



**Northern Illinois
University**

A HIGH-PRECISION EMISSION COMPUTATIONAL MODEL FOR ULTRACOLD ELECTRON SOURCES

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What is Ultracold?



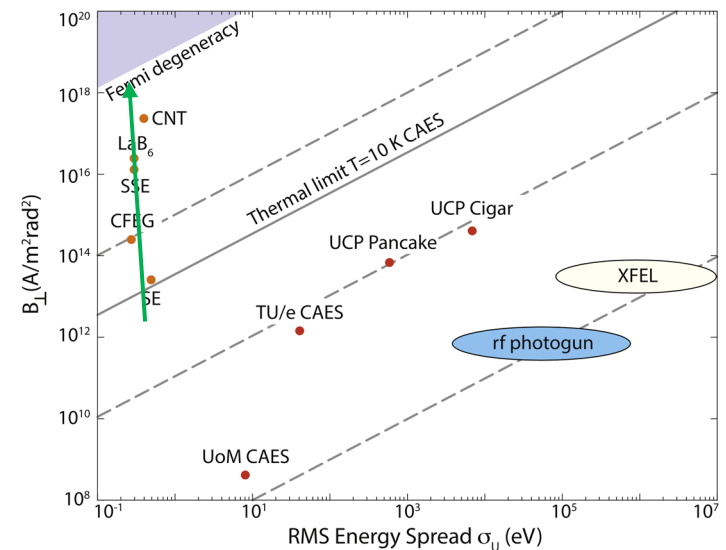
Not specifically a thermal temperature (though they are related)

Near the quantum degeneracy limit

- High Brightness
- Low Emittance

Applications:

- Injectors for XFELs
- High-intensity accelerator sources
- UEM/UED
 - High spatial resolution with short time scales



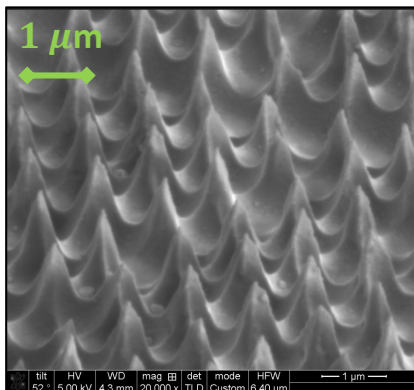
A. McCulloch, J. Phys. B: At. Mol. Opt. Phys. **49** 164004 (2016)

Sharp-tip (nanoscale) emitters have yielded extremely low emittances

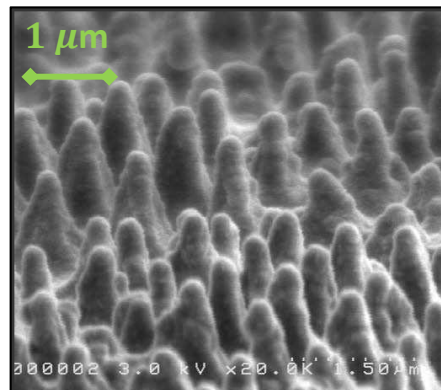
- However, they also have very low current
- Arrays of nano-emitters can yield higher currents
 - Patterned beams
- Single nanotip model in Gmsh
 - Adaptive high-order 2D mesh

C. Geuzaine and J.-F. Remacle, Int. J. Numer. Meth. Eng. **79** 11 (2009).

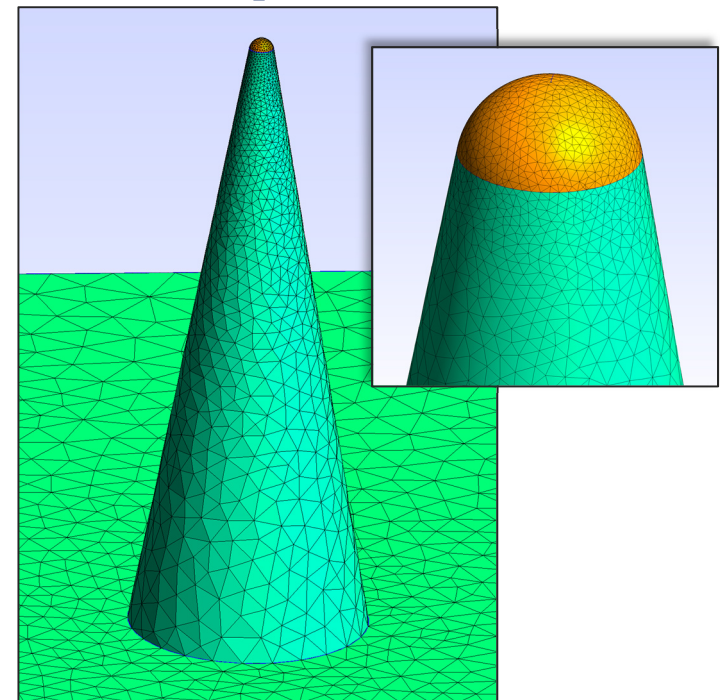
Si Nanotip Array



After UNCD Coating



Nanotip Emitter



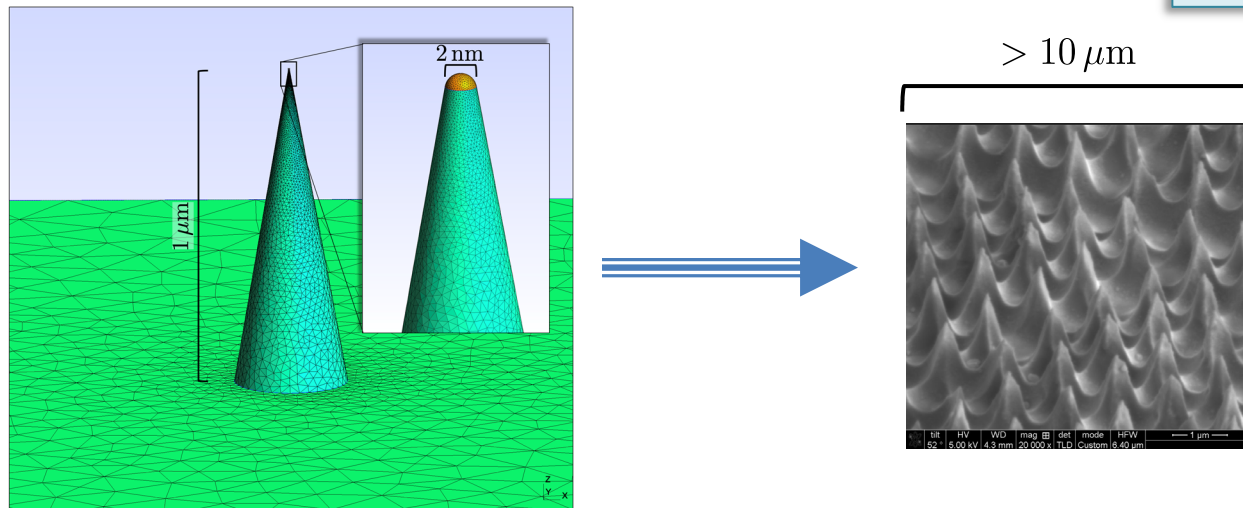
Modelling Limitations



PIC/FEM modelling tools are inefficient

- Needle-shaped tip (nanometer scale variation)
- Array of hundreds of tips (10s of micrometers)

4 orders of magnitude!



- Develop a High-Precision Emission (HiPE) computational model
 - Meet accuracy and efficiency requirements

Simulation Workflow (HiPE)



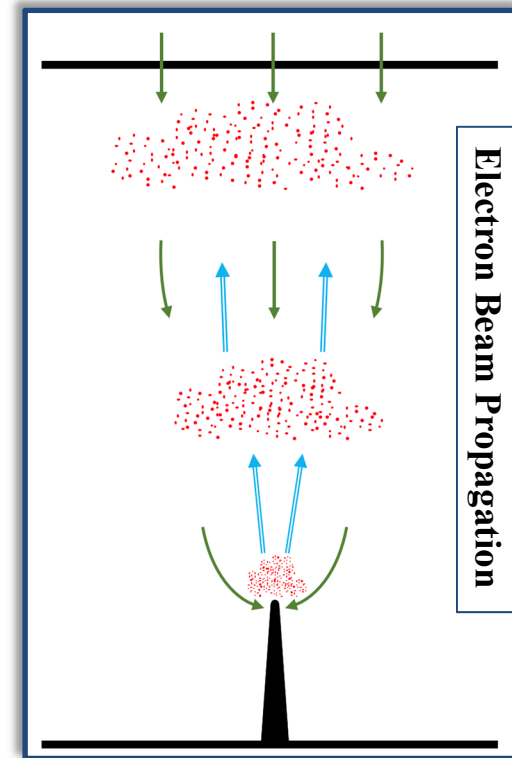
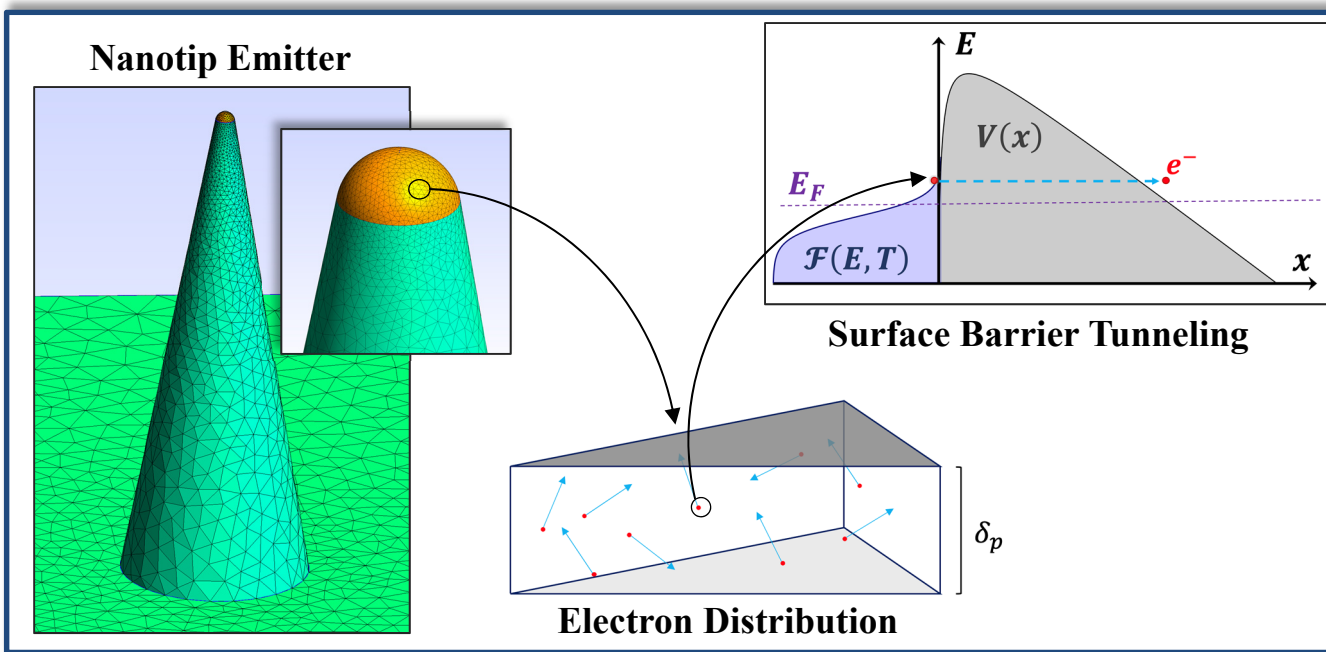
Three fundamental processes:

- Initial Electron Distribution and Transport
- Electron Emission (field)
- Propagation of Electrons
 - Dynamics governed by τ

Programmed in COSY INFINITY:

- Robust suite of beam physics routines
- Language level differential algebra implementation

M. Berz and K. Makino, Michigan State University, Aug. 2017.



Initial Electron Distribution and Transport



Sample the statistical electron distribution

- Electron density (n) given by Fermi-Dirac statistics and the density of states
- Sampling efficiently by factoring in the momentum angle distribution
 - Only sample N relevant electrons
- Parallelizability by parametric mapping from unit triangle to surface element

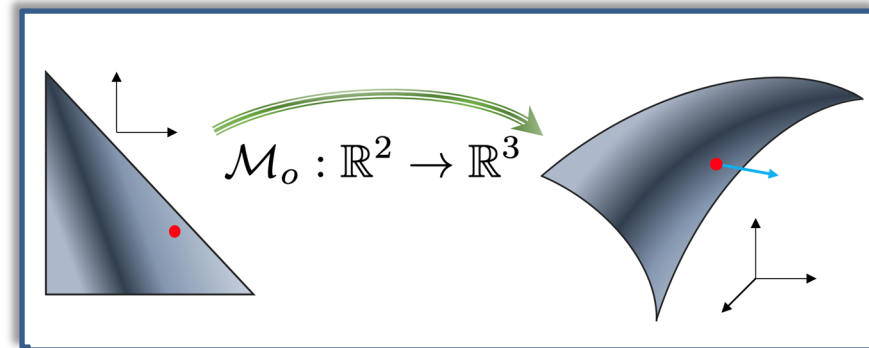
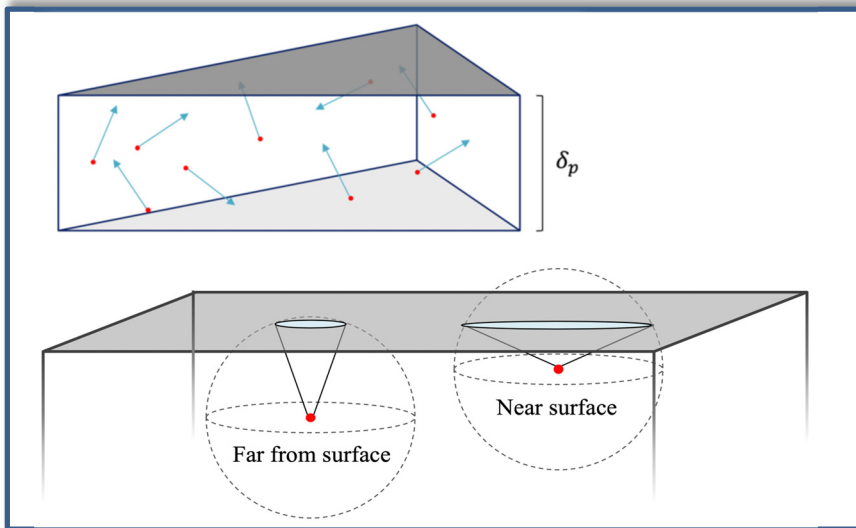
$$\mathcal{F}(E) = \frac{1}{\exp\left[\frac{E-\mu}{k_B T}\right] + 1}$$

$$g(E) = \frac{(2m^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{E - E_c}$$

$$n(E) = g(E)\mathcal{F}(E)$$

$$P(E) = \frac{1}{2} - \sqrt{\frac{mz_i^2}{8E\tau^2}}$$

$$N = \int_0^\infty P(E)n(E)\delta_p\tau dE$$

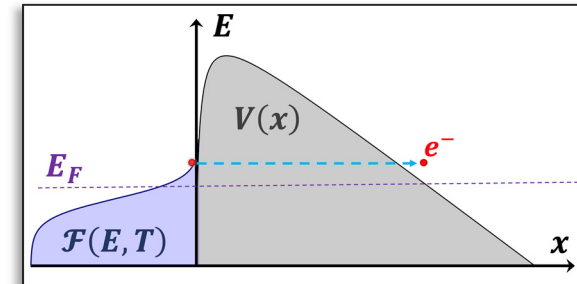


Electron Emission and PISCS



When are electrons emitted?

- Transmission coefficient gives probability that electron tunnels through the surface barrier
 - Depends on the external electric potential

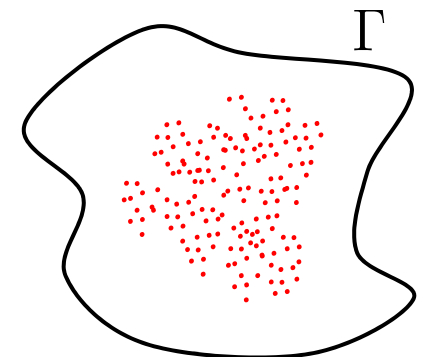
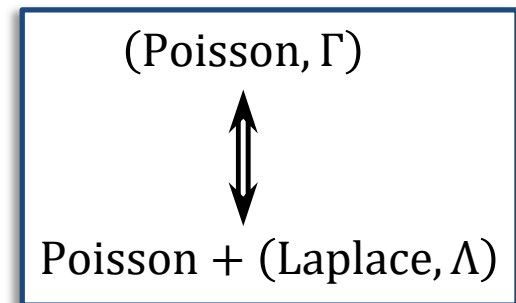


Potentials and fields calculated using PISCS

- Poisson Integral Solver with Curved Surfaces
- Utilizes an adaptive fast multipole method to accurately/efficiently solve the N -body force problem

A. Gee, PhD dissertation, Northern Illinois University, 2018 .

S. Abeyratne, A. Gee, and B. Erdelyi, *Commun. Nonlinear. Sci.* **72** (2019).



$$T(E) = \exp \left[- \frac{2}{\hbar} \int \sqrt{2m(V(x) - E)} dx \right]$$

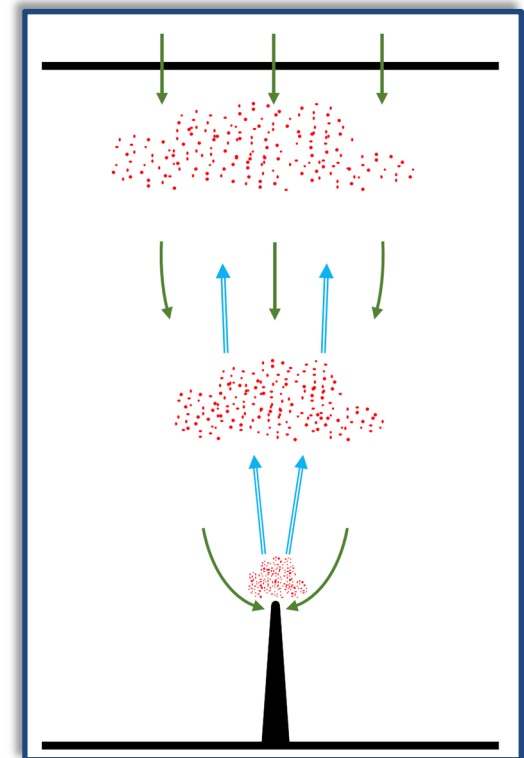
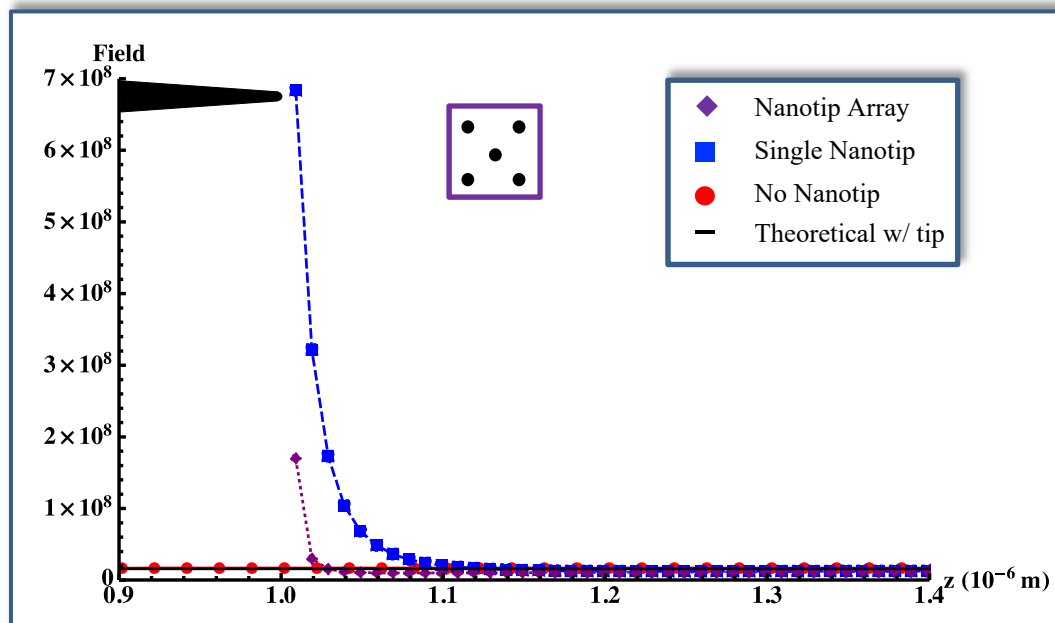
Propagation of Electrons Utilizing SIMO and PISCS



Dynamics of electron beam near nanotip array with SIMO integrator

- Accurate and [relatively] efficient collisional N-body numerical integrator
- Functional form of external fields given by PISCS
 - Series expansion using DA
- Nanotip(s) cause significant field modification

A. Al Marzouk and B. Erdelyi, *SIAM J. Sci. Comput.*, **40** 6 (2018).



Simulating electron emission from a nanotip array is a significant computational challenge!

- Important physical processes scale from
 - $\mathcal{O}(\text{nm})$ to $\mathcal{O}(> 10 \mu\text{m})$
 - $\mathcal{O}(< \text{fs})$ to $\mathcal{O}(\text{ns})$

HiPE is designed to efficiently meet these accuracy requirements

- High-order surface parametrization
- Particle-level model of electron emission
- Accurate computation of electric potentials and fields (PISCS)
- Accurate beam dynamics near cathode (SIMO and PISCS)



Research Group Sponsors



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Sumana Abeyratne

Anthony Gee

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Alister Tencate

Thanks for Your Attention!

Questions?

[Beam Physics Code Repository](https://www.niu.edu/beam-physics-code/projects/index.shtml)

<https://www.niu.edu/beam-physics-code/projects/index.shtml>