Analysis of BPM requirements with Bayesian Gaussian regression

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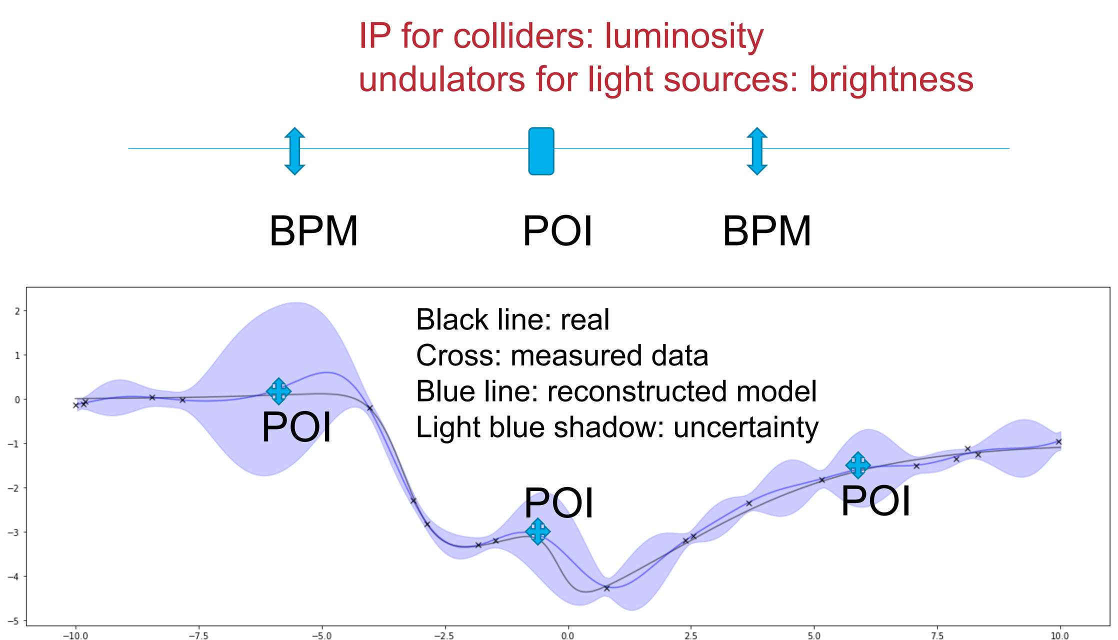
Problem



Methodology

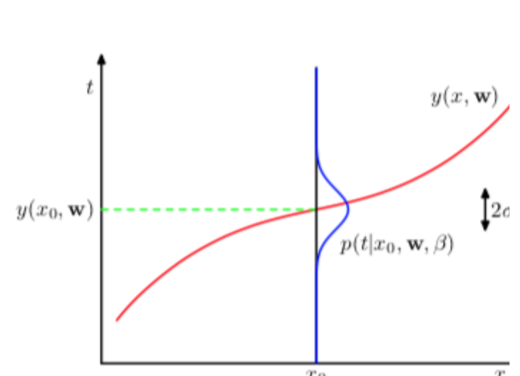
A Bayesian Gaussian regression approach can determine the probability distribution of the predictive errors at point of interest (POI), which can be used to conversely analyze the BPM system requirements

Bayes' Theorem

$$p(\Delta\boldsymbol{K}|\boldsymbol{\beta}) = \frac{p(\boldsymbol{\beta}|\Delta\boldsymbol{K})p(\Delta\boldsymbol{K})}{p(\boldsymbol{\beta})} \propto p(\boldsymbol{\beta}|\Delta\boldsymbol{K})p(\Delta\boldsymbol{K}).$$

Measured Optics

$$\mathcal{N}(\beta|\bar{\beta},\sigma_\beta^2) = \frac{1}{\sqrt{2\pi}\sigma_\beta}\exp\left[-\frac{(\beta-\bar{\beta})^2}{2\sigma_\beta^2}\right].$$



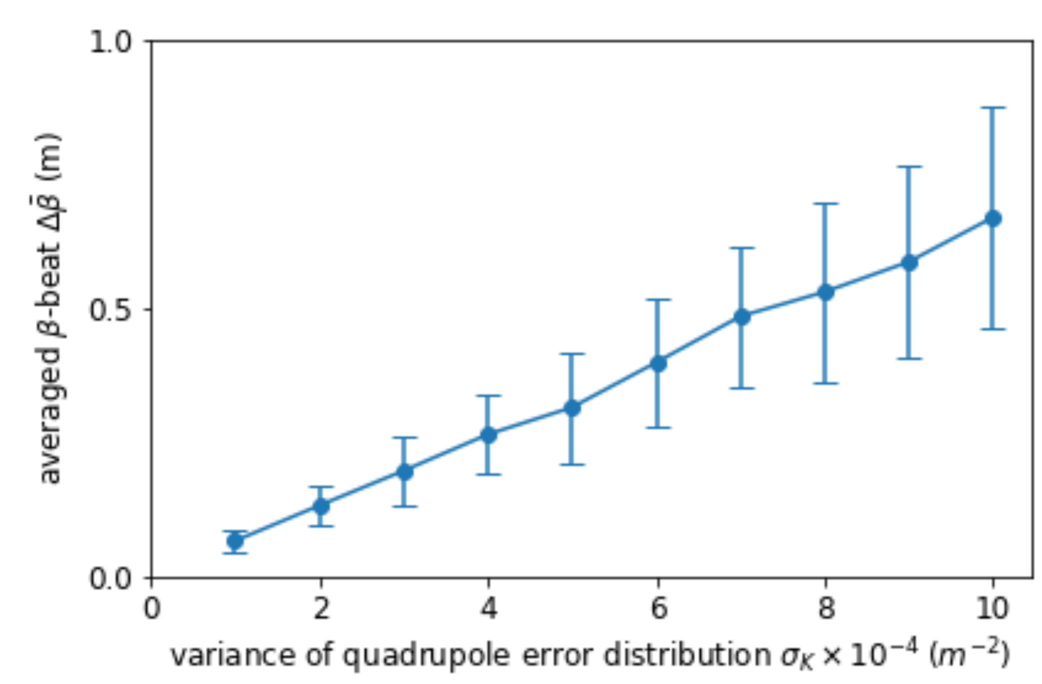
Quad's Errors

$$\mathcal{N}(\Delta\boldsymbol{K}|0,\sigma_K^2) = \frac{1}{\sqrt{2\pi}\sigma_K}\exp\left[-\frac{\Delta\boldsymbol{K}^2}{2\sigma_K^2}\right]$$

Works Cited

- Y. Li et al., arXiv:1904.05683 (2019)
- Y. Li et al., Phys. Rev. Accel. Beams **22** (2019)

Prior from Model



$$\sigma_{\Delta K} \sim \kappa \cdot \Delta\beta$$

Predictive errors

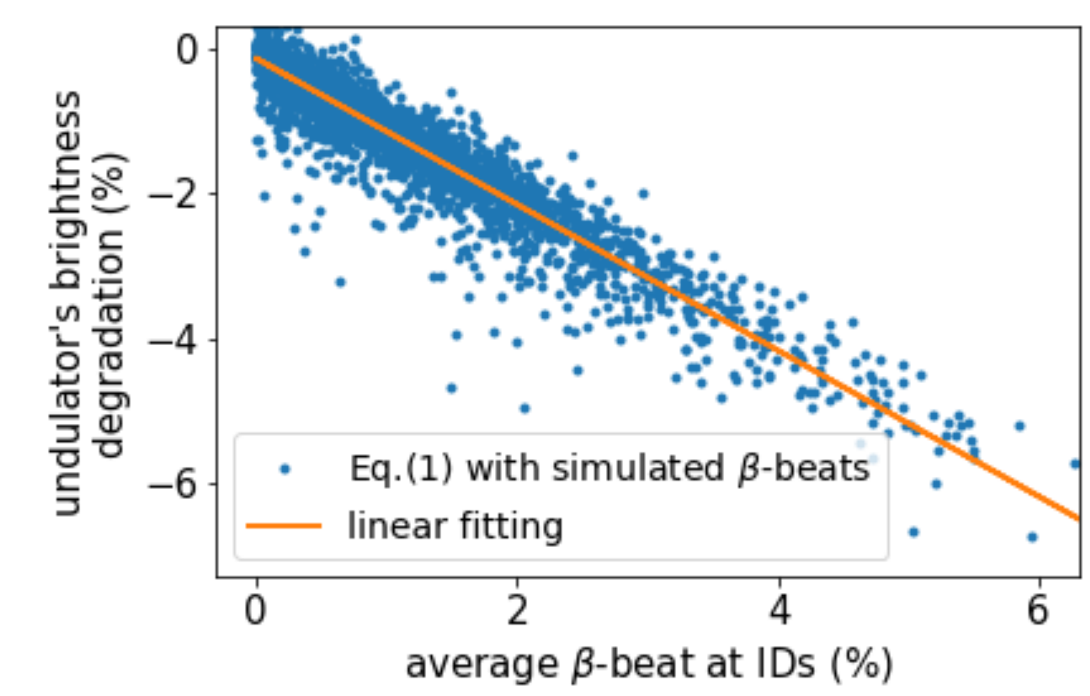
$$\boldsymbol{m}_* = \sigma_\beta^{-2}\boldsymbol{M}_*\boldsymbol{A}^{-1}\boldsymbol{M}^T\Delta\bar{\boldsymbol{\beta}}$$

$$\boldsymbol{\Sigma}_*^2 = \boldsymbol{M}_*\boldsymbol{A}^{-1}\boldsymbol{M}_*^T,$$

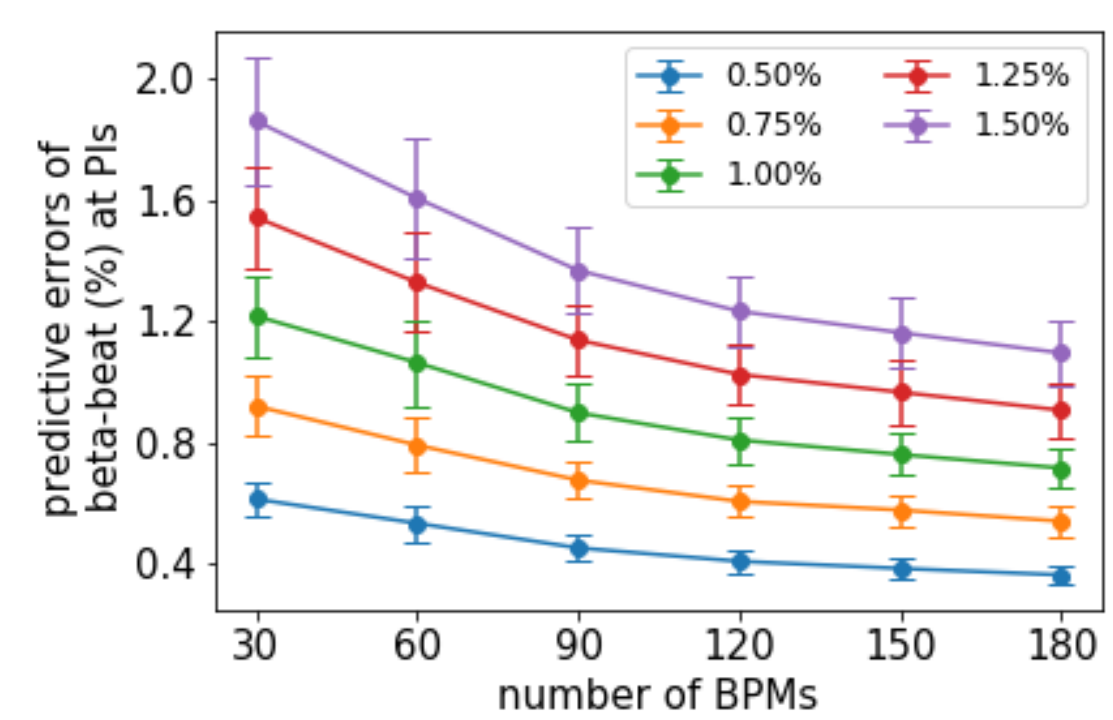
$$\boldsymbol{A} = \left[\sigma_\beta^{-2}\boldsymbol{M}^T\boldsymbol{M} + \sigma_{\Delta K}^{-2}\boldsymbol{I}\right]$$

- From BPM errors to model errors, then predictive errors at POIs
- Inverse problem: How are the BPM system technical requirements determined in order to observe whether a ring achieves its desired performance or not.

NSLS-II brightness



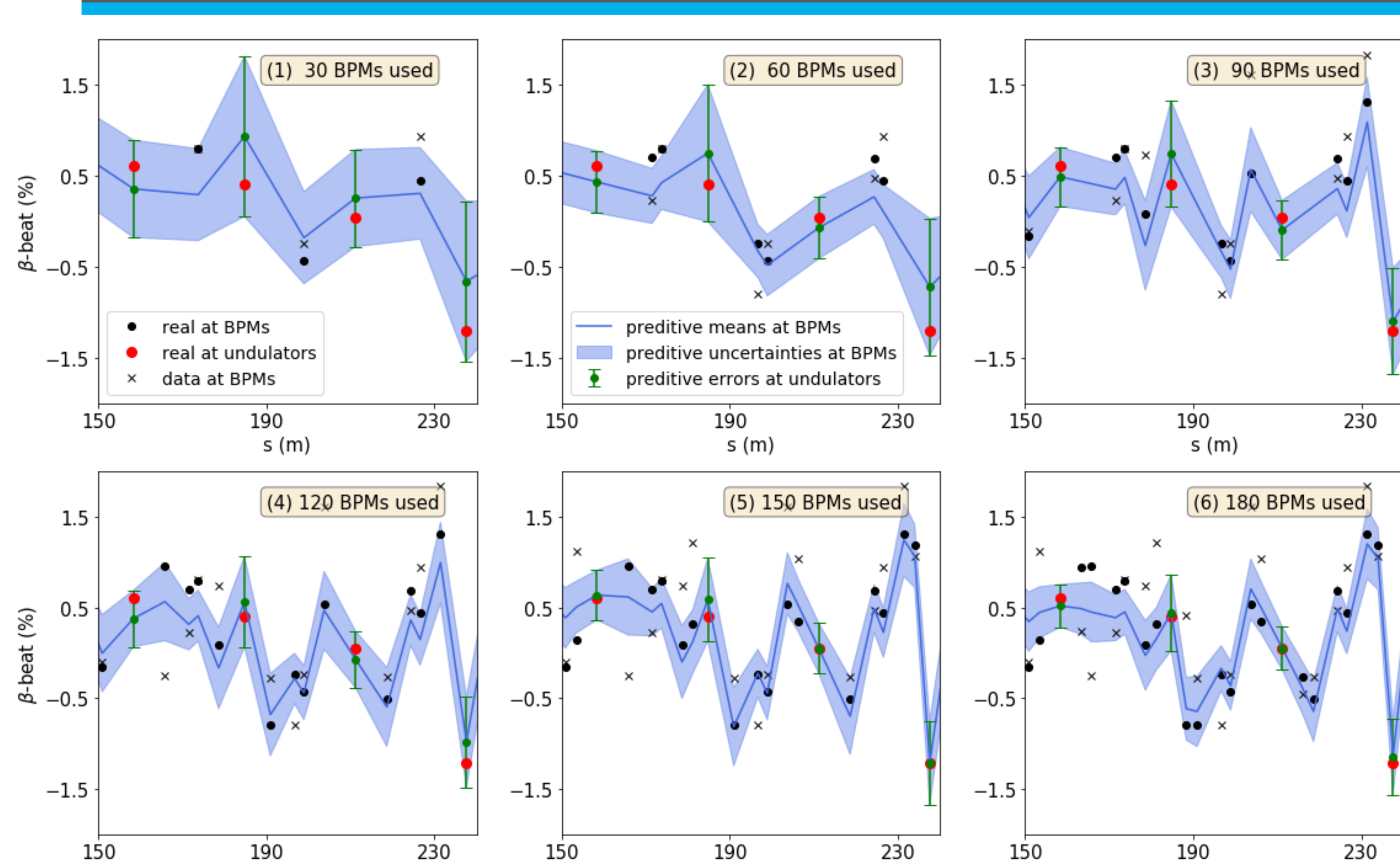
BPMs' resolution?



Summary:

- BPMs' quality is more important than their quantity, an optimal balance exists
- Specifications of BPM system can be quantitatively determined based on its ultimate performance

How many BPMs needed?



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