Analysis of beam position monitor requirements with Bayesian Gaussian regression

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Outline

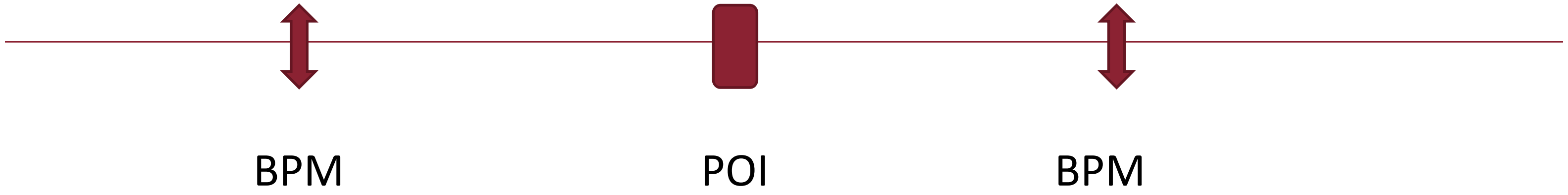
- Motivation
- Bayesian Gaussian regression
- An example: NSLS-II's brightness prediction with BPM's turn-by-turn functionality
- Summary

Motivation

- A storage ring's ultimate performance is determined beam properties at some particular points of interest (POI)
- BPMs can not be located at POIs
- Our goal is to predict beam behavior at POIs using BPM data

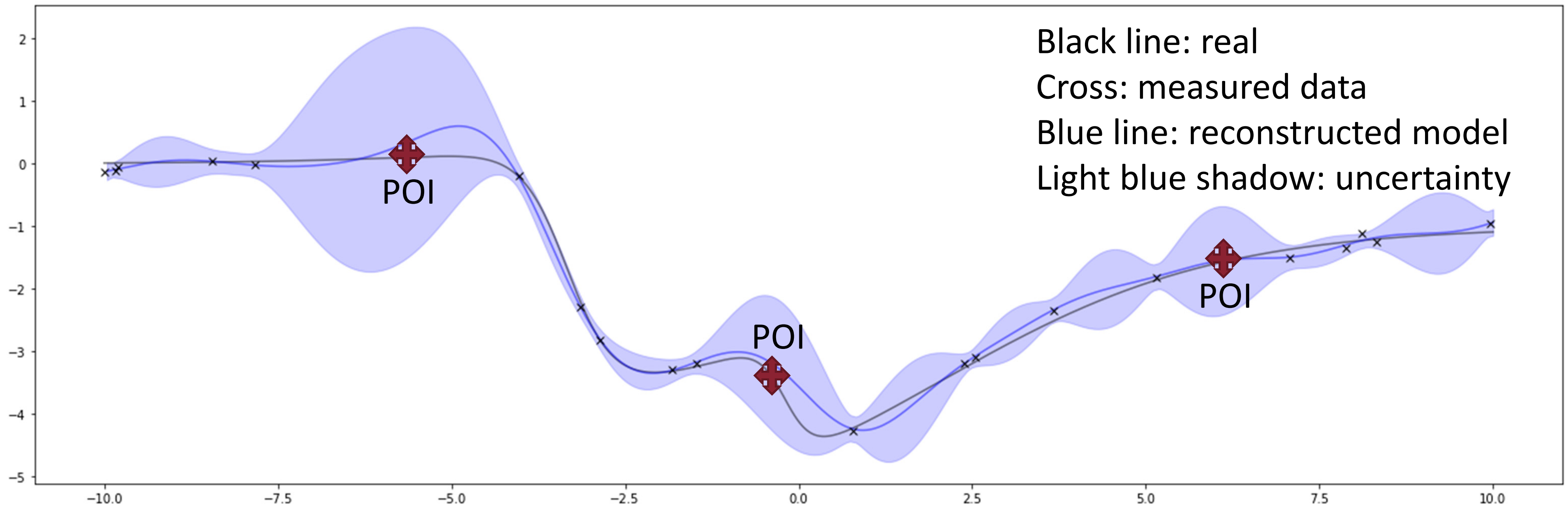
- What are the required BPM's specifications (quality, quantity and location)

IP for colliders: luminosity
undulators for light sources: brightness



Bayesian Gaussian regression

- Discrete measurements have uncertainties (resolution, failure)
- How confident while predicting beam performance at POIs



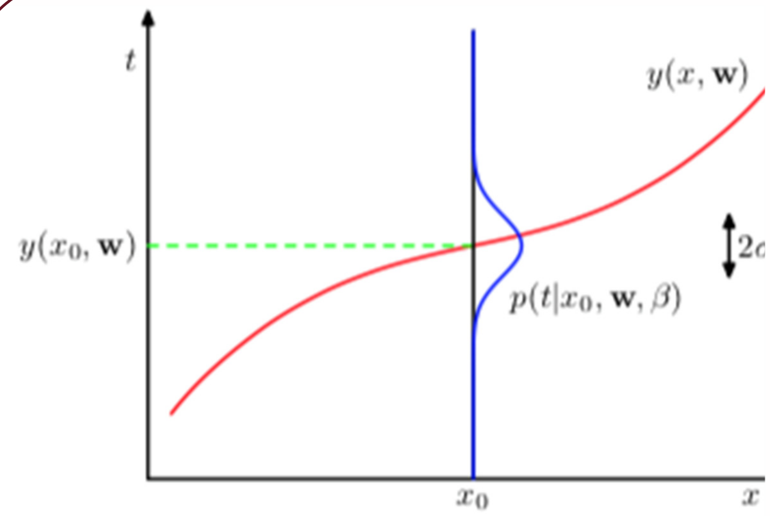
Bayesian Gaussian regression

- Supervised learning:
  - from BPMs' data to accelerator model
  - predict beam's ultimate performance at POIs
- Design models provide prior, likelihood
- Measurement errors Gaussian distributed

posterior, likelihood, prior, marginal

$$p(\Delta \boldsymbol{K}|\boldsymbol{\beta}) = \frac{p(\boldsymbol{\beta}|\Delta \boldsymbol{K})\, p(\Delta \boldsymbol{K})}{p(\boldsymbol{\beta})}$$

$$\propto p(\boldsymbol{\beta}|\Delta \boldsymbol{K})\, p(\Delta \boldsymbol{K}).$$



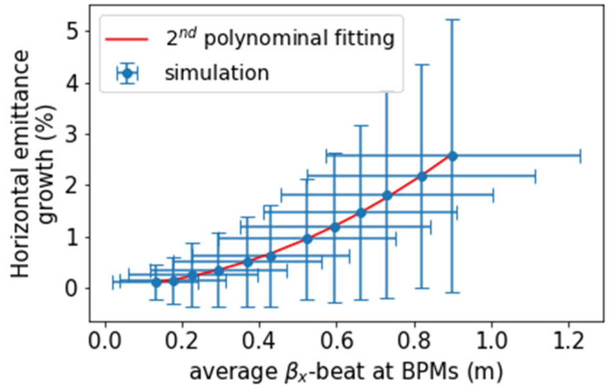
Example: NSLS-II's undulator brightness performance

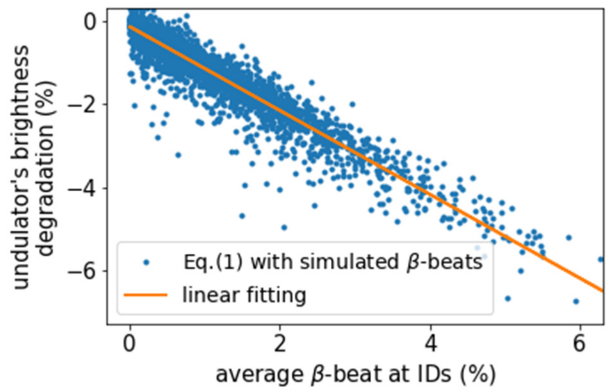
- e/photon-Beam 's transverse profile at undulators

$$\mathcal{B} \propto \frac{1}{\Sigma_x \Sigma'_x \Sigma_y \Sigma'_y}$$

$$\Sigma_{x,y} = \sqrt{\epsilon_{x,y}\beta_{x,y} + \eta^2_{x,y}\sigma^2_\delta + \sigma^2_{ph}}$$

$$\Sigma'_{x,y} = \sqrt{\epsilon_{x,y}\gamma_{x,y} + \eta'^2_{x,y}\sigma^2_\delta + \sigma'^2_{ph}}$$





1% beta-beat at POIs ~ 1.5% brightness degradation at NSLS-II

Standard Gaussian regression formulae

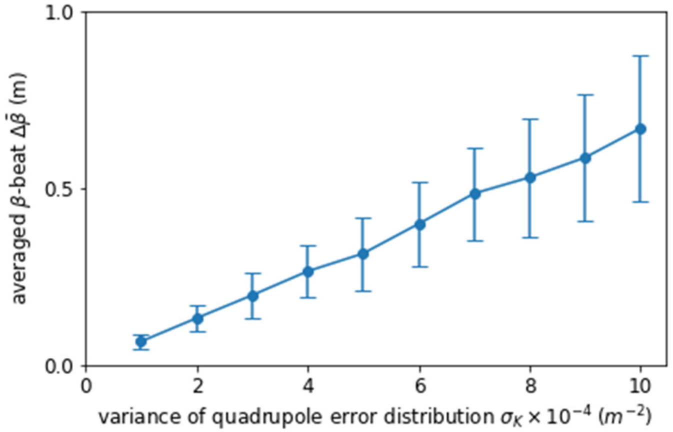
- BPM resolution and model re-construction

$$\mathcal{N}(\boldsymbol{\beta}|\bar{\boldsymbol{\beta}}, \sigma_\beta^2) = \frac{1}{\sqrt{2\pi}\sigma_\beta} \exp\left[-\frac{(\boldsymbol{\beta}-\bar{\boldsymbol{\beta}})^2}{2\sigma_\beta^2}\right]$$
$$\approx \frac{1}{\sqrt{2\pi}\sigma_\beta} \exp\left[-\frac{(\Delta\boldsymbol{\beta}-\boldsymbol{M}\Delta\boldsymbol{K})^2}{2\sigma_\beta^2}\right]$$

- Prior based on design model

$$p(\Delta\boldsymbol{K}) = \mathcal{N}(\Delta\boldsymbol{K}|0, \sigma_{\Delta K}^2)$$
$$= \frac{1}{\sqrt{2\pi}\sigma_{\Delta K}} \exp\left[-\frac{\Delta\boldsymbol{K}^2}{2\sigma_{\Delta K}^2}\right]$$

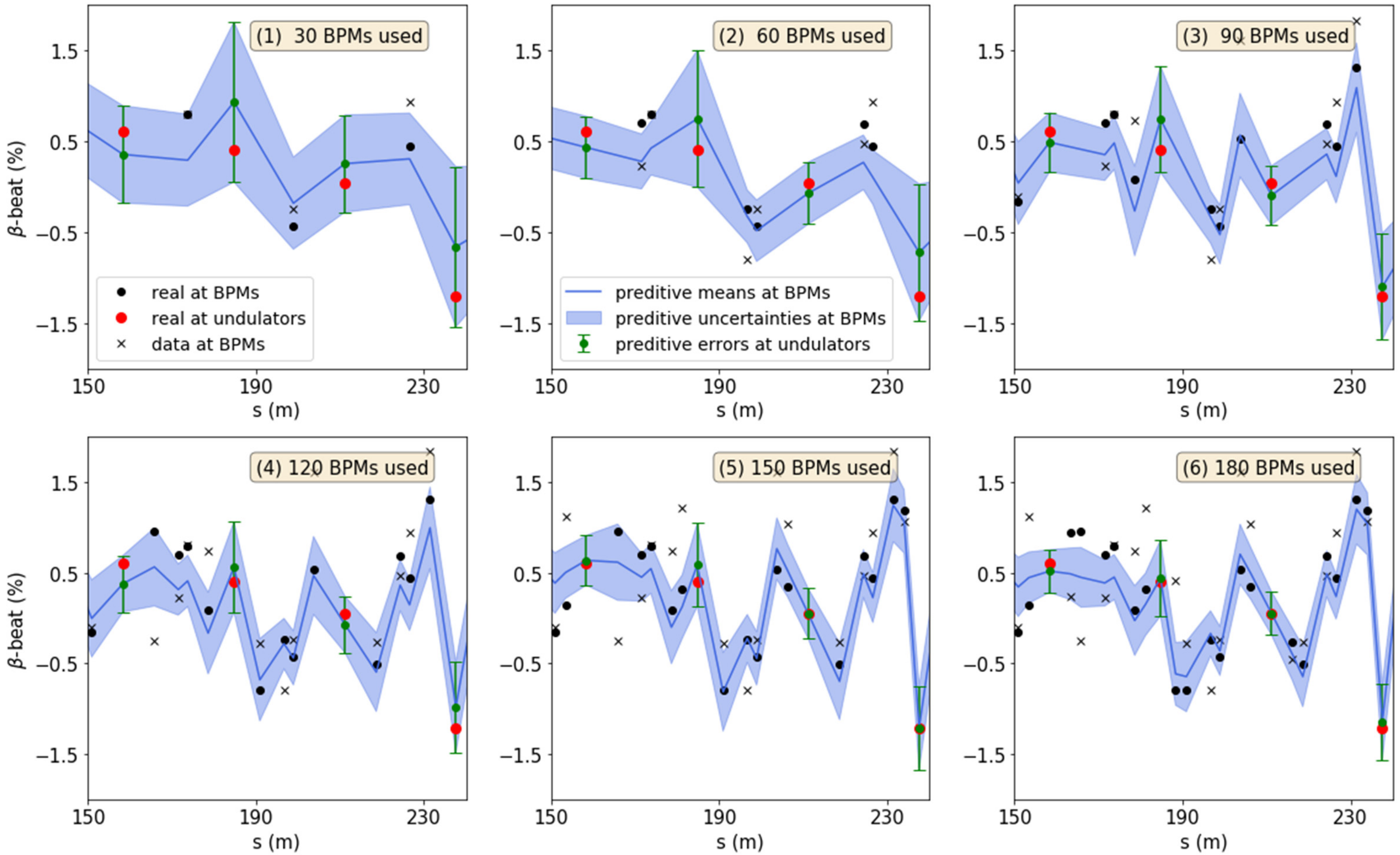
$$\sigma_{\Delta K} \sim \kappa|\Delta\beta| = \kappa|\bar{\beta} - \beta_0|$$

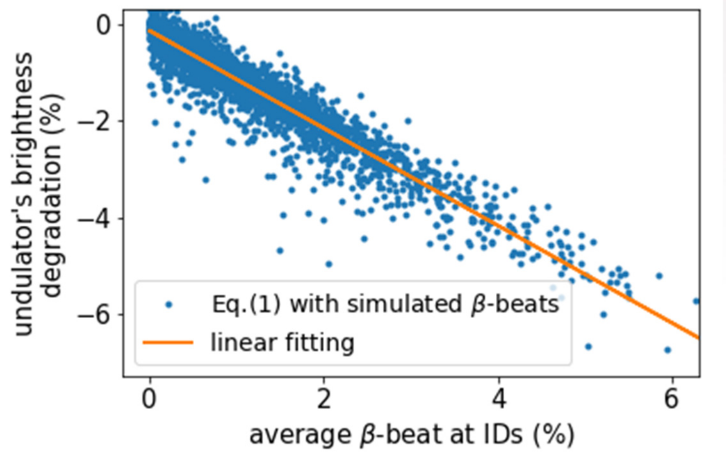


Prediction

$$\boldsymbol{m}_* = \sigma_\beta^{-2}\boldsymbol{M}_*\boldsymbol{A}^{-1}\boldsymbol{M}^T\Delta\bar{\boldsymbol{\beta}}$$
$$\boldsymbol{\Sigma}_*^2 = \boldsymbol{M}_*\boldsymbol{A}^{-1}\boldsymbol{M}_*^T,$$
$$\boldsymbol{A} = \left[\sigma_\beta^{-2}\boldsymbol{M}^T\boldsymbol{M} + \sigma_{\Delta K}^{-2}\boldsymbol{I}\right]$$

Performance prediction: BPMs' quantity

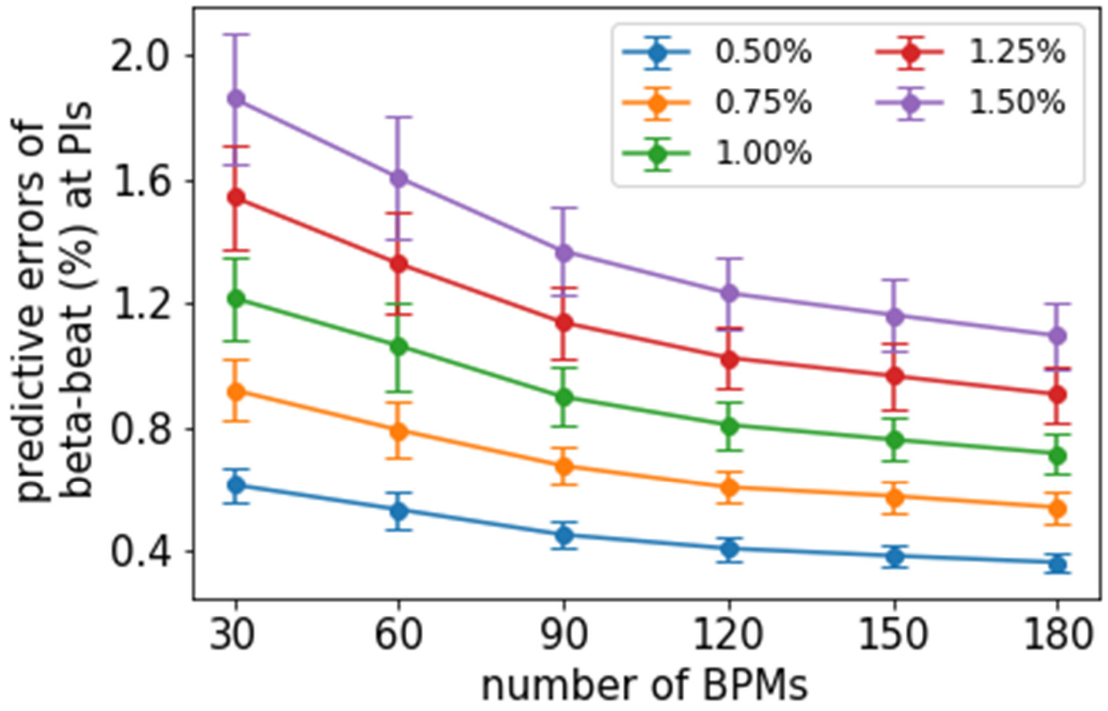




180 RF BPMs installed at NSLS-II.

Are BPMs sufficient or redundant?

Performance prediction: BPMs' quality



BPMs' quality (resolution) is more important than their quantity

Lattice model is also critically important (see ref. [Hao]

Summary

- Using Bayesian Gaussian regression, the model with uncertainty can be re-constructed
- BPMs' specification can be determined by the required resolution of the ultimate performance: brightness/luminosity
- This method should be more effective than using trial-test simulations

- References

➢ Y. Li et al. arXiv: 1904.05683, 2019

➢ Y. Li et al. PRAB 22, 012804, 2019

➢ Y. Hao et al. arXiv: 1902:11157, 2019

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