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DIFFUSION AND NONLINEAR PLASMA EFFECTS IN MICROBUNCHED ELECTRON COOLING*

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Abstract

The technique of microbunched electron cooling (MBEC) is an attractive scheme for enhancing the brightness of hadron beams in future high-energy circular colliders [1]. To achieve the required cooling times for a realistic machine configuration, it is necessary to boost the bunching of the cooler electron beam through amplification sections that utilize plasma oscillations. However, these plasma sections also amplify the intrinsic noise of the electron beam, leading to additional diffusion that can be very detrimental to the cooling. Moreover, they can exhibit nonlinear gain behavior, which reduces performance and limits the applicability of theory. In this paper, we study both of these important effects analytically with the aim of quantifying their influence and keeping them under control.

INTRODUCTION

In MBEC, the hadron beam first imprints an energy modulation on a co-propagating (cooler) electron beam in a segment of the machine known as the modulator. This energy modulation is then converted into a density modulation (bunching) after the e-beam passes through a dispersive chicane section with strength $R_{56}^{(e,1)}$ (Fig. 1). In the meantime, the hadrons are transported through their own—separate—section of the lattice, which also includes a chicane with strength $R_{56}^{(h)}$. The bunched electron beam then once again interacts with the hadrons in a subsequent section of the machine (the kicker), in a way that can ultimately lead to a significant reduction in the hadron energy spread and transverse emittance, after many passages through the cooling section. In order to accelerate this process and ensure that the cooling timescale is small enough for practical purposes, additional amplification stages are typically required, in which the bunching of the electron beam is boosted through the space charge (or plasma) effect. Each such plasma stage consists of a drift space followed by a chicane of strength $R_{56}^{(e,j)}$ ($j = 2, ..., M+1$, where M is the total number of stages). For simplicity, in this paper we assume that all stages have the same length L_d. In [2–4] we derived the cooling timescales using a technique that tracks the microscopic fluctuations in the hadron and electron beams. The main results can be summarized as follows: the characteristic cooling times for the energy spread and the emittance—normalized by the ring revolution period T and labeled by N_c^η and N_c^ϵ (respectively)—are given by $1/N_c^\eta = A_0' I_\eta'$ and $1/N_c^\epsilon = A_0' I_\epsilon'$, where

$$A_0' = \frac{4 I_e L_m L_k r_h}{\Sigma^3 \pi \gamma^3 I_A \sigma_e \sigma_h} \times \left(\frac{1}{\sigma_e} \sqrt{\frac{2 I_e}{\gamma I_A}} \right)^M \tag{1}$$

is a pre-factor and the cooling integrals I_η' and I_ϵ' are expressed by

$$\begin{aligned} I_\eta'/(2(q_h - q_s)) &= I_\epsilon'/q_s = (-1)^M \times q_{e,1} q_{e,2} \ldots q_{e,M+1} \\ &\times \int_0^\infty d\hat{k} \hat{k}^2 \exp(-\hat{k}^2((q_h - q_s)^2 + q_r^2/2)/2) \\ &\times H^2(\hat{k}, r) \exp(-\hat{k}^2 (q_{e,1}^2 + q_{e,2}^2 + \ldots + q_{e,M+1}^2)/2) \\ &\times \left(\frac{\hat{k} H_1(r_p \hat{k})}{r_p} \right)^{M/2} \sin^M (r_p \frac{\Omega_p L_d}{c} \sqrt{\frac{2 \hat{k} H_1(r_p \hat{k})}{r_p}}). \end{aligned} \tag{2}$$

In the expressions given above, γ is the relativistic factor (common for the co-propagating hadron/electron beams), L_m and L_k are the lengths of the modulator and kicker sections, $r_h = (Ze)^2/m_h c^2$ is the classical radius of the hadrons, I_e is the electron beam current and $I_A = m_e c^3/e \approx 17$ kA is the Alfven current. Moreover, σ_h and σ_e are, respectively, the rms energy spread values for the hadron and electron beams (assuming a Gaussian energy distribution for both). As far as the transverse properties of the beams are concerned, we again adopt Gaussian profiles and assume that a) at the modulator and kicker, the interacting beams have an identical, elliptical cross section characterized by a horizontal rms size Σ and a size aspect ratio r b) at the plasma stages, the e-beam is round with a common rms size $r_p \Sigma$. The squeeze factor r_p is also involved in the definition of the plasma frequency Ω_p, which is given by $\Omega_p = (c/r_p \Sigma)(I_e/\gamma^3 I_A)^{1/2}$.

In Eq. (1), $q_h = R_{56}^{(h)} \sigma_h \gamma / \Sigma$ is the scaled hadron chicane strength and $q_{e,j} = R_{56}^{(e,j)} \sigma_e \gamma / \Sigma$ are the normalized strengths of the various electron chicanes. In order to describe the mechanism of emittance cooling, we need to take into account the betatron motion of the hadron beam from the modulator to the kicker [4] (for simplicity, we only consider the vertical component of this motion). Including this effect is reflected in the parameters q_s and q_r, which are given by $q_s = S \sigma_h \gamma / \Sigma$ and $q_r = \gamma R \sqrt{\epsilon} / \Sigma$, where

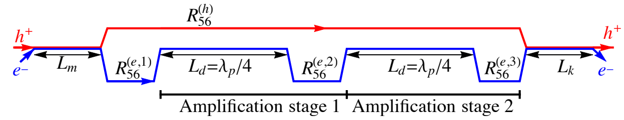

Figure 1: MBEC configuration with two amplification stages (the length L_d is a free parameter but, in practice, its value is $\sim \lambda_p$, where λ_p is the plasma wavelength).

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$S = D_0^2 \sin\mu/\beta_0$ and $R^2 = 8D_0^2 \sin^2(\mu/2)/\beta_0$. In these definitions, ϵ is the vertical emittance, D_0 (β_0) is the common vertical dispersion (beta function) at the modulator/kicker, while μ is the phase advance for the hadron transfer line that links these two locations. Lastly, the important function $H(\hat{k}, r)$, which is directly related to the Fourier transform of the space charge interaction function, is defined by $H(\hat{k}, r) = \hat{k}\int_0^\infty d\tau\tau \exp(-\hat{k}^2\tau^2/4)/\sqrt{(\tau^2+4)(\tau^2+4r^2)}$, while $H_1(\hat{k}) = H(\hat{k}, r=1)$. Some representative plots of the H-function are given in Fig. 2.

DIFFUSION AND SATURATION

The scaled cooling times N_c^η and N_c^ϵ determine the transverse/longitudinal cooling rates via $d\sigma_h^2/dt = -\sigma_h^2/(N_c^\eta T)$ and $d\epsilon/dt = -\epsilon/(N_c^\epsilon T)$. However, these statements are only true in an approximate fashion, if one ignores the various diffusion effects that can degrade the cooling. In general, including the latter would dampen the cooling rates according to $d\sigma_h^2/dt = -\sigma_h^2/(N_c^\eta T) + 2D_\eta$ and $d\epsilon/dt = -\epsilon/(N_c^\epsilon T) + D_\epsilon$, where D_η and D_ϵ are the appropriate diffusion coefficients (which are positive quantities). One prominent source of diffusion is the intrinsic noise in the hadron beam. Following the treatment of [3,4], one can show that its contribution to the longitudinal diffusion rate is given by

$$\begin{aligned} D_\eta^{(h)} &= \frac{4\sigma_h^2}{\pi T}\frac{I_h I_e^2 r_h^2 L_m^2 L_k^2}{I_A^3\gamma^7 r_e \Sigma^5\sigma_e^2\sigma_h^2}\times\frac{1}{\sigma_e^{2M}}\left(\frac{2I_e}{\gamma I_A}\right)^M \\ &\times q_{e,1}^2 q_{e,2}^2 \ldots q_{e,M+1}^2 \int_{-\infty}^{\infty} d\hat{k}\hat{k}^2 H^4(\hat{k}, r) \\ &\times \exp(-(q_{e,1}^2 + q_{e,2}^2 + \ldots + q_{e,M+1}^2)\hat{k}^2) \\ &\times \left(\frac{\hat{k}H_1(r_p\hat{k})}{r_p}\right)^M \sin^{2M}\left(r_p\frac{\Omega_p L_d}{c}\sqrt{\frac{2\hat{k}H_1(r_p\hat{k})}{r_p}}\right), \end{aligned} \tag{3}$$

while its transverse counterpart is expressed by $D_\epsilon^{(h)} = (D_0^2/\beta_0)D_\eta^{(h)}$. In these expressions, we clarify that I_h is the current of the hadron beam and $r_e = e^2/m_e c^2$.

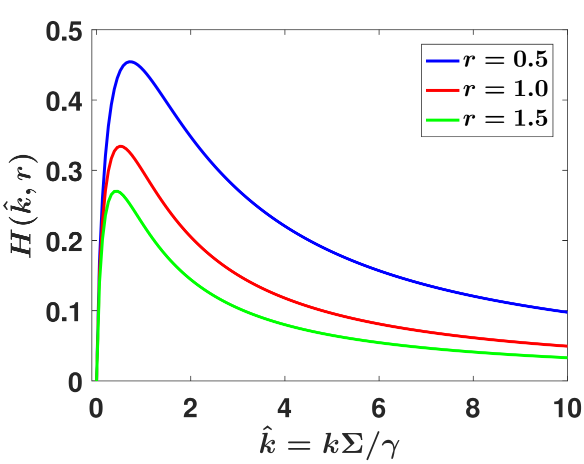

Figure 2: Plots of the function $H(\hat{k}, r)$ for positive $\hat{k}$ and different values of the r-parameter.

Table 1: Parameters of the eRHIC Collider with a Hypothetical MBEC Cooling Section

Proton relative energy spread, σ_h	4.6×10^{-4}
Electron relative energy spread, σ_e	1×10^{-4}
Relativistic factor, γ	293
Peak electron beam current, I_{e0} [A]	30
Peak hadron beam current, I_{h0} [A]	23
Electron rms bunch length, $\sigma_z^{(e)}$ [mm]	4
Proton rms bunch length, $\sigma_z^{(h)}$ [cm]	5
Revolution period T [s]	1.2×10^{-5}
Hor./vert. proton emittance ϵ_x/ϵ_y [nm]	9.2/1.3
Modulator and kicker lengths L_m, L_k [m]	50

An additional source of diffusion is the shot noise in the cooler electron beam itself. This noise can be significantly amplified by the presence of amplification stages meant to boost the bunching. According to [3], the contribution of this effect to the longitudinal diffusion rate can be quantified by the expression

$$\begin{aligned} D_\eta^{(e)} &= \frac{2\sigma_h^2}{\pi T}\frac{I_e r_h^2 L_k^2}{Z^2 I_A\gamma^3 r_e\Sigma^3\sigma_h^2}\times\frac{1}{\sigma_e^{2M}}\left(\frac{2I_e}{\gamma I_A}\right)^M \\ &\times q_{e,2}^2 q_{e,3}^2 \ldots q_{e,M+1}^2 \int_0^\infty d\hat{k} H^2(\hat{k}, r) \\ &\times \exp(-(q_{e,2}^2 + \ldots + q_{e,M+1}^2)\hat{k}^2) \\ &\times \left(\frac{\hat{k}H_1(r_p\hat{k})}{r_p}\right)^M \sin^{2M}\left(r_p\frac{\Omega_p L_d}{c}\sqrt{\frac{2\hat{k}H_1(r_p\hat{k})}{r_p}}\right). \end{aligned} \tag{4}$$

The total energy spread-related diffusion coefficient is $D_\eta = D_\eta^{(h)} + D_\eta^{(e)}$. As far as the emittance diffusion rate is concerned, we approximate $D_\epsilon \approx D_\epsilon^{(h)}$. For diffusion to be negligible, the diffusion-to-cooling ratios $(T/\sigma_h^2)D_\eta N_c^\eta$ and $(T/\epsilon)D_\epsilon N_c^\epsilon$ should be much smaller than unity. However, there is one extra complication stemming from the finite longitudinal size of the hadron/electron beams. In particular, let us assume that both beams have a Gaussian longitudinal profile, so that $I_e = I_{e0}\exp(-z^2/2(\sigma_z^{(e)})^2)$ and $I_h = I_{h0}\exp(-z^2/2(\sigma_z^{(h)})^2)$, where $\sigma_z^{(e)}$ and $\sigma_z^{(h)}$ are the rms bunch lengths and the z is the longitudinal position measured from the common centroid of both bunches. The cooling and diffusion rates are then local quantities (i.e. functions of z) and a proper averaging becomes necessary. This is accomplished by using the hadron probability distribution $\lambda_h(z) = \exp(-z^2/2(\sigma_z^{(h)})^2)/\sqrt{2\pi}\sigma_z^{(h)}$ as a weighting function [3], so that the bunch average of a local quantity $F(z)$ is defined by $\langle F\rangle_z = \int_{-\infty}^{\infty} dz\lambda_h(z)F(z)$.

Finally, we address the issue of possible nonlinear behavior in the amplification cascade, an effect which can be important if the gain is large enough. Such a deviation from linearity can be tracked by the ratio $I_{\rm sat}^2 \equiv \langle\delta n^2\rangle/n_{0e}^2$, where δn is the density modulation of the electron beam due to the plasma oscillations, n_{0e} is the background electron den-

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sity and the brackets denote statistical averaging. Linear behavior is defined by $I_{\rm sat}^2 \ll 1$, while nonlinear behavior (or saturation) starts to occur when $I_{\rm sat}^2 \approx 1$. Again using [3, 4], the saturation ratio $I_{\rm sat}$ can be expressed by

$$
\begin{aligned}
I_{\rm sat}^2 &= \frac{4Z^2 I_h r_e L_m^2}{\pi I_A \sigma_e^2 \Sigma^3 \gamma^3} \times \frac{1}{\sigma_e^{2M}} \left(\frac{2I_e}{\gamma I_A}\right)^M \\
&\times q_{e,1}^2 q_{e,2}^2 \ldots q_{e,M+1}^2 \int_0^\infty d\hat{k}\hat{k}^2 H^2(\hat{k}, r) \\
&\times \exp(-(q_{e,1}^2 + \ldots + q_{e,M+1}^2)\hat{k}^2) \\
&\times \left(\frac{\hat{k}H_1(r_p\hat{k})}{r_p}\right)^M \sin^{2M}\left(r_p \frac{\Omega_p L_d}{c}\sqrt{\frac{2\hat{k}H_1(r_p\hat{k})}{r_p}}\right). \qquad (5)
\end{aligned}
$$

Since $I_{\rm sat}$ is also a local quantity, we may take its maximum value along the bunch—$I_{\rm sat}^{\rm max}$—as a rather conservative measure of nonlinearity.

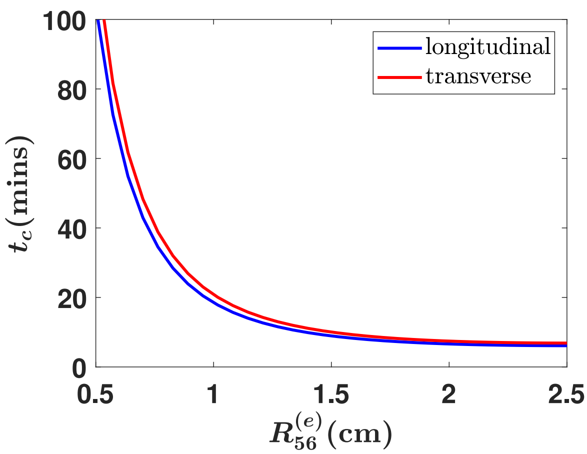

Figure 3: Bunch-averaged cooling times vs the common value of the electron chicane strengths.

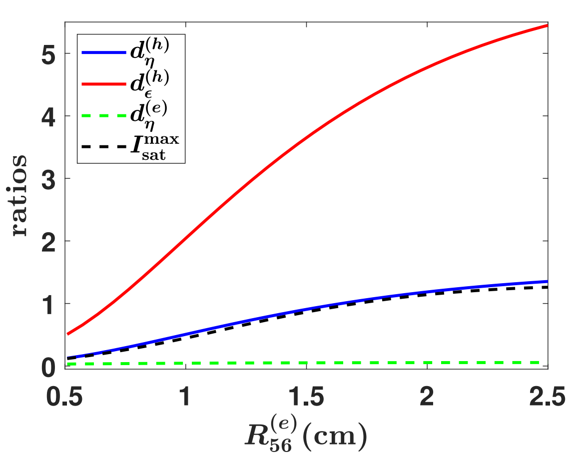

Figure 4: Diffusion and saturation ratios versus the common e-chicane strength. As far as the former is concerned, we define $d_\eta^{(h,e)} = (T/\sigma_h^2)\left\langle D_\eta^{(h,e)}\right\rangle_z / \langle 1/N_c^\eta \rangle_z$ etc.

NUMERICAL STUDY

Using Eqs. (1)–(5), we performed a numerical study of diffusion and saturation effects for a parameter set that is representative of the prospective eRHIC collider (Table 1). For $\beta_x = \beta_y = \beta_0 = 50$ m, the horizontal size is Σ is 680 µm, while the vertical size without dispersion is $r_0\Sigma = 250$ µm (we have also assumed a squeeze factor of 0.2). A preliminary numerical optimization procedure for the emittance cooling rate yields a minimum (bunch-averaged) cooling time of 4.5 mins for $D_0 = 1.3$ m, $\mu = 0.37$ and $R_{56}^{(h)} = 1.25$ cm, $R_{56}^{(e)} = 2.5$ cm, while $L_d \approx 80$ m (results also reported in [4]). Though attractive, such a low cooling timescale is, in fact, likely to be limited by diffusion and saturation effects. To start with, this optimum point corresponds to zero cooling for the energy spread so, in practice, we must choose a dispersion that is smaller than the optimum (say 80%) in order to obtain cooling in both degrees of freedom. In Fig. 3 we plot the cooling times versus what we assume is a common value for the electron chicane strengths, keeping all other parameters constant. In Fig. 4 we also plot the various diffusion ratios, along with the maximum value of the saturation ratio, as functions of the e-chicane strength. The basic conclusion is that, while the 5 min value for the cooling time is indeed rather unrealistic due to diffusion and saturation, a more reasonable figure ≈ 1 h is achievable by using weaker electron chicanes and smaller dispersion.

CONCLUSIONS

We have reviewed the analytical expressions for the MBEC cooling and diffusion rates, generalizing the latter so as to include the effects of hadron betatron motion and elliptical beam cross section. Moreover, we have updated the expression for the saturation ratio, which is a quantitative measure of the proximity to nonlinear behavior in the plasma cascade. Using these formulas, we evaluated a prospective MBEC configuration for the eRHIC collider, concluding that a cooling time of about 1 hour appears feasible with an appropriate choice of parameters that also keep diffusion and saturation effects under control.

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