. Measured radiation spectra (with random spectra subtracted) for 54.5-MeV electrons channeled along the (a) (100), (b) (110), and (c) (111) planes in a 23-μm-thick natural diamond.

Table I. Measured characteristics of positron channeling radiation from diamond crystals.[a]

Plane	Type IIa			Type Ia		
	Peak energy (keV)	Relative intensity	Width (keV)	Peak energy (keV)	Relative intensity	Width (keV)
(111)	54.5 ± 0.3	0.74 ± 0.03	14.1 ± 0.3	53.7 ± 0.3	0.43 ± 0.02	8.9 ± 0.3
(110)	65.3 ± 0.3	1.00	12.0 ± 0.4	63.2 ± 0.4	0.24 ± 0.02	11.4 ± 0.6
(100)	66.6 ± 0.8	0.32 ± 0.03	17 ± 1	67 ± 4	0.04 ± 0.02	20 ± 6

[a]For a positron energy of 54.4 MeV (γ = 107.4).

channeling, using particles with 54.5-MeV energy. The Type-IIa diamond has no platelets and the Type-Ia diamond contains platelets. For the three major planes the photon energy is constant for both types of diamond, but there are significant differences in intensity and linewidth. For the (100) planes, the emission peak near 67 keV almost disappears when platelets are present. This is because the platelets distort the existing planes and introduce additional planes which interfere with the channeling. For the (110) and (111) planes the platelet appears as a stacking fault, which is far less inhibiting to channeling. Indeed, for the (111) planes there is only ≈ 40% reduction in the intensity of channeling radiation. More noteworthy, however, is the reduction in linewidth from 26% for the Type-IIa diamond to 17% for the Type-Ia one, a result that is contrary to what one would expect from the introduction of a defect. The most likely explanation, which, however, requires further study, is that the stacking fault preferentially removes population from eigenstates that contribute the most towards line-broadening. The results of this measurement raise the possibility that channeling radiation might provide one with a powerful diagnostic tool for impurities and defects in crystals.

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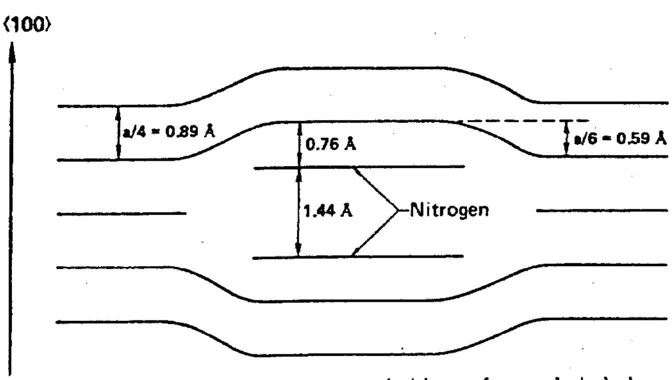

Figure 5. Schematic representation of a platelet in a Type-Ia diamond crystal, according to the model of Ref. 12. The double layer of nitrogen atoms precipitated on the (100) planes constitute a possible scattering center for charged particles channeled along the (100) direction and distort the (100) planes of carbon atoms on both sides of the platelet. The lattice constant a = 3.567 Å

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