

ANALYSIS OF ALLISON SCANNER PHASE PORTRAITS USING ACTION-PHASE COORDINATES *

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Abstract

Allison scanners provide detailed information on the beam transverse phase space. An effective way for analyzing the beam distribution from these measurements is to use action-phase coordinates, where beam propagation in a linear lattice is reduced to advancing the phase. This report presents such analysis for measurements performed with a 2.1 MeV, 5 mA H⁻ beam in the MEBT of the PIP2IT test accelerator at Fermilab. In part, with the choice of calculating the Twiss parameters over the high intensity portion of the beam, the beam core is found to be phase-independent with intensity decreasing exponentially with action, while the beam tails exhibit a clear phase dependence that is stable over the beam line.

INTRODUCTION

To improve comparisons of beam phase space measurements performed with different focusing, the phase portraits can be viewed in action-phase coordinates where the action J and phase ψ are defined as

$$J = \frac{1}{2} (\gamma x^2 + 2\alpha x x' + \beta x'^2) \quad (1)$$

$$\psi = \arctan \left(\frac{\alpha x + \beta x'}{x} \right) \quad (2)$$

where α , β , and γ are the Twiss parameters and x and x' are the position and angle coordinates.

Table 1: PIP2IT MEBT Allison Scanner Dimensions

Parameter	Value
Slit size	0.2 mm
Slit separation	320 mm
Slit thickness	0.04 mm
Plate voltage	± 1000 V
Plate length	300 mm
Plate separation	5.6 mm

The benefit of using this coordinate system that is the intensity distribution over action does not change assuming negligible non-linear effects [1]. Also, the phase is proportional to the betatron phase advance. Thus under linear optics, beam transport results in only a shift in phase. Distributions can therefore be compared even between measurements with different beamline configurations resulting in

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different Twiss parameters. This allows for measurements of distortions and tail growth due to non-linear forces.

This approach is detailed below and illustrated by measurements of the phase space of a 2.1 MeV H⁻ beam in the PIP2IT beamline [2]. The measurements were taken using an Allison scanner [3] with dimensions given in Table 1. The measured 2D distributions, called phase portraits, were converted to action-phase coordinates by calculating the action and phase for each pixel in the scan based on the measured ensemble.

DISTRIBUTION OVER ACTION

For negligible space charge, the beam density is expected to be Gaussian in position and angle [4]. In this case the intensity follows a Boltzmann distribution in action

$$I = I_0 e^{-J/\epsilon_c} \quad (3)$$

where ϵ_c is referred to as the central slope. At higher actions, it was found that the intensities deviate from the distribution in Eq. (3). The action where the intensities deviate significantly from Eq. (3) is found by binning the intensities by action, typical bin size of 0.05 mm mrad, and calculating the mean and standard deviation of the intensity in each bin. The ‘transition action’ J_t is defined as the action where the average intensity deviates from Eq. (3) by more than three times the standard deviation of the mean.

$$I(J_t) - I_0 e^{-J_t/\epsilon_c} = 3\sigma_{\text{int}}(J_t) \quad (4)$$

The transition action defines the separation of the Gaussian core from the non-Gaussian beam tails. The fraction of the beam in the core is typically between 70-90%.

At larger actions the beam becomes phase dependent and splits in phase into two ‘branches’ separated by approximately π (Fig.1 middle). Currently, no satisfactory description or source of the phase dependent tails has been found, and the tails are only characterized by the average phase of the ‘upper’ branch, $\psi \in (0, \pi]$, and the maximum action.

In the initial attempts to transform the measured $x - x'$ phase portraits into $J - \psi$ coordinates two issues were found. The first issue was the distributions over action were not constant under changes to the optics. The second peculiarity was the distributions showed a phase dependence even at low actions which was not believed to accurately describe the beam. To address these issues, special care must be taken when defining the central slope and Twiss parameters used to determine J and ψ . The corrections for these complications are described below.

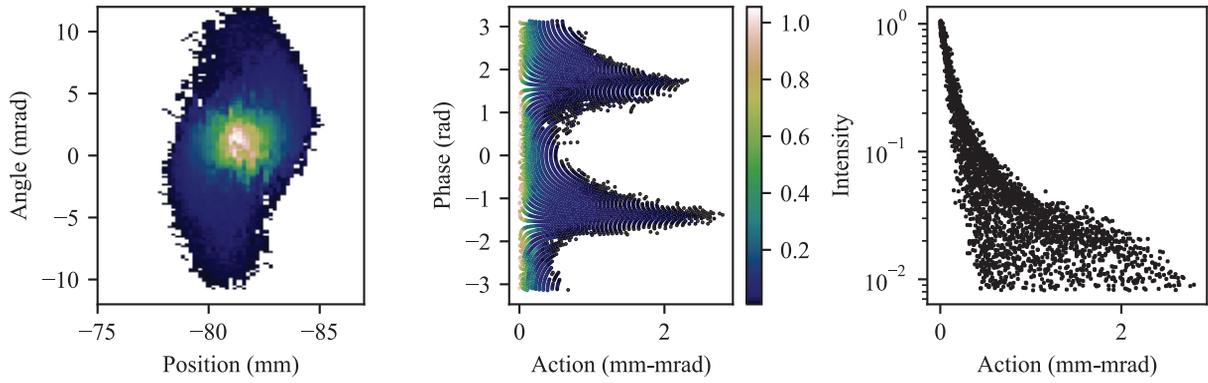


Figure 1: Measured phase space at the end of the MEBT in $x-x'$ coordinates (left) and $J - \psi$ coordinates (center and right).

Allison Scanner Phase Dependence

An Allison scanner will add some small phase dependence to all measured phase portraits due to the finite size of the slits. For example, if a pure 2D Gaussian is measured with an Allison scanner of slit to slit length ℓ and slits y_1 and y_2 that are $2d$ wide the measured intensity distribution is given by integrating over both slits:

$$I_{\text{meas}}(x, x') = \frac{1}{4d^2} \int_{-d}^d \int_{-d}^d \exp\left(-\frac{1}{2\epsilon_c} \left[\gamma(x + y_1)^2 + 2\alpha(x + y_1)\left(x' + \frac{y_2 - y_1}{\ell}\right) + \beta\left(x' + \frac{y_2 - y_1}{\ell}\right)^2 \right]\right) dy_1 dy_2. \quad (5)$$

The integrand was expanded to second order in y_1 and y_2 and the resulting measured distribution up to order d^2 is

$$I_{\text{meas}}(x, x') = \exp\left(-\frac{1}{2\epsilon_c} [\gamma x^2 + 2\alpha x x' + \beta x'^2]\right) \left(1 + \frac{d^2}{6\epsilon_c^2} \left[\epsilon_c \left(\frac{2\alpha}{\ell} - \frac{2\beta}{\ell^2} - \gamma\right) + 2\left(\frac{\alpha x + \beta x'}{\ell}\right)^2 + (\alpha x' + \gamma x)^2 - 2\left(\frac{\alpha x + \beta x'}{\ell}\right)(\alpha x' + \gamma x) \right]\right). \quad (6)$$

This results in a $\cos(2\psi)$ phase dependence for all actions. At large J , when the parameters in Table 1 are used, this variation is approximately 2% of the measured intensity variation in the branches and was generally ignored. However, at low J , this phase dependence needs to be accounted for when defining action.

The effect of the slits can be seen by varying the strength of a quadrupole magnet directly upstream of the Allison scanner to change the Twiss parameters at the Allison scanner (Fig. 2 top). If the slit effect is not accounted for, then the central slope decreases linearly with the quadrupole current (Fig. 2 bottom). After the correction, the central slope is constant within $\pm 5\%$.

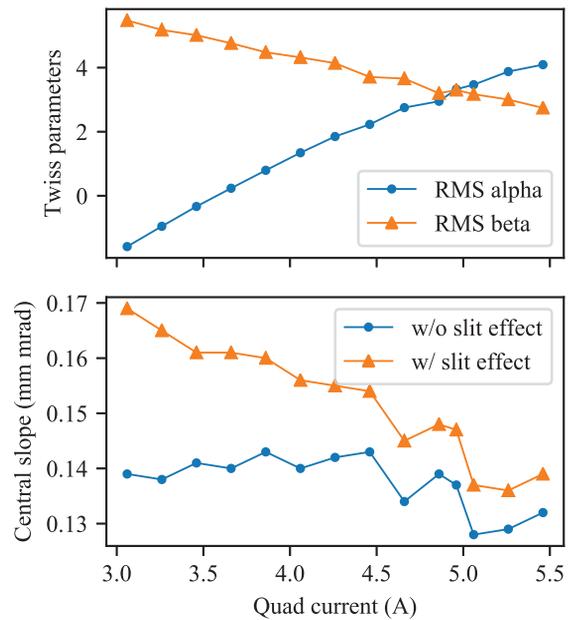


Figure 2: Top: Variation of parameters with quadrupole current. Bottom: The central slope is constant when accounting for the slit size.

Central Twiss Parameters

The action and phase can be defined using any definition of the Twiss parameters and the distribution will remain constant under linear optics. One choice is to use all pixels in the phase portrait to define the Twiss parameters (referred here as ‘rms Twiss parameters’). In this case, the beam tails will affect the analysis resulting in a definition of action that does not well represent the core or the tails of the beam and results in the intensities at low action appearing to be phase dependent.

Alternatively, the action can be defined using pixels in the ‘central’ portion of the beam. The central portion was found by removing the lower intensity pixels of the beam then

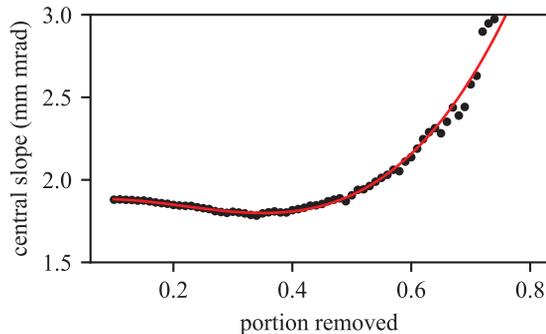


Figure 3: Central slope as a function of the portion of the beam removed. The curve is fit to a cubic polynomial.

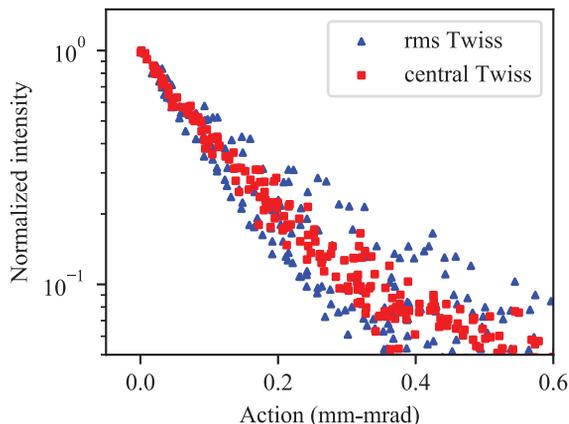


Figure 4: Comparison of intensity scatter at low action when using rms Twiss parameters and central Twiss parameters to define action.

fitting Eq. (6) to find the Twiss parameters and central slope. The fraction removed was scanned from 30-60% of the total intensity in 1% steps. Generally, the central slope increases at large and small cuts (Fig. 3). The increase at small cuts is attributed to the tails affecting the fit and at large cuts poor statistics increases the central slope significantly when the number pixels is below ~ 30 . To avoid both of these effects, the central slope was fit to a cubic polynomial and the cut was chosen to be the point closest to the minimum of the fitted curve.

When these ‘central’ Twiss parameters are used the spread of intensities at low action decreases (Fig. 4) resulting in a phase independent Gaussian core.

STABILITY OVER THE BEAMLINE

The stability of the distribution in action allows for direct comparisons of phase portraits taken at different locations. For example, over the course of 18 months, the Allison scanner in the PIP2IT MEBT was moved to 3 locations along the 10 m beamline: at the beginning of the MEBT measuring the horizontal phase space, in the middle of the

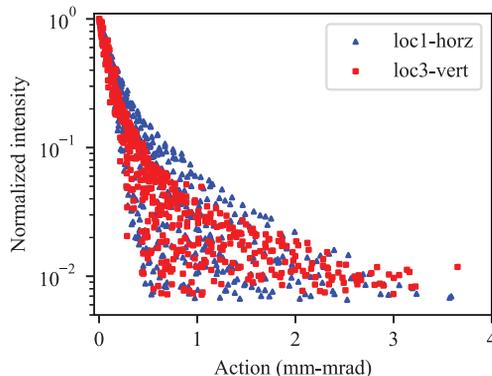


Figure 5: Comparison of intensity distribution over action at beginning and end of the MEBT.

Table 2: Allison Scanner Position Scan

location	rms ϵ	ϵ_c	% in core
1 - horz	0.20±0.013	0.146±0.003	88±2.5
2 - vert	0.19±0.015	0.117±0.013	71±11
3 - vert	0.22±0.024	0.123±0.011	72±10

Values are averaged over 10 scans at each location. Errors are rms.

MEBT measuring the vertical plane, and at the end of the MEBT measuring the vertical plane. Ten phase portraits were chosen at each location and were averaged to minimize day to day variations (Table 2). The measurements were taken of a 5 mA beam with the same LEBT and RFQ settings.

The distributions from location 2 and location 3, which measure the same plane, were similar within rms errors showing no significant effects from non-linear effects and minimal tail growth through the MEBT. Comparing the horizontal distribution, location 1, and vertical distribution, locations 2 and 3, show the distributions over action were different (Fig. 5). Despite having the same rms emittance, the horizontal distributions have a larger central slope and fraction in the core than the vertical ones. This deviation is believed to be a disparity between the two planes and not caused by evolution along the beamline because of the stability between locations 2 and 3. However, both planes were not measured at a single location for a direct comparison.

SUMMARY

The phase portraits measured by the Allison scanner was analyzed using action phase coordinates. After choosing the central Twiss parameters and accounting for the effects of the slits’ size, the core is shown to be Gaussian and phase independent with tails deviating from this distribution and splitting into two branches in phase. The stability of action under linear optics allows for comparison of phase spaces measured under different focusing configurations. Moving the Allison scanner throughout the PIP2IT MEBT showed no significant tail growth or beam evolution due to non-linear forces.

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