

ELECTRICAL BEHAVIOR OF LONG LINAC TANKS AND A NEW TANK-COUPLING SCHEME\*  
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Abstract

The electrical behavior of long linac tanks can be represented by the behavior of a finite chain of coupled harmonic oscillators.<sup>1</sup> A theory is given of chains of oscillators of two kinds, which is appropriate for semi-resonant or resonant coupling structures.<sup>2</sup> The perturbation theory given previously is extended to predict the effects of errors in the coupling cells and in the main cells.<sup>3</sup> Some advantages of  $\pi/2$  mode operation of the double chain are discussed. A new accelerating scheme is presented in which the coupling structures are resonant and the tanks are resonantly coupled. The theory, numerical calculations, and measurements on laboratory models<sup>3</sup> all indicate that tolerances on power amplifier voltage and phase can be relaxed by as much as a factor of five.

Introduction

At Los Alamos for the past three years we have been studying the electrical behavior of the resonant linac tanks made from loaded guide. We have made laboratory measurements of the steady state and transient response of linac tanks containing from one to as many as 40 resonant cells, coupled one to the next. A model has been developed which represents the behavior of such a tank in terms of a chain of coupled oscillators. The model has a very strong resemblance to lattice dynamics, since the individual resonators can be thought of as vibrating atoms, or to electrical network theory, where each cavity is represented by an equivalent circuit. The results relate the frequency spectrum, Q's and wave functions of the tank to the individual properties of the cavities. A perturbation theory has been developed which treats the effects of errors in cavity fabrication and alignment. The predictions and the laboratory measurements are generally in excellent agreement.

In parallel with this approach we wrote a computer program for the MANIAC to solve the network problem represented by the tank. This was quite successful, and produced a wealth of information, again in general agreement.

About a year ago we became interested in the behavior of the very long tanks, driven at a number of points. The MANIAC program became extravagant of computer time for tanks of more than a few hundred cells, and so one of us (BK) devised a program for Stretch, the large computer at Los Alamos, which successfully computed a tank with 8000 cells.

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Analysis of Tank Response

Consider an accelerator tank, with cells  $0, 1, \dots, N$ . For each cavity mode with resonant frequencies  $\omega_0$  there are  $N+1$  modes of the tank with resonant frequencies  $\omega_q$ ,  $q = 0, 1, \dots, N$ . The tank is driven at the point (cell number)  $rn$ . The drive frequency is  $\omega$  and the (complex) amplitude is  $a(rn)$ . The response function or Green's Function<sup>2</sup>, namely the amplitude at the  $p^{\text{th}}$  cell is

$$G = \sum_{q=0}^N \varphi^q(p) \varphi^q(rn) a(rn) d(q) \quad (1)$$

where the  $\varphi$  are the wave functions

$$\varphi^q(p) = \frac{1}{\sqrt{N}} e^{i\pi pq/N} \quad (2)$$

$$\text{and } d = \omega_0^{-2} (\omega_q^{-2} - \omega^{-2})^{-1}$$

The problem will be recognized as the discrete analog of the problem of the response of the plucked violin string, and the solution  $G(p)$  gives the familiar<sup>4</sup> cusp at  $p=rn$ . Now let this structure of tank and drive become one period of a long tank. There are now to be  $\nu$  drives symmetrically located at points  $r(n-\frac{1}{2})$ ,  $n=1 \dots \nu$  ( $\nu r=N$ ).

The response function is obtained from Eq.(1) by summing over the index  $n$ . In the case of identical drives the cusp pattern is repeated to form the scalloped pattern of Fig. 1. However, we will be especially interested in the problem of what happens when the drives differ in amplitude and phase. For this discussion it becomes convenient to express the set of drive amplitudes in terms of their Fourier components  $\alpha(s)$  as follows

$$a(rn) = \frac{1}{\sqrt{\nu}} \sum_{s=0}^{2\nu-1} \alpha(s) e^{-i\pi ns/\nu} \quad (3)$$

Substituting in Eq. (1) the response becomes

$$G(p) = \sum_{s=0}^{2\nu-1} \alpha(s) g_s(p) \quad (4)$$

This expression effectively separates the response into products of Fourier coefficients  $\alpha$  which contain the effects of errors, and functions  $g$  which are independent of errors and susceptible to direct computation. In the example to be discussed below  $\nu=45$ , and  $s=0 \dots 89$ .

The function  $g_s$  can be written more explicitly as follows:

$$g_s = \frac{1}{\sqrt{\nu}} \sum_{q=0}^N d(q) \varphi^q(p) \sum_{n=1}^{\nu} \frac{e}{\sqrt{N}} e^{i\pi \left( \frac{rnq}{rv} - \frac{ns}{v} - \frac{q}{2} \right)}$$

$$g_s = \frac{1}{\sqrt{r}} \sum_{k=0}^{\left(\frac{r}{2}-1\right)} d(s+2kv) \varphi^{s+2kv} \left( p - \frac{r}{2} \right)$$

$$g_s = \frac{1}{\sqrt{Nr}} e^{\frac{i\pi s(p-r/2)}{N}} \sum_{k=0,1,\dots,(r/2-1)} d(s+2kv) e^{\frac{2\pi ikv(p-r/2)}{N}} \quad (5)$$

The coefficient  $g_s(p)$  is the contribution to the response from the  $s$ th component in the drive pattern. In particular the real part of  $g_0$  is the response to a perfect set of drives. Note  $g_0$  has the period  $r$ , and peaks at the drive points  $rn$ , giving the scalloped pattern of Fig. 1. We can compute the  $g_s$  knowing the dispersion curve, which gives the  $d$ 's. If we are on resonance  $g_0$  is the largest,  $g_1$  is smaller, etc. Note that  $g_1$  and  $g_{3s}$  generate the longest wavelength modulation (tilt), and  $g_2$  and  $g_{3s}$  wavelength half as large, etc. This separation of effects should be helpful in adjusting drive levels, since each Fourier component in drive imbalance produces a corresponding component in the imbalance of the accelerator response.

#### Effect of Errors and Fluctuations in the Drive

If the  $a(rn)$  fluctuate randomly about a mean value  $a$ , and the fluctuations in  $a(rn)$  are uncorrelated with the fluctuations in  $a(rm)$   $m \neq n$ , then the  $\alpha(s)$  fluctuate but the  $g_s$  do not. We can write  $\langle x \rangle \equiv$  expectation value of  $x$ . We find

$$\langle G \rangle = \sum_s g_s \langle \alpha(s) \rangle$$

$$\langle \alpha(s) \rangle = a \delta(s)$$

$$\langle |a(n)|^2 \rangle = a^2 (1 + \epsilon^2)$$

$$\langle (G - G^0)^2 \rangle = a^2 \epsilon^2 \sum_s |g_s|^2 \quad (6)$$

Since  $g_s$  contains  $\nu^{-\frac{1}{2}}$  the effect of the mean square error  $\epsilon^2$  is reduced by the factor  $1/\nu$ , and also by the effect of the energy denominator. We estimate from these formulae a suppression of about 10% for  $g_1 \alpha(1)$  over  $g_0$  at resonance for the parameters of Fig. 1. This is about what is seen.

#### Numerical Calculations. General Discussion

The method of computer calculation is described by Lucas, et al.<sup>5</sup> Both short and long tanks have been considered. The results of these computations verify that the  $\pi/2$  mode structure basically is very tolerant of errors,<sup>6,7</sup> in both main and coupling cells. The principal cause of diminished  $Q$  of the tanks is the presence of errors in the main cell resonant frequencies. Errors in the coupling strengths result in local unflattening of the tank. Finally errors in the coupling cell resonant frequencies contribute to displacement of the phases of the fields in the main cells. This separation of effects should be very useful in initial alignment of the tanks. We emphasize that the tolerances are quite generous. These features persist when we consider the very long resonantly coupled tank with multiple drive. The additional desirable features which come from the resonant coupling and the multiple drives are diminished sensitivity to phase and amplitude errors in the drives, and the elimination of power splitters.

The numerical results from Stretch shown in Figs. 1 to 4 were obtained with the following parameters for the accelerator.

TABLE I  
Accelerator Parameters

Coupling strength $k$	= .04
$Q$ of main cells	20,000
$Q$ of coupling cells	10,000
Number of cells $N$	= 7469
Drive frequency $\omega/2\pi$	= 805.0 Mc
$\pi/2$ - mode	
Number of drives $\nu$	= 45
Number of cells/drive $r - 1$	= 165

A possible mechanical arrangement is shown in Fig. 5. Figure 1 shows the computed response for the case of no errors in the drives of the accelerator. The maximum droop in field, shown on the curve as the depth of the scallops is 1.3%. This is perfectly acceptable. The number of cells per drive, namely 165, require exactly one Mw 805 Mc power amplifier per feed point. This means no power splitters between amplifier and accelerator.

Figures 2, 3, and 4 refer to the case of 10% and 10<sup>0</sup> errors in drive phase and amplitude, respectively. In Fig. 2 there are no accelerator errors. The stars plotted below are the drive amplitudes fed into the computation. The upper half of the figure displays a solid curve of main cell amplitudes, showing the effects of the errors. The maximum excursion in amplitude is 1.3%. The long wavelength trend (tilt) is not serious for the motion of the protons. The

wavelength of phase oscillations is about 500 cells, or 3 feedpoints, at 200 Mev, and increases at higher energies.

Figures 3 and 4 refer to 10% and 10 degree errors in drive. The accelerator has random errors of 80kc in the individual cells and 0.1% in coupling strength. The maximum resulting amplitude excursion in Fig. 3 is now 5.8%, and the maximum phase error is 4.1 degrees. The phase response is shown as the solid curve in Fig. 4. The maximum phase error is 6.5 degrees. A peculiarity of Fig. 4 is that the sign of the phase is displayed as positive. Actually the phase changes sign at the points where the curve touches the horizontal axis.

We expect  $g_{15}$  and  $g_{35}$  to be the principal contributors to driving the phase oscillation. These components are suppressed by the action of the energy denominator, since the nearest mode is 15 away from resonance. Indeed, the oscillations with wavelengths of about 500 cells appear in Figs. 2 and 3 to be effectively suppressed.

#### Concluding Remarks

We have discussed the behavior of the long super tank formed by resonantly coupling together a large number of individual tanks. It requires few mechanical changes from the conventional arrangement. At the present time however, the phase and amplitude control by servos looks quite feasible<sup>8</sup> and the solution discussed here is not required.

Phase and amplitude control of the RF system would still be required for the resonantly coupled accelerator, to take care of the effects of beam loading. However, the tolerances are much less stringent, and the control might well be done in the master reference amplifier rather

than at the power amplifier modules. Presumably considerable economies could be realized. A detailed study of costs would be undertaken before choosing one system or the other.

#### References

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- <sup>2</sup>D. E. Nagle, Proceedings of the MURA Conference on Linear Accelerators, Stoughton, Wisconsin, July 20-24, 1964, p21. Available from U. S. Dept. of Commerce, Springfield, Va. T.I.D. 4500.
- <sup>3</sup>E. Knapp, *ibid*, p.31.
- <sup>4</sup>cf. e.g, Webster, "Partial Differential Equations of Mathematical Physics", Teubner, Leipzig; 1933.
- <sup>5</sup>B. Knapp, et. al, "Resonantly Coupled Accelerating Structures for High Current Proton Linacs" (BB-14), this conference.
- <sup>6</sup>D. E. Nagle, P-11/2, Oct. 1963, and P-11/3, Oct. 1964, (unpublished).
- <sup>7</sup>D. E. Nagle, P-11/DN/BK/EK-1, Dec. 1964 (unpublished).
- <sup>8</sup>R. A. Jameson, et. al, (B-8), this conference.

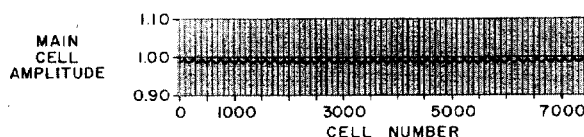


Fig. 1. Response of the accelerating field amplitude in an accelerator of 7470 cells (3600 accelerating cells). The operation is  $\pi/2$ -mode, at resonance, and there are 45 equal drives. There are no errors in the drives or in the accelerator. See Table I for further parameters.

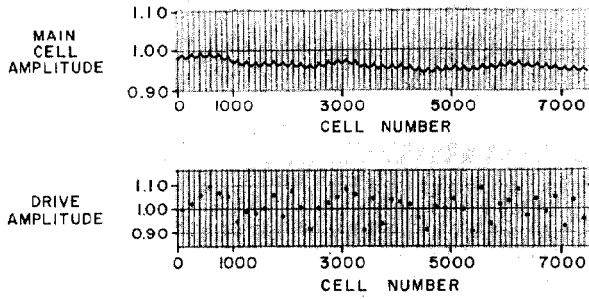


Fig. 2. Response for random errors in the drives of 10% in amplitude and 10 degrees in phase. Accelerator parameters as in Table I. The maximum resulting droop is 1.3%.

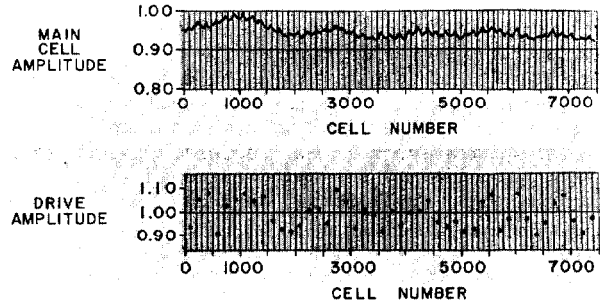
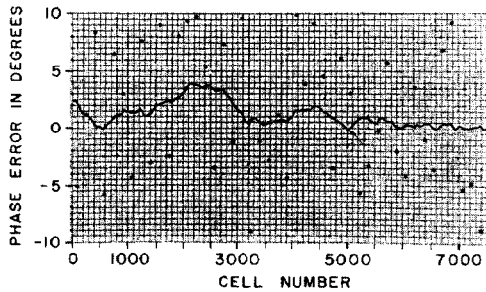
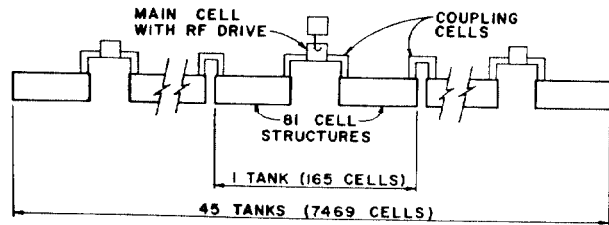


Fig. 3. Amplitude response to random errors in drive of 10% in amplitude and 10 degrees in phase. The accelerator has random errors of 80ke in cell resonant frequencies and 0.01% in coupling strength. The maximum resulting droop is 5.8%.



— PHASE ERROR IN MAIN CELL  
 \* — DRIVE PHASE

Fig. 4. Phase response of the main cells. Errors of 10% and 10 degrees in the drives, of  $10^{-4}$  in the cell frequencies, and  $10^{-3}$  in coupling strength. The maximum resulting phase displacement is 4.1 degrees.



ACCELERATOR CONFIGURATION  
 USED FOR COMPUTER STUDIES

Fig. 5. Layout of resonantly coupled accelerator.