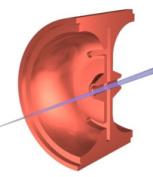




清华大学  
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# The Single-Shot Wakefield Measurement System

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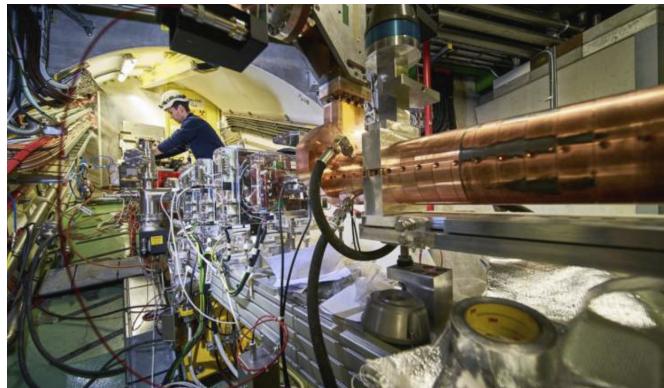
*Shenzhen Minjie Medical Technology Co., Ltd, China*

# Outline

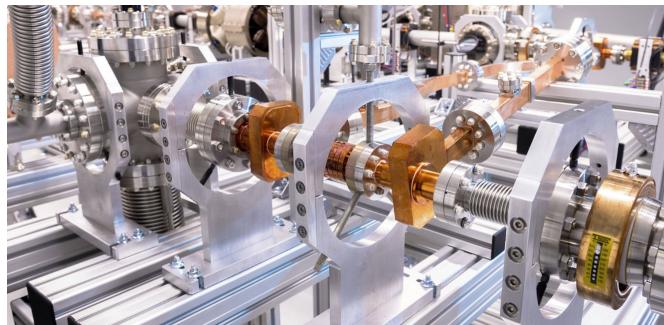
- Introduction and Motivation
- Theory of Single-Shot Wakefield Mapping Method
- Beam Dynamic Simulation
- Experiment Demonstration
- Conclusion

# Introduction and Motivation

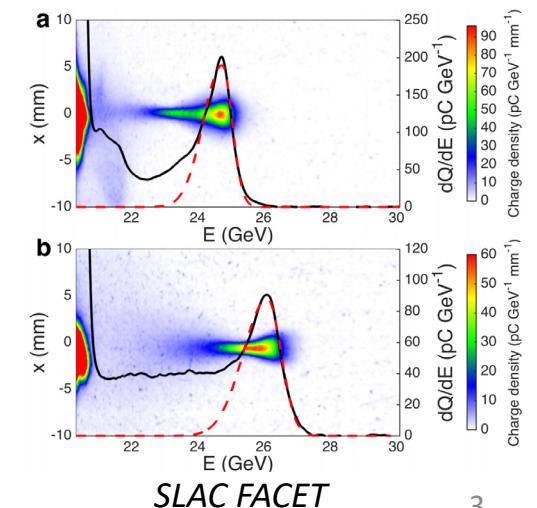
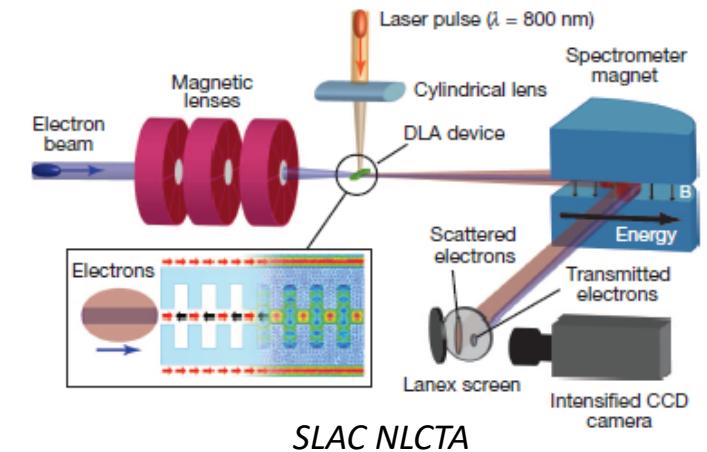
- Wakefield accelerators
  - Driven source
    - Charged beam driven
    - Laser driven
  - Acceleration structure
    - Dielectric
    - Metallic
    - Plasma
  - Acceleration method
    - Collinear
    - Power extraction
  - Benefit
    - High gradient, potential for future colliders.



Cern AWAKE

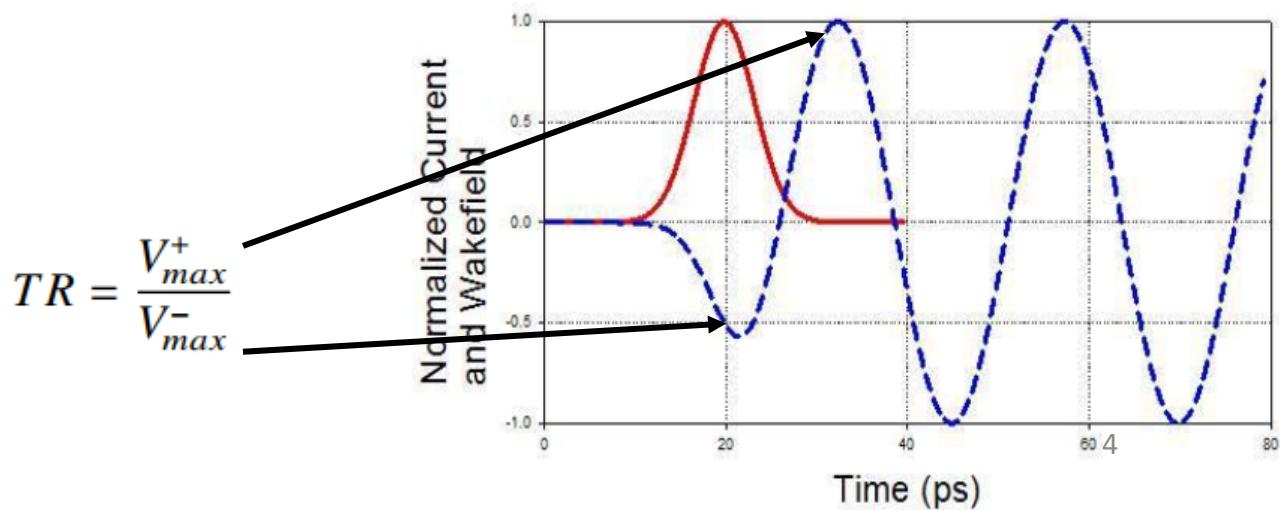
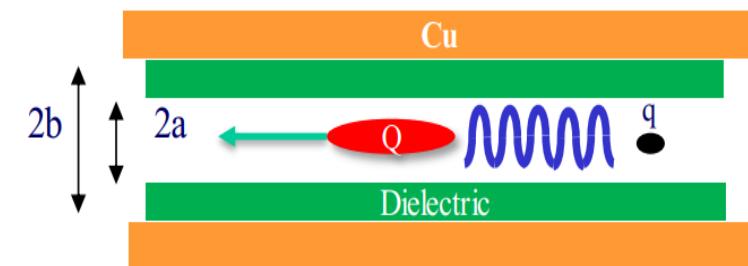


AWA TBA



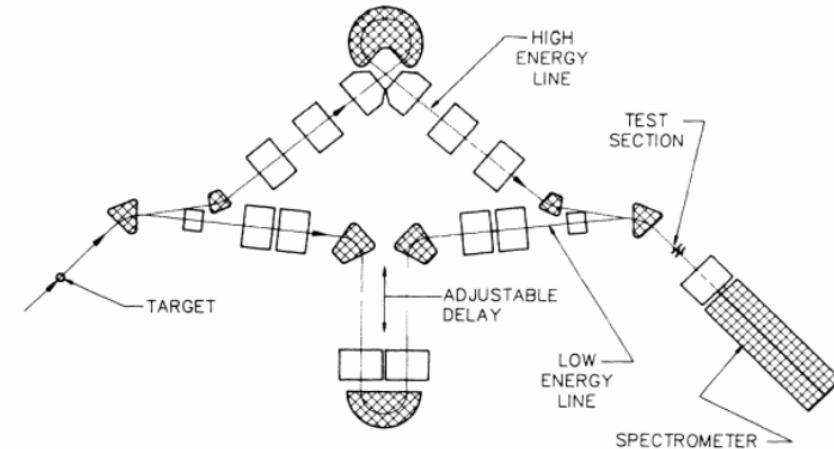
# Introduction and Motivation

- Wakefield characteristics
  - Short pulse ( $\sim$ ns).
  - High frequency, up to terahertz.
  - High gradient, up to GV/m.
- How to Measure the wakefield?
  - Gradient?
  - Transformer ratio?
    - The ratio of peak acceleration field behind drive bunch to the peak deceleration field inside drive bunch.
    - determines the maximum energy gain of the witness bunch when the drive bunch loses all of its energy in a CWA scheme.

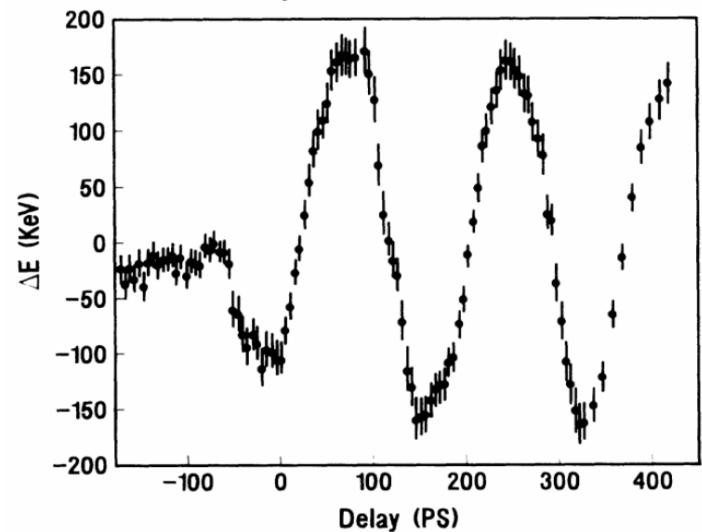


# Introduction and Motivation

- Traditional measurement method
  - Low charge, short witness beam with dedicated delay scanning system. The wakefield gradient can be derived from the witness beam energy gain.
  - Two major drawbacks:
    - The duration of witness beam and the jitter of the delay both smear out the wakefield measurement.
    - The short-range wakefield (inside drive bunch) is especially difficult to measure.
- Motivation
  - Develop a more precise/convenient wakefield measurement system without scanning tool.
  - Capability of measuring both the short and long range wakefield.



\*Figueroa H, et al. 1988

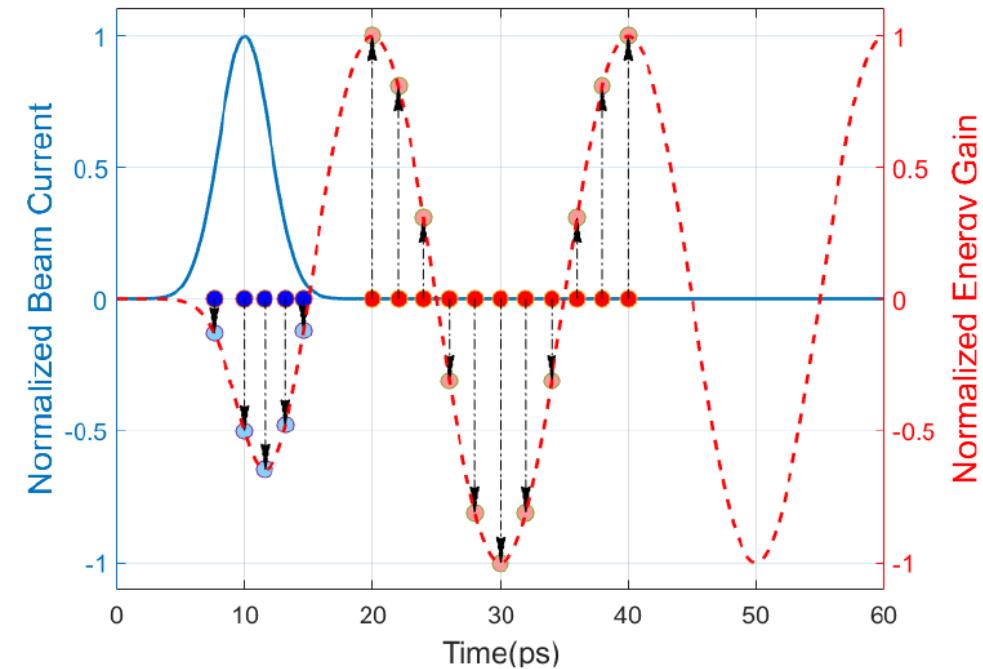


# Outline

- Introduction and Motivation
- Theory of Single-Shot Wakefield Mapping Method
- Beam Dynamic Simulation
- Experiment Demonstration
- Conclusion

# Theory of Single-Shot Wakefield Mapping Method

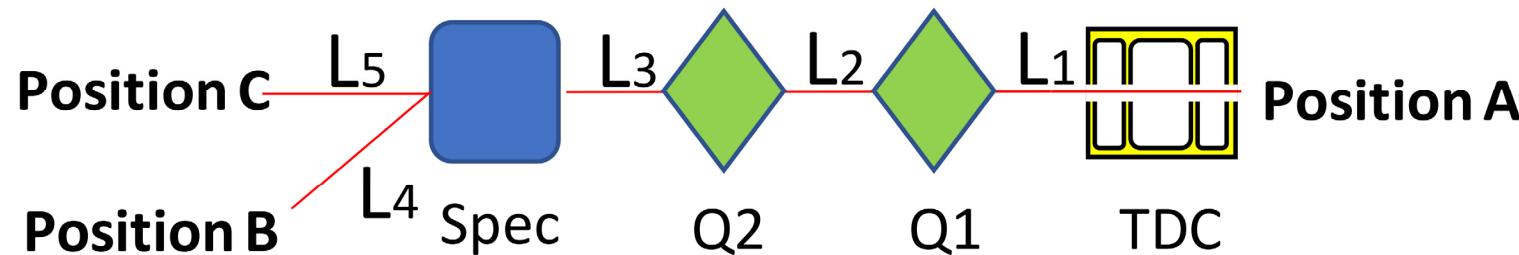
- Main idea of single-shot mapping method
  - long witness bunch instead of scanning.
  - longitudinal phase space measurement instead of sole spectrum measurement.
- Long witness bunch and its interaction with wakefield
  - Witness length comparable to one wavelength (region of interest).
  - Energy gain of drive and witness at each time slice represents the wakefield.



*Illustration of the interaction between drive and long witness. Solid blue line indicates the bunch current profile, dashed red line is its wakefield. The navy blue/sky blue dots represent the electrons inside drive bunch with/without wakefield, the cardinal/light red dots are the electron inside a long witness bunch with/without wakefield.*

# Theory of Single-Shot Wakefield Mapping Method

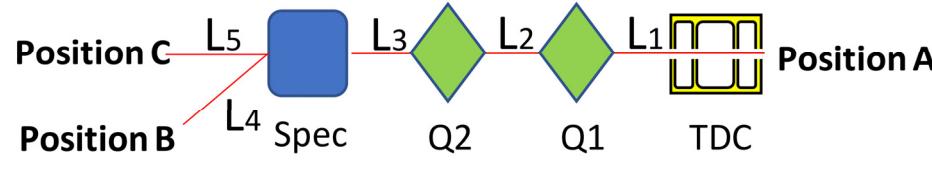
- Longitudinal phase space mapping
  - Spectrometer bends the beam in horizontal direction (x).
  - Transverse deflecting cavity kicks beam in vertical direction (y).
  - Quadrupoles are used to focus the beam to improve the resolution.
  - Energy is projected into x direction and time is projected into y direction.



LPS measurement system layout, Spec, Q, TDC, L stand for spectrometer, quadrupole, transverse deflecting cavity and drift with length L. Position A represents the location at beginning, B and C refer to beam observation stations at bending and straight section after spectrometer, separately.

# Theory of Single-Shot Wakefield Mapping Method

- Longitudinal phase space mapping – thin-lens first order beam dynamics



- Transport matrix from A to B:

$$M_{AB} = M_D(L_4) \cdot M_{Sf}(\theta) \cdot M_S(\theta) \cdot M_{Sf}(0) \\ \cdot M_D(L_3) \cdot M_Q(f_2) \cdot M_D(L_2) \\ \cdot M_Q(f_1) \cdot M_D(L_1) \cdot M_T(0)$$

$$= \begin{bmatrix} R_{11B} & R_{12B} & R_{13B} & 0 & 0 & R_{16B} \\ R_{21B} & R_{22B} & R_{23B} & 0 & 0 & R_{26B} \\ 0 & 0 & R_{33B} & R_{34B} & R_{35B} & 0 \\ 0 & 0 & R_{43B} & R_{44B} & R_{45B} & 0 \\ R_{51B} & R_{52B} & R_{53B} & 0 & 1 & R_{56B} \\ 0 & 0 & \kappa & 0 & 0 & 1 \end{bmatrix}$$

- Projection formula:

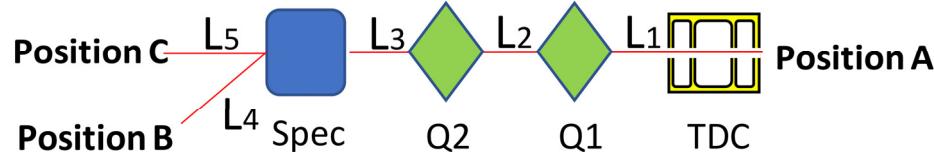
$$\begin{cases} x_B &= R_{11B}x_A + R_{12B}x'_A + R_{13B}y_A + R_{16B}\delta_A \\ y_B &= R_{33B}y_A + R_{34B}y'_A + R_{35B}z_A \end{cases}$$

- Optimization goal:

- ✓ Maximize the factor of  $\delta_A$  and  $z_A$ .
- ✓ Minimize the influence of initial transverse phase space ( $x_A, x'_A, y_A, y'_A$ ) by  $Q_1$  and  $Q_2$ .
- ✓  $R_{13B}$  and  $R_{33B}$  can not be optimized to the minimum value at the same time so that a slit with a small vertical opening is needed to locate at Position A to eliminate the influence of  $R_{13B}y_A$ .
- How to optimize  $Q_1$  and  $Q_2$  in practical?

# Theory of Single-Shot Wakefield Mapping Method

- Longitudinal phase space mapping – thin-lens first order beam dynamics



- Transport matrix from A to C with TDC and Spec off:

$$M_{AC} = M_D(L_5) \cdot M_D(b) \cdot M_D(L_3) \cdot M_Q(f_2) \\ \cdot M_D(L_2) \cdot M_Q(f_1) \cdot M_D(L_1)$$

$$= \begin{bmatrix} R_{11C} & R_{12C} & 0 & 0 & 0 & 0 \\ R_{21C} & R_{22C} & 0 & 0 & 0 & 0 \\ 0 & 0 & R_{33C} & R_{34C} & 0 & 0 \\ 0 & 0 & R_{43C} & R_{44C} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- If  $L_4$  and  $L_5$  satisfy the equation:

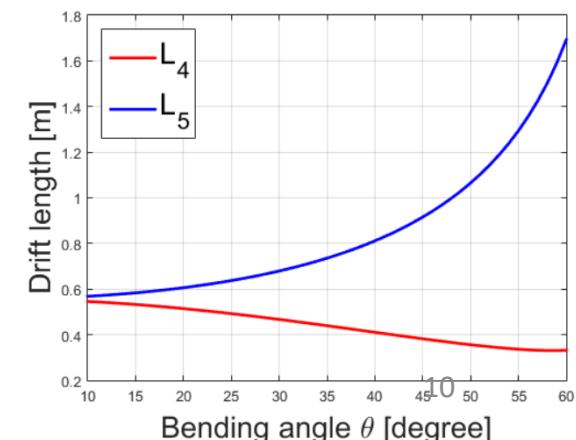
$$\begin{cases} L_5 = \frac{L_4}{\cos^2 \theta} + \rho \tan \theta - \rho \sin \theta \\ L_5 = \frac{\rho[\theta \rho + L_4(1 - \theta \tan \phi_e)]}{\rho(1 - \theta \tan \phi_i) - L_4[\tan \phi_i + \tan \phi_e(1 - \theta \tan \phi_i)]} - \rho \sin \theta \end{cases}$$

- Thus,

- The equation on the right indicates that no matter how to select the bending angle of spectrometer, once the  $L_4$  and  $L_5$  have the specific relationship, the settings of  $Q_1$  and  $Q_2$  to achieve the smallest beam size at position C, with TDC and SPEC off, is the optimized setting of  $Q_1$  and  $Q_2$  to minimize the initial transverse phase space influence at Position B while TDC and SPEC are on.

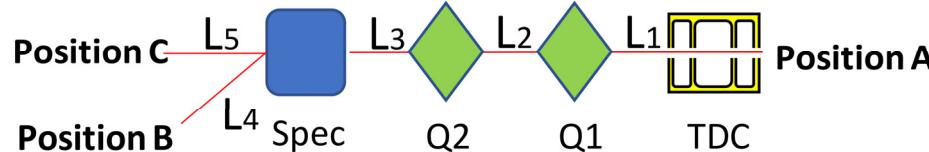
$$\begin{cases} R_{11B} = \zeta_x R_{11C}; R_{12B} = \zeta_x R_{12C} \\ R_{33B} = \zeta_y R_{33C}; R_{34B} = \zeta_y R_{34C} \end{cases}$$

$$\begin{cases} \sigma_{x_B}^2 = \boxed{\zeta_x^2 \sigma_{x_C}^2} + R_{16B}^2 \sigma_{\delta_A}^2 + R_{13B}^2 \sigma_{y_A}^2 \\ + 2R_{11B} R_{16B} \sigma_{x_A \delta_A} + 2R_{12B} R_{16B} \sigma_{x'_A \delta_A} \\ + 2R_{11B} R_{13B} \sigma_{x_A y_A} + 2R_{12B} R_{13B} \sigma_{x'_A y_A} \\ + 2R_{13B} R_{16B} \sigma_{y_A \delta_A} \\ \sigma_{y_B}^2 = \boxed{\zeta_y^2 \sigma_{y_C}^2} + R_{35B}^2 \sigma_{z_A}^2 \\ + 2R_{33B} R_{35B} \sigma_{y_A z_A} + 2R_{34B} R_{35B} \sigma_{y'_A z_A} \end{cases}$$



# Theory of Single-Shot Wakefield Mapping Method

- Longitudinal phase space mapping – thick-lens first order beam dynamics



- Considering the non-zero length of TDC and quadrupoles, transport matrix from A to B:

$$M_{AB} =$$

$$\begin{bmatrix} R_{11_B} & R_{12_B}^* & R_{13_B} & R_{14_B}^* & R_{15_B}^* & R_{16_B} \\ R_{21_B} & R_{22_B}^* & R_{23_B} & R_{24_B}^* & R_{25_B}^* & R_{26_B} \\ 0 & 0 & R_{33_B} & R_{34_B}^* & R_{35_B}^* & 0 \\ 0 & 0 & R_{43_B} & R_{44_B}^* & R_{45_B}^* & 0 \\ R_{51_B} & R_{52_B}^* & R_{53_B} & R_{54_B}^* & R_{55_B}^* & R_{56_B} \\ 0 & 0 & \kappa & \frac{\kappa L_T}{2} & \frac{\kappa^2 L_T}{4} & 1 \end{bmatrix}$$

- Applying the relationship between  $L_4$  and  $L_5$ :

$$\left\{ \begin{array}{l} \sigma_{x_B}^2 = \zeta_x^2 \sigma_{x_C}^2 + R_{13_B}^2 \sigma_{y_A}^2 + R_{14_B}^{*2} \sigma_{y'_A}^2 + R_{15_B}^{*2} \sigma_{z_A}^2 + R_{16_B}^2 \sigma_{\delta_A}^2 \\ + 2R_{11_B} R_{16_B} \sigma_{x_A \delta_A} + 2R_{12_B}^* R_{16_B} \sigma_{x'_A \delta_A} + 2R_{13_B} R_{16_B} \sigma_{y_A \delta_A} + 2R_{14_B}^* R_{16_B} \sigma_{y'_A \delta_A} \\ + 2R_{15_B}^* R_{16_B} \sigma_{z_A \delta_A} + 2R_{11_B} R_{15_B}^* \sigma_{x_A z_A} + 2R_{12_B}^* R_{15_B}^* \sigma_{x'_A z_A} \\ + 2R_{13_B} R_{15_B}^* \sigma_{y_A z_A} + 2R_{14_B}^* R_{15_B}^* \sigma_{y'_A z_A} + 2R_{11_B} R_{14_B}^* \sigma_{x_A y'_A} + 2R_{12_B}^* R_{14_B}^* \sigma_{x'_A y'_A} \\ + 2R_{13_B} R_{14_B}^* \sigma_{y_A y'_A} + 2R_{11_B} R_{13_B} \sigma_{x_A y_A} + 2R_{12_B}^* R_{13_B} \sigma_{x'_A y_A} \\ \sigma_{y_B}^2 = \zeta_y^2 \sigma_{y_C}^2 + R_{35_B}^{*2} \sigma_{z_A}^2 \\ + 2R_{33_B} R_{35_B}^* \sigma_{y_A z_A} + 2R_{34_B}^* R_{35_B}^* \sigma_{y'_A z_A} \end{array} \right.$$

- The equation above illustrates that:
  - ✓ The energy and temporal resolution is affected by the spectrometer bending angle and deflecting cavity kick strength.
  - ✓ The finite length of deflecting cavity deteriorates the resolution of energy measurement.

# Theory of Single-Shot Wakefield Mapping Method

- Longitudinal phase space mapping – thick-lens first order beam dynamics

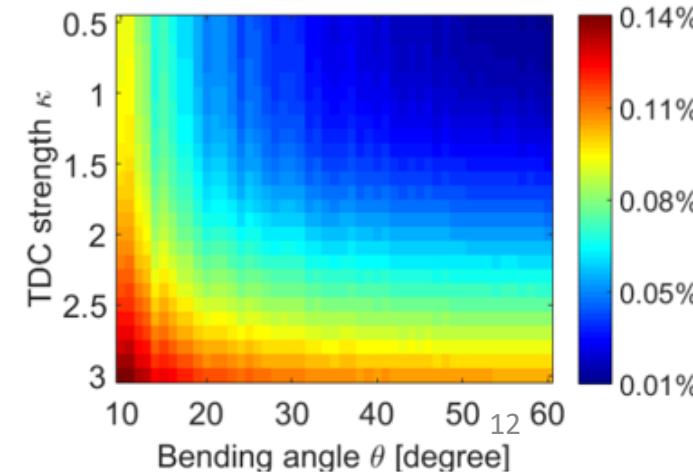
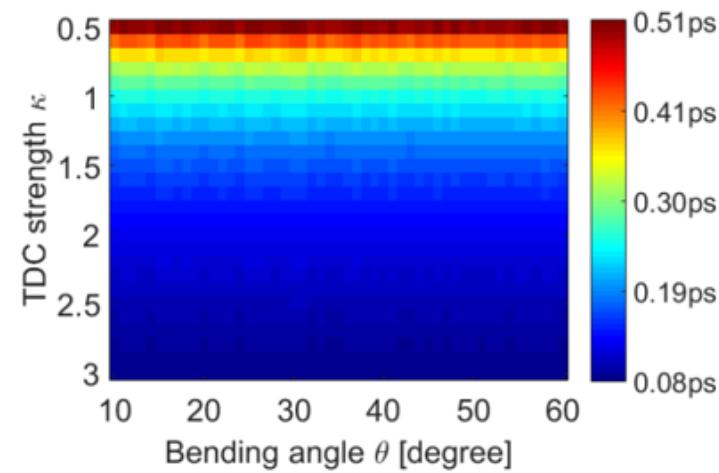
- resolution:

$$\begin{cases} \sigma_\delta &= \frac{\sigma_{x_{other}}}{R_{16B}^*} \\ \sigma_t &= \frac{\sigma_{y_{other}}}{v_c R_{35B}^*} \end{cases}$$

- “Other” refer to the terms contributing to the beam size other than energy or time.

$$\left\{ \begin{array}{l} \sigma_{x_{other}}^2 = \zeta_x^2 \sigma_{x_C}^2 \\ \quad + R_{13B}^2 \sigma_{y_A}^2 + R_{14B}^{*2} \sigma_{y'_A}^2 + R_{15B}^{*2} \sigma_{z_A}^2 \\ \quad + 2R_{11B} R_{15B}^* \sigma_{x_A z_A} + 2R_{12B}^* R_{15B}^* \sigma_{x'_A z_A} + 2R_{13B} R_{15B}^* \sigma_{y_A z_A} + 2R_{14B}^* R_{15B}^* \sigma_{y'_A z_A} \\ \quad + 2R_{11B} R_{14B}^* \sigma_{x_A y'_A} + 2R_{12B}^* R_{14B}^* \sigma_{x'_A y'_A} + 2R_{13B} R_{14B}^* \sigma_{y_A y'_A} \\ \quad + 2R_{11B} R_{13B} \sigma_{x_A y_A} + 2R_{12B}^* R_{13B} \sigma_{x'_A y_A} \\ \sigma_{y_{other}}^2 = \zeta_y^2 \sigma_{y_C}^2 \end{array} \right.$$

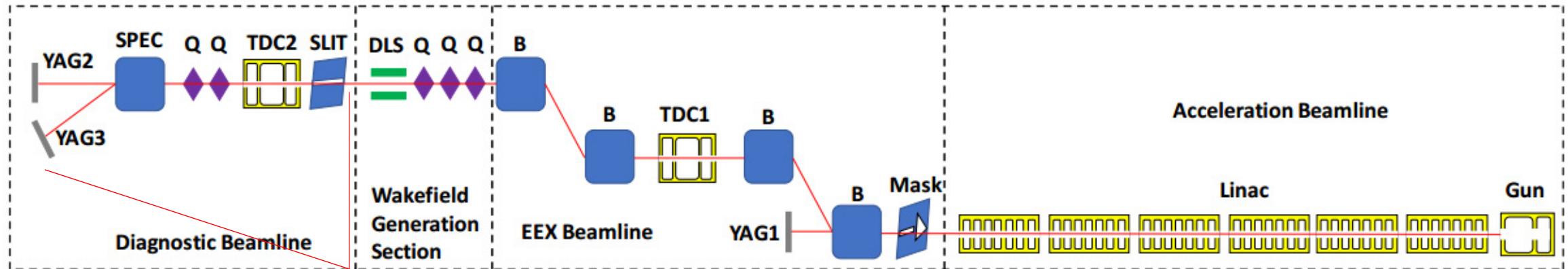
- The bending angle  $\theta$  and deflecting kick strength  $\kappa$  affect the resolution significantly.
- As stated before, a slit with a small vertical opening is needed to eliminate the affection on energy measurement coming from initial vertical phase space.
- Using the beam parameters of Argonne Wakefield Accelerator, the resolution can be numerically calculated.
- For the actual  $\theta$  and  $\kappa$  of AWA, the resolution of energy and time is 0.06% and 0.124 ps.



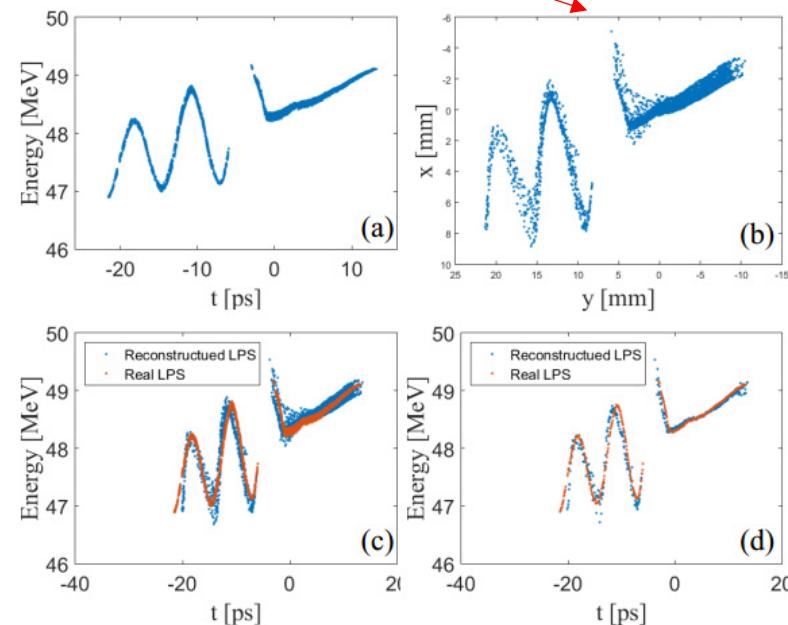
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- Introduction and Motivation
- Theory of Single-Shot Wakefield Mapping Method
- Beam Dynamic Simulation
- Experiment Demonstration
- Conclusion

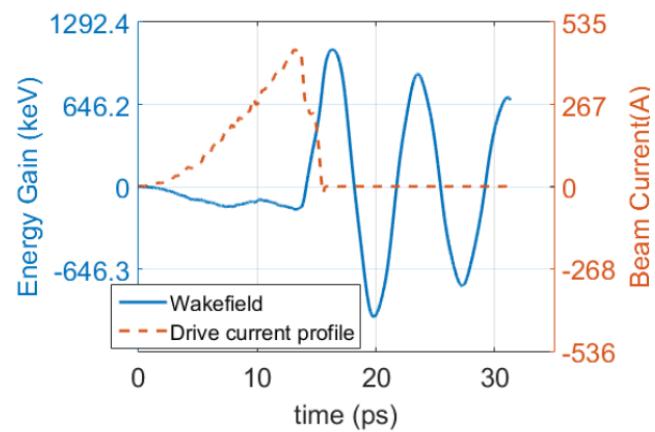
# Beam Dynamic Simulation



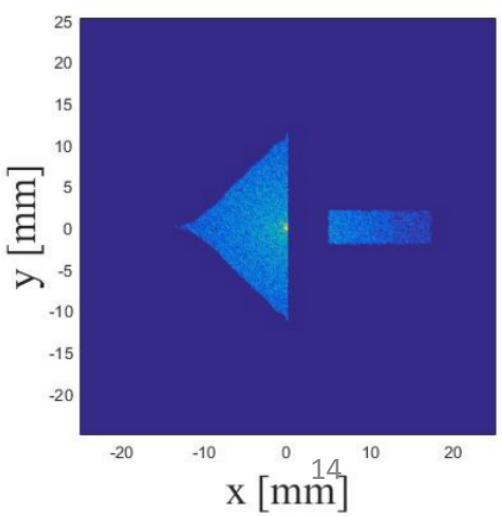
(a) Longitudinal phase space of drive and witness beam behind the wakefield generation section.  
(b) x-y plane beam image at YAG3 beam observation station.  
(c) Comparison between reconstructed and real LPS.  
(d) Comparison of mean LPS profile.



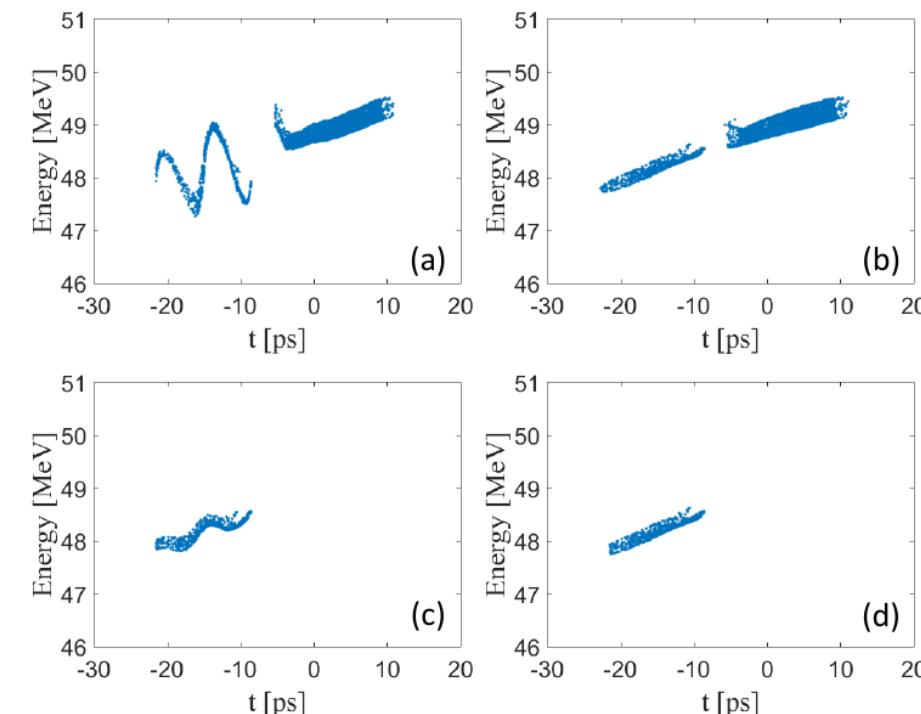
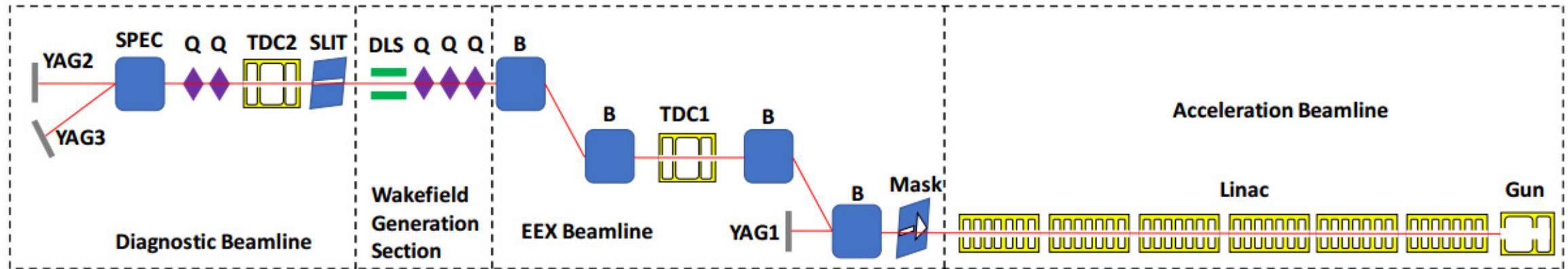
- Drive bunch longitudinal profile and its wakefield in dielectric-lined slab.



- Beam intercepted by the mask to generate drive bunch and long witness bunch in transverse plane

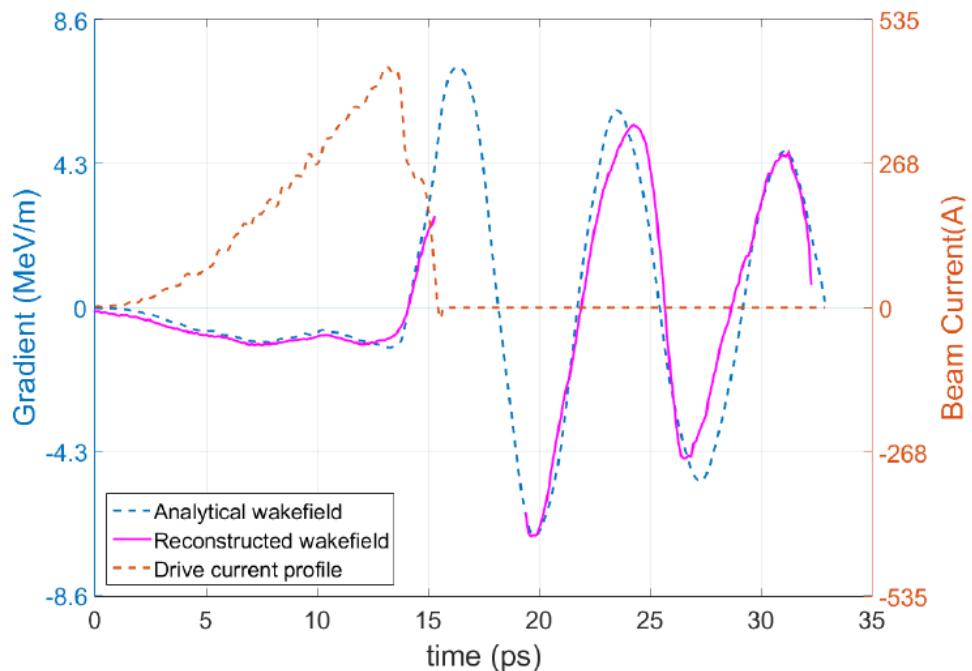
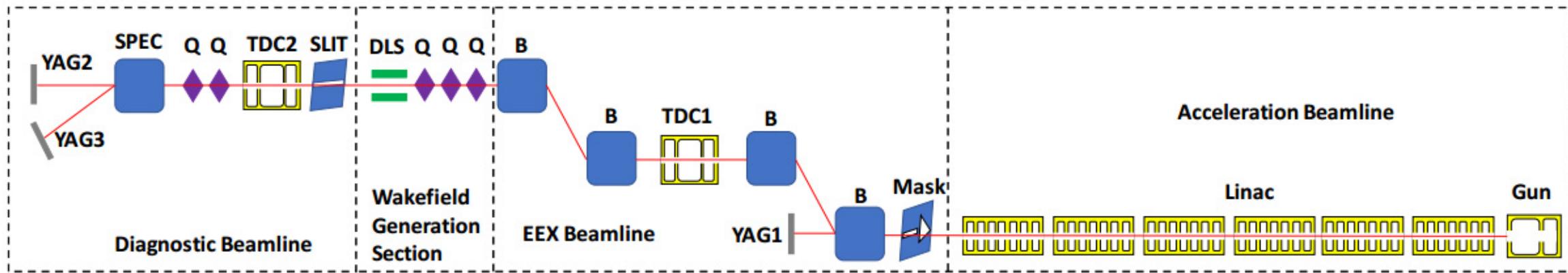


# Beam Dynamic Simulation



- Wakefield reconstruction method in the simulation:
  - ✓ Take the beam x-y plane image at YAG3 position when the condition is:
    - (a) both drive and witness beam exist, turn on wakefield effect of the dielectric-lined slab (DLS).
    - (b) both drive and witness beam exist, turn off wakefield effect of the DLS.
    - (c) block the drive beam, turn on wakefield effect of the DLS.
    - (d) block the drive beam, turn off wakefield effect of the DLS.
  - ✓ Take the energy gain between (c) and (d) to derive the self-wakefield of witness beam.
  - ✓ Take the energy gain between (a) and (b) with the subtraction of witness beam self-wakefield, the drive beam wakefield is achieved.

# Beam Dynamic Simulation

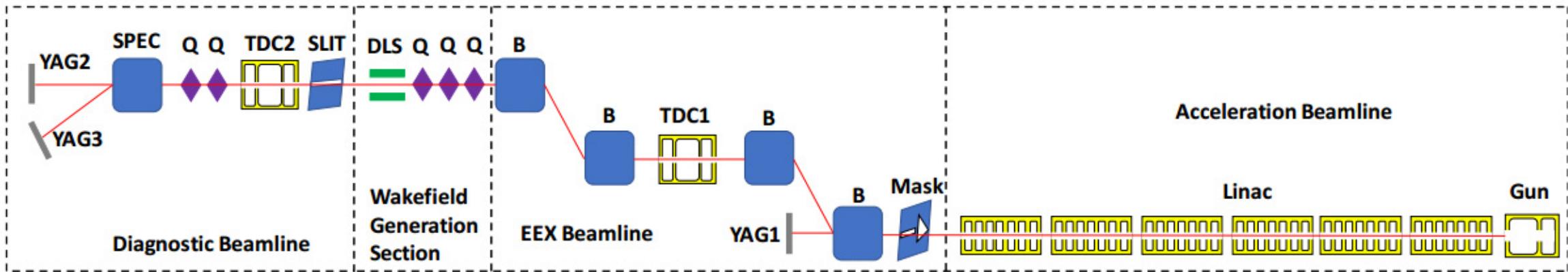


- Wakefield reconstruction result in simulation:
  - ✓ With the long witness beam and LPS mapping system, the wakefield can be measured by single-shot from beam x-y cross section images.
  - ✓ Both short and long range wakefield information are achieved.
  - ✓ High resolution.

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# Experiment Demonstration



## Diagnostic

- Mapping the longitudinal phase space.

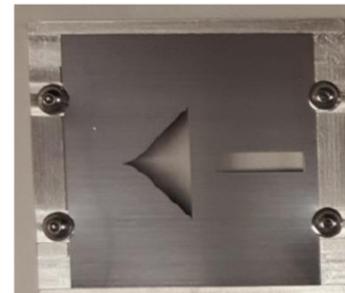
## Wakefield Generation Section

- Electron bunch travels through the dielectric-lined slab to excite wakefield.

## EEX Beamline

- Exchange the transverse and longitudinal profile of the bunch.

## Mask

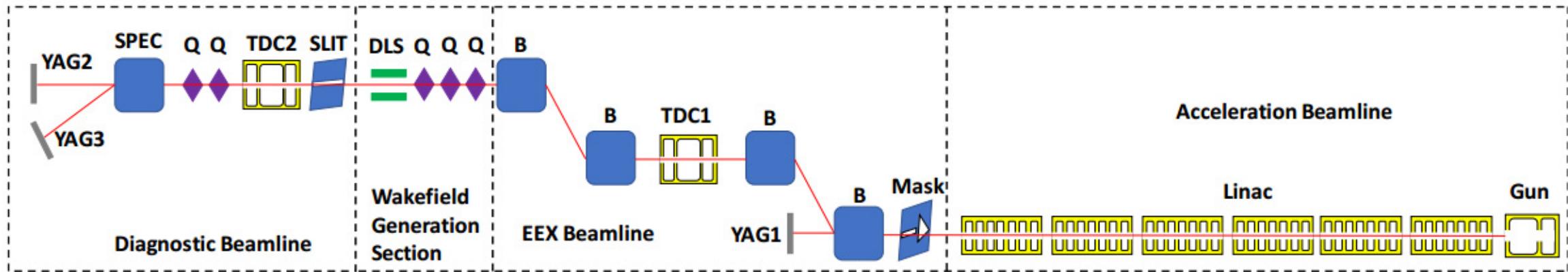


- Tailor the electron bunch transverse phase space

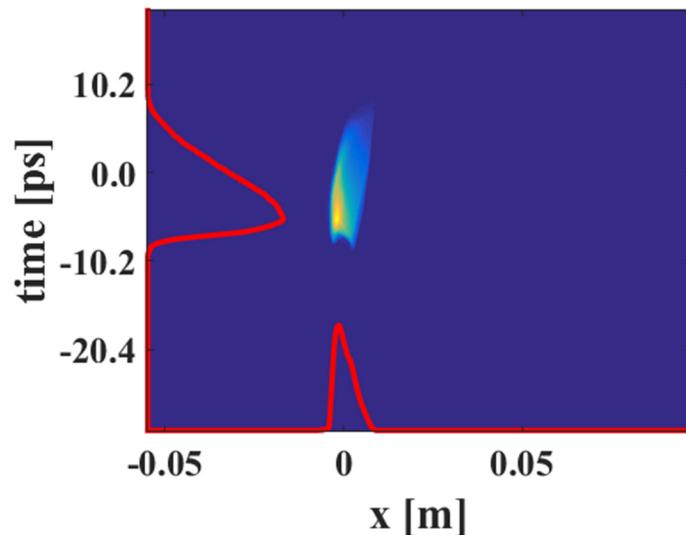
## Acceleration Beamline

- Photo-injector: Generates a 20-nC, 8 MeV electron bunch.
- Linac: accelerates the bunch to 48 MeV.

# Experiment Demonstration

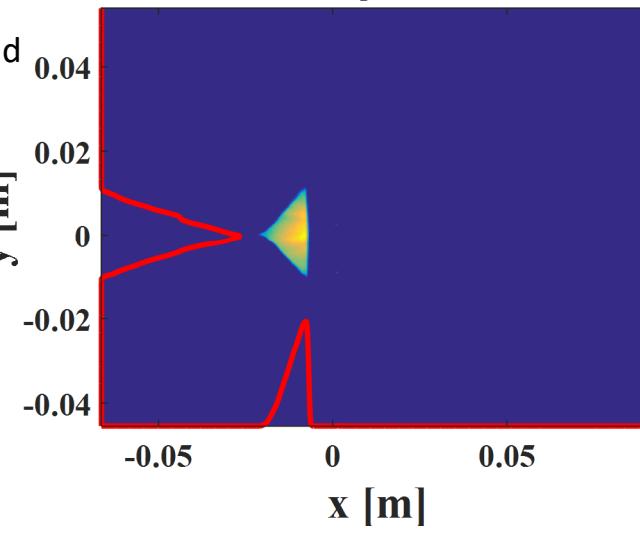


Drive bunch @EEX exit

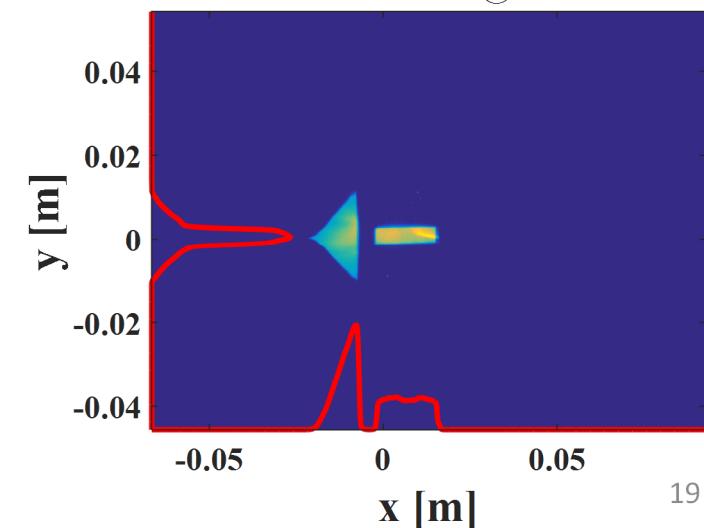


Transverse and longitudinal Exchange

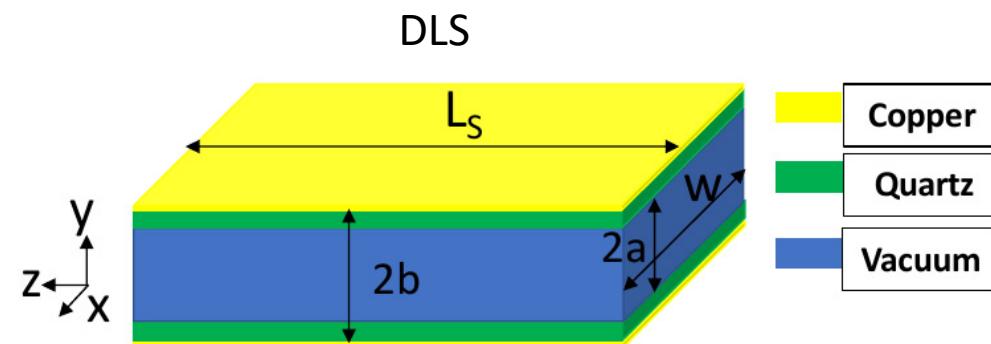
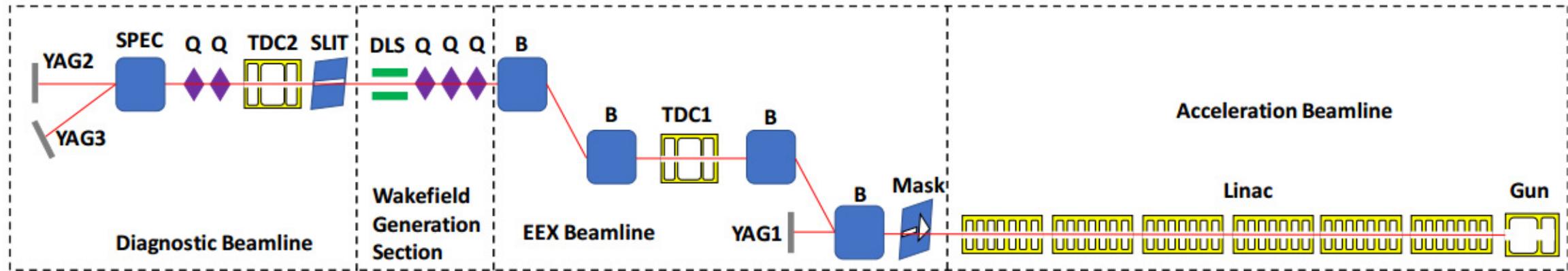
Drive Bunch @ EEX Entrance



Drive And Full Witness @ EEX Entrance



# Experiment Demonstration

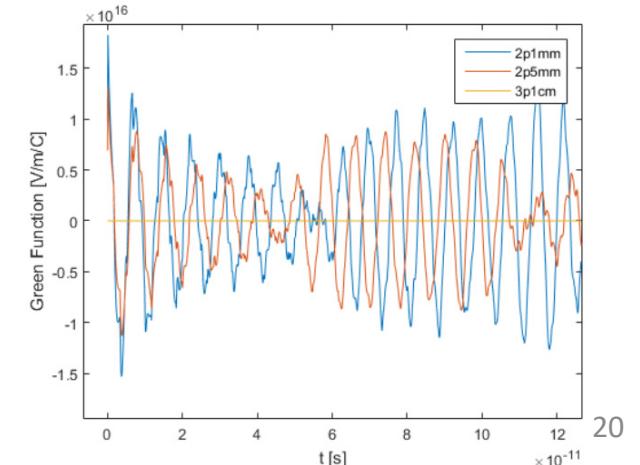


- Quartz,  $\epsilon_r = 3.75$ , thickness=150  $\mu\text{m}$ , length=15cm, width=1.27cm, copper coating.
- Gap between two jaws ranges from 0 to 3.1 cm.
- Fundamental modes: @121.9 and @130.7 GHz.

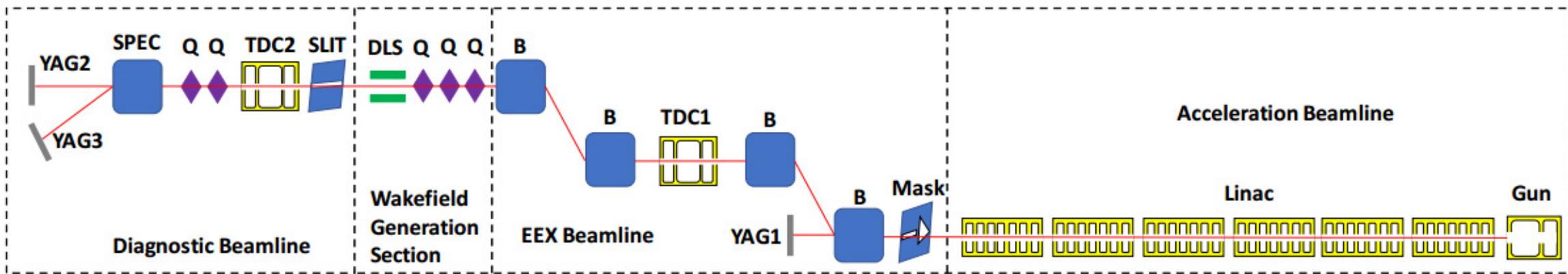


## ▪ Wake function

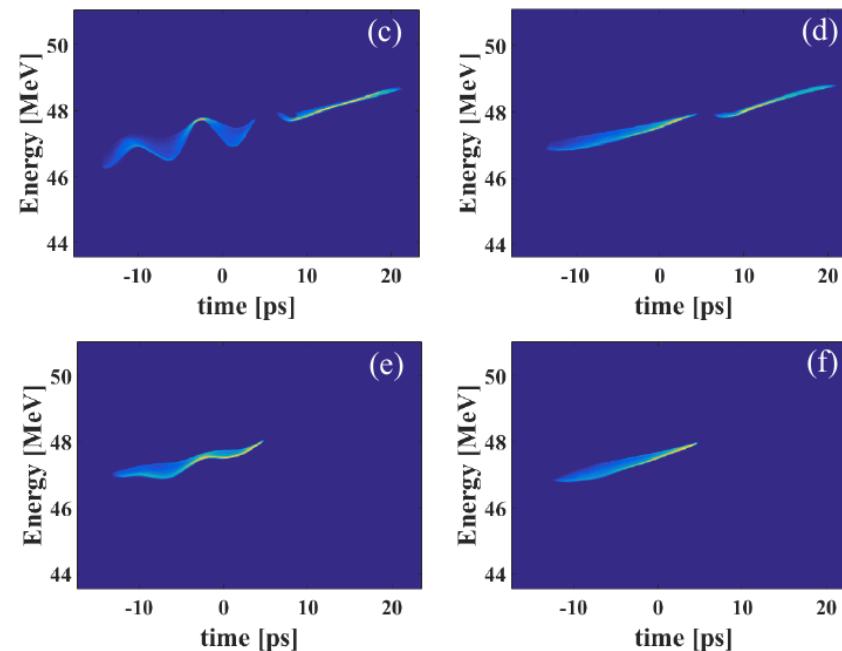
$$G(z) = \sum_{m=1}^M \sum_{n=1}^N \frac{\omega_{mn}}{4} \left(\frac{R}{Q}\right)_{mn} \frac{1}{1 - v_g/c} \cos \beta_{mn} z$$



# Experiment Demonstration

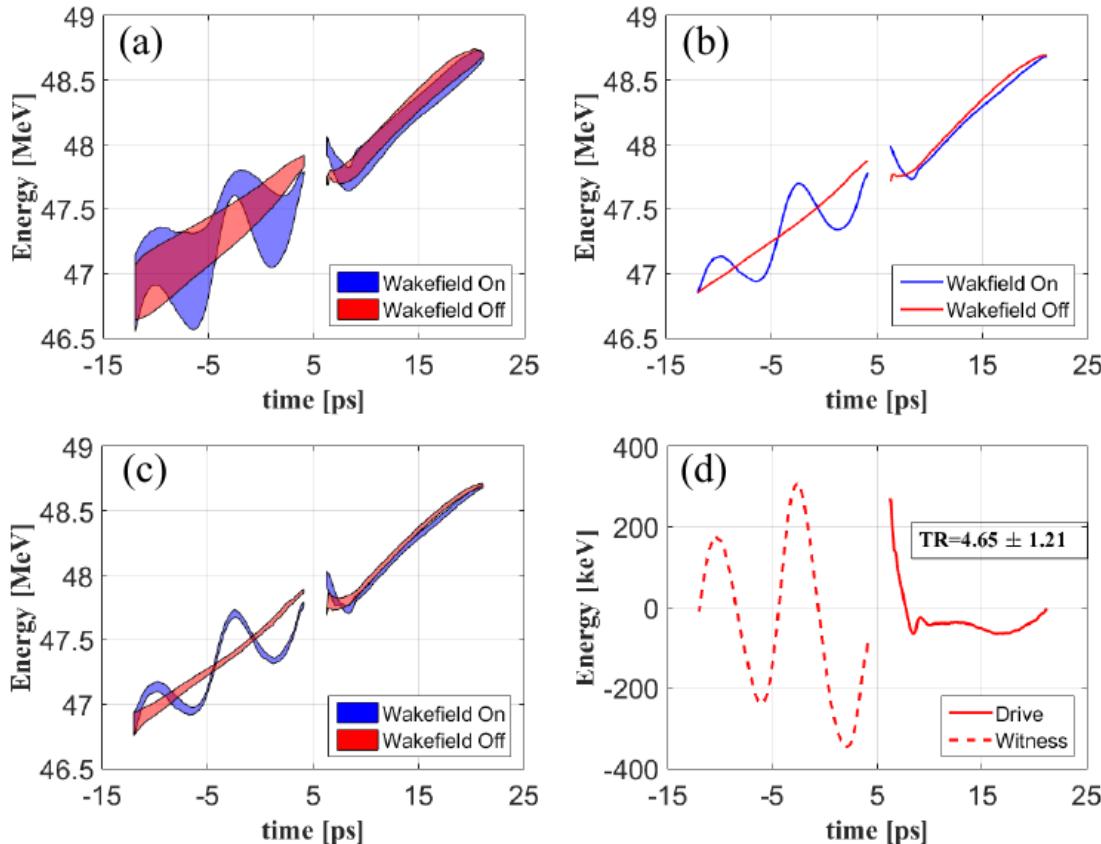


- (c-d) Drive and witness bunch at YAG3 with wakefield on/off.
- (e-f) Witness bunch at YAG3 with wakefield on/off.



- The wakefield can be reconstructed from beam x-y cross section images in four cases.
- Wakefield off indicates the dielectric slab is fully open, and there is no wakefield. Wakefield on represents the structure is closed with a gap 2.5 mm where wakefield is strong.
- Drive bunch is blocked to study the witness beamloading due to its self-wakefield, this beamloading is subtracted from the total energy gain in order to calculate the drive bunch wakefield.

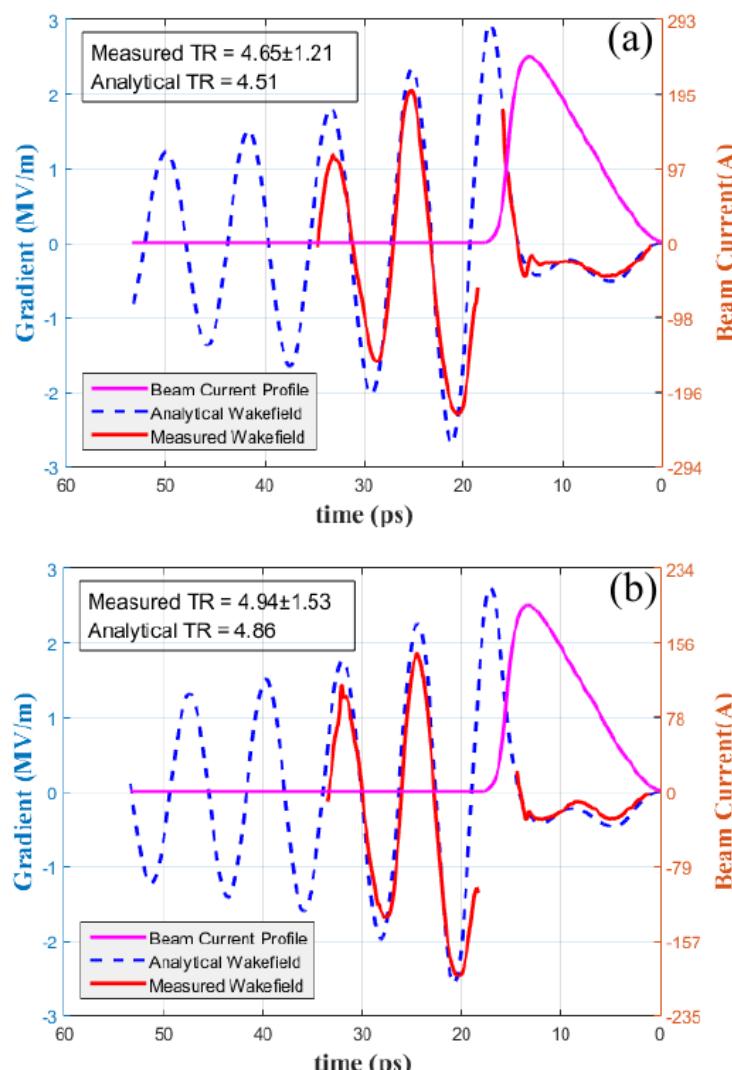
# Experiment Demonstration



Reconstructed longitudinal phase space and measured wakefield: (a) Longitudinal phase space full envelope. (b) Average profile in LPS. (c) Average profiles error bar in LPS. (d) Measured energy gain induced by wakefield.

- When the gap size is 2.5 mm, the total charge of drive bunch travelling through is  $2.08\text{nC}$ , and the witness charge is  $1.06\text{nC}$ .
- Figure (a) shows the single shot phase space profile of wakefield on and off.
- Figure (b) is average energy profile of LPS for both cases.
- Figure (c) shows the fluctuation between shot and shot.
- Figure (d) is the measured wakefield of drive bunch.

# Experiment Demonstration



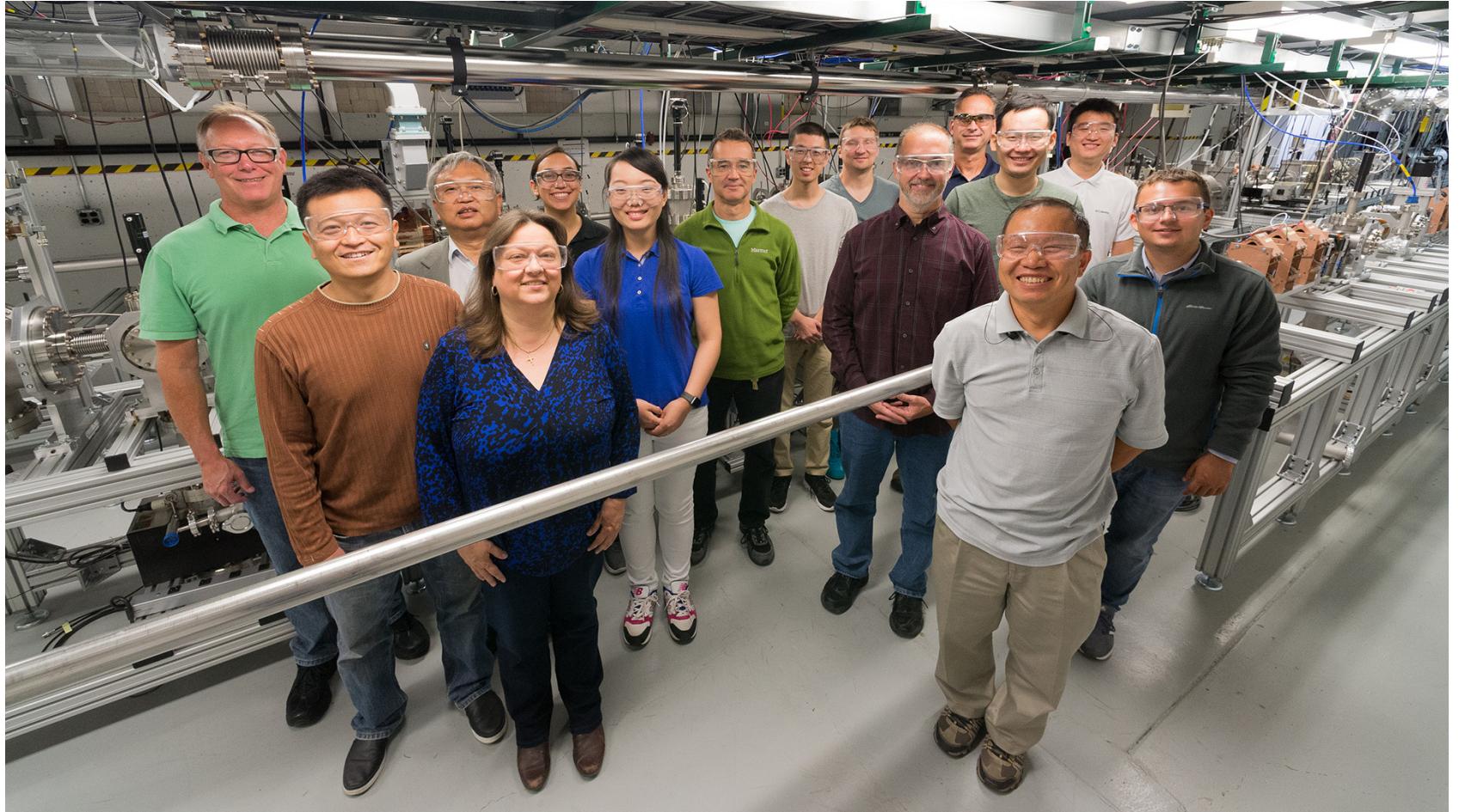
- Both short and long range wakefield are measured and compared with analytical result as shown in figure (a) and (b), whose gap size is 2.5 mm and 2.1 mm.
- The ideal wakefield can be derived from the following equation:
$$W(t) = \int_0^t \sum_{m=1}^{80} \sum_{n=1}^{10} I(\tau) G_{mn}(t - \tau) d\tau$$
- The Transformer ratio value is compared with analytical one, which shows a good agreement.
- The measured wakefield strength is slightly lower than analytical value, reasons:
  - The beam size at x direction (width direction) is non-negligible.
  - The strength of wake function derived from dielectric fill wave guide model is overestimated than the realistic slab model.

# Outline

- Introduction and Motivation
- Theory of Single-Shot Wakefield Mapping Method
- Beam Dynamic Simulation
- Experiment Demonstration
- Conclusion

# Conclusion

- Single-shot wakefield measurement system consists of two main elements:
  - Long witness bunch (longer than the region of interest)
  - Longitudinal phase space mapping system
- Single-shot wakefield measurement system features:
  - No need for dedicated scanning tool
  - Capability of measuring both short and long range wakefield
  - Capability of measuring THz wakefield
- Both simulation and experiment demonstrate that it can measure the wakefield precisely.
- Single-shot wakefield measurement system may be a general tool for wakefield measurement, especially for Transformer Ratio measurement.



Thank you for your attention