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ELECTRON HEATING BY IONS IN COOLING RINGS*

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Abstract

Hadron beam cooling at high energy is a critical technique for Electron-Ion Colliders (EIC). We consider using an electron storage ring for the EIC at BNL. For such a cooler, the electron beam quality plays an important role since it directly determines the cooling rate. Besides the effects of IBS, space charge and synchrotron damping, which are calculable with well known methods, the heating effect by ions also needs to be carefully considered in electron beam dynamics. In this paper, we present an analytical model to calculate the heating rate by ions and give some example calculations. In addition, this model was benchmarked by applying it on the IBS calculation.

INTRODUCTION

Brookhaven National Laboratory (BNL) is proposing an electron ion collider (EIC), based on the existing and highly optimized RHIC ion-ion collider [1]. In order to achieve the full luminosity of eRHIC some beam cooling is required. We will consider an electron cooler based on a storage ring designed to balance emittance growth rates due to intrabeam scattering (IBS). The challenges of such a cooler include long cooling section without solenoids, bunched electron beam cooling at high energy and keeping the low temperature of electron beam for a long time. Recently, the LEReC project has successfully demonstrated hadron cooling using a bunched electron beam at RHIC with no magnetic field in the cooling section [2, 3]. In our design we use a series of wiggler magnets to keep the low temperature of electron beam. The electron beam dynamics are dominated by IBS, radiation damping and heating due to ions. The first two effects have well known models to make estimates [4, 5]. The electron heating by ions is a newer effect [6, 7] which has largely been estimated using conservation of energy arguments.

In this paper we review the conservation of energy approach by applying the Landau/Spitzer formula for thermal equilibration. Next we develop a gas model based on the full Landau collision integral that allows for different temperatures in all three-dimensions. The two models are compared using a beam tracking simulation, and the heating effect by ions is estimated based on eRHIC design. We also applied the gas model to IBS calculation and compared it with the Bjorken-Mtingwa IBS model. The results of the two IBS calculation show a good agreement.

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HEATING MODELS

Spitzer formula

This model is based on the energy exchange between two charged particles during encounter [8]. Considering both the electron and ion beam have Maxwellian velocity distribution but with different kinetic temperatures T_e and T_i, the heating or cooling rate for electron beam can be obtained by

$$\frac{dT_e}{dt} = \frac{T_i - T_e}{\tau_{eq}} \tag{1}$$

where τ_{eq} is the time of equipartition

$$\tau_{eq} = \frac{3m_i m_e (4\pi\varepsilon)^2}{8\sqrt{2\pi} n_i Z^2 e^4 ln\Lambda}\left(\frac{kT_e}{m_e} + \frac{kT_i}{m_i}\right)^{3/2}. \tag{2}$$

This formula gives the average temperature changes of a beam, but it is just a one-demensional formula. So, it cannot accurately estimate the energy change in real conditions. Generally, this formula is perfect for the beam with the same or similar temperatures in each dimension, but not correct when the beam has different temperatures in three-dimensions. In order to get a more accurate formula, we developed a new model called gas model that considered the three-dimensional distribution of beams.

Gas model

We start with the Boltzmann transport equation [9]

$$\frac{\partial f}{\partial t} + \vec{v}\cdot\nabla f + \frac{\vec{F}}{m}\cdot\frac{\partial f}{\partial \vec{v}} = C(f) \tag{3}$$

where $f = f(\vec{x}, \vec{v})$ is the beam distribution function in phase space, $\vec{F}$ is the external force on particles and $C(f)$ is the collision integral. From Eq. (3), we know that the evolution of the beam distribution depends on particle diffusion, external force and the collisions between particles. Our cooling is in a drift section so only coulomb collisions are relevant. We include electron-electron collisions in the IBS rates and only consider electron-ion collisions here [10],

$$\frac{\partial f_e}{\partial t} = C_{ei} \tag{4}$$

where

$$C_{ei} = \frac{\gamma_{ei}}{2}\frac{\partial}{\partial v_\alpha}\int U_{\alpha,\beta}\left(f_i'\frac{\partial f_e}{\partial v_\beta} - \frac{m_e}{m_i}f_i\frac{\partial f_i'}{\partial v_\beta'}\right)d^3v' \tag{5}$$

and the scattering tensor $U_{\alpha,\beta}$ and constant γ_{ei} are

$$U_{\alpha,\beta} = \frac{u^2\delta_{\alpha,\beta} - u_\alpha u_\beta}{u^3}, \quad \gamma_{ei} = \frac{e_e^2 e_i^2 ln\Lambda}{4\pi\varepsilon_0^2 m_e^2} \tag{6}$$

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where v' and f_i' represent the ion velocity and its distribution, $u = v_e - v_i'$ is the velocity difference between electron and ion, $\delta_{\alpha,\beta}$ is Kronecker delta and $ln\Lambda$ is Coulomb logarithm. Assuming the electron beam distribuion have a very small change after each pass of the cooling section, then the rms velocity change of electrons can be written by

$$\int d^3v \frac{\partial f_e}{\partial t} v_{x,y,s}^2 = \langle \dot{v_{x,y,s}^2} \rangle n_e \quad (7)$$

where $n_e = n_e(x, y, s)$ is the electron beam density. Because $m_e << m_i$, we ignore the second term in the Eq. (5) and finally get

$$\langle \dot{v_x^2} \rangle n_e = \gamma_{ei} \int d^3v \int d^3v' f_e(v) f_i(v') \{ \frac{u^2 - u_x^2}{u^3} \frac{v_x^2}{\sigma_{v_x}^2} - \frac{u_x u_y}{u^3} \frac{v_x v_y}{\sigma_{v_y}^2} - \frac{u_x u_s}{u^3} \frac{v_x v_s}{\sigma_{v_s}^2} \} \quad (8)$$

Here the heating rate is a six-dimensional integral of the beam velocities, which is a time-consuming equation in calculation. Considering that both the electron beam and ion beam have a Gaussian velocity distribution

$$f(v) = \frac{n}{\sqrt{(2\pi)^3}\sigma_{v_x}\sigma_{v_y}\sigma_{v_s}} \exp(-\frac{v_x^2}{2\sigma_{v_x}^2} - \frac{v_y^2}{2\sigma_{v_y}^2} - \frac{v_s^2}{2\sigma_{v_s}^2}) \quad (9)$$

put Eq. (9) into Eq. (8) and after some manipulations, the equation of heating rate can be simplified to

$$\begin{aligned} \langle \dot{v_x^2} \rangle = & \frac{\gamma_{ei} n_i}{(2\pi)^3 \sigma_{v_{ex}} \sigma_{v_{ey}} \sigma_{v_{es}} \sigma_{v_{ix}} \sigma_{v_{iy}} \sigma_{v_{is}}} \int d^3u \{ \\ & \frac{u^2 - u_x^2}{u^3} \frac{1}{\sigma_{v_{ex}}^2} I_{x,2} \cdot I_{y,0} \cdot I_{s,0} \\ & - \frac{u_x u_y}{u^3} \frac{1}{\sigma_{v_{ey}}^2} I_{x,1} \cdot I_{y,1} \cdot I_{s,0} \\ & - \frac{u_x u_s}{u^3} \frac{1}{\sigma_{v_{es}}^2} I_{x,1} \cdot I_{y,0} \cdot I_{s,1} \}. \end{aligned} \quad (10)$$

The heating rates in vertical and longitudinal have the same form with Eq. (10), in which $I_{m,n} = I(\frac{1}{2\sigma_{v_{em}}^2}, \frac{1}{2\sigma_{v_{im}}^2}, u_m, n)$ is the simplified integral based on the Gaussian velocity distribution, and it can be directly calculated by

$$\begin{cases} I(a,b,c,0) = \sqrt{\frac{\pi}{a+b}} exp(-\frac{ab}{a+b}c^2) \\ I(a,b,c,1) = -\sqrt{\frac{\pi}{a+b}} \frac{bc}{a+b} exp(-\frac{ab}{a+b}c^2) \\ I(a,b,c,2) = [\frac{1}{2(a+b)} + \frac{b^2c^2}{(a+b)^2}] I(a,b,c,0). \end{cases} \quad (11)$$

From above, we gave the details of the two models to calculate the electron heating rate by the ions. One thing needs to be reminded is that all the equations above are based on the particle reference frame. It's easy to apply these models into a simulation code just like the IBS heating rate on the beam.

SIMULATION

A program was written to integrate the ordinary differential equations derived above. Firstly, we compare the two models by tracking the evolution of a 150 MeV electron beam (N=3×10^{11}) interacting with protons (N=13.6×10^{10}). In the simulation, only the heating effect by the protons and radiation damping effect were considered. For the purpose of comparison, both proton and electron beam initially have the similar temperature in each dimension. The initial temperatures of proton beam that assumed as invariants are $T_x/T_y/T_s = 0.83/0.83/0.78$ keV, which is corresponding to $\epsilon_x/\epsilon_y = 3.1/3.1$ nm, $dp/p = 9 \times 10^{-4}$. The electron beam is $T_x/T_y/T_s = 0.29/0.29/0.28$ eV. The cooler parameters and the radiation integrals are listed in Table 1. The evolution of the heating rates of the two models are shown in Fig. 1. We can see that the Spitzer model and gas model have good agreement at the beginning, and that is due to the similar temperature in each dimension for proton and electron beam. As the process going on, the radiation damping effect changes the balance of the temperatures in three-dimensions, at which the Spitzer model is no longer suitable. So, a difference between the two models occurs in Fig. 1.

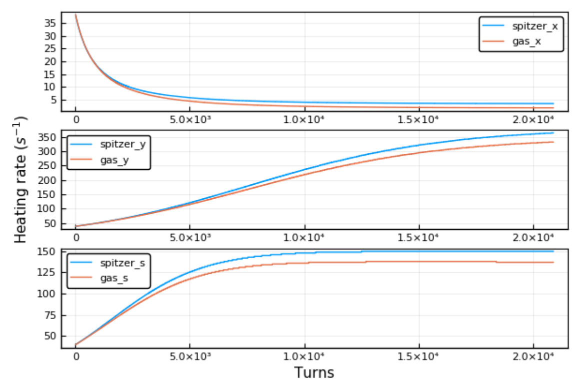

Figure 1: Evolution of the heating rates by ions. The initial beam temperature: Ion: $T_x/T_y/T_s = 0.83/0.83/0.78$ keV and electron: $T_x/T_y/T_s = 0.29/0.29/0.28$ eV (Corresponding to ion: $\epsilon_x/\epsilon_y = 3.1/3.1$ nm, $dp/p = 9 \times 10^{-4}$ and electron: $\epsilon_x/\epsilon_y = 2.0/2.0$ nm, $dp/p = 7.5 \times 10^{-4}$.

Moreover, the program is used to estimate the electron beam distribution based on the eRHIC design parameters [11]. In this calculation, the IBS effect, radiation damping and heating effect by ions are all considered. The cooler and beam parameters are listed in Table 1 and the results are shown in Fig. 2. Based on the calculation, the final electron beam status can be estimated, which is important to estimate the cooling effect on ions and the luminosity on eRHIC. Beside that, we also compared the heating rates due to ions and IBS. It shows that the ratio between the heating rate by ions and IBS heating rate can reach to 1/4 in longitudinal. Therefore, the heating effect by ions plays an imporant role in the electron beam distribution and this effect should be carefully considered in such a cooler ring design.

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Table 1: Parameters of the Cooler and Beams

Name	proton	electron
Circumference (m)	430	
Cooling length (m)	200	
$\beta_{x,y}$ @ cooling section (m)	300	
$\alpha_{x,y}$ @ cooling section	0	
Radiation integrals (I1-I3)	0.43, 4.9×10^3,4.5×10^4	
Radiation integrals (I4-I5)	-20, 1.3×10^3	
Energy	γ= 293.1	γ= 293.1
Bunch intensity (10^{10})	13.6	30
RMS emittance h/v (nm)	20/6.1	2.0/2.0
RMS dp/p	6.6×10^{-4}	5.0×10^{-4}
RMS bunch length (m)	0.07	0.07

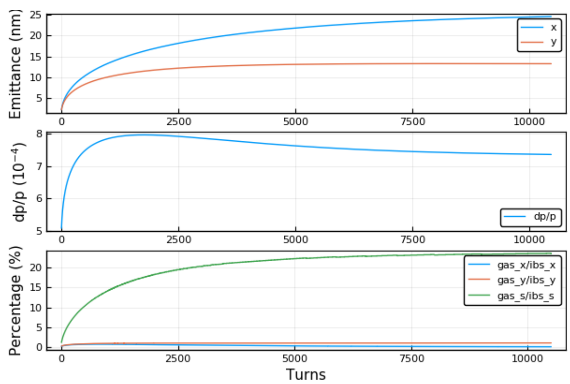

Figure 2: Evolution of the electron beam parameters and the ratio between the heating rate by ions and IBS heating rate.

APPLICATION ON IBS

As we can see from Eq. (4), the gas model can also be applied to electron-electron collisions, which exactly is the IBS effect. There are successful IBS models, such as the Martini model and the Bjorken-Mtingwa model. We can benchmark these with the gas model. Using the same method as before, we get the IBS heating rate formula based on the gas model

$$\langle \dot{v_x^2} \rangle = \frac{\gamma_{ei} n_e}{(2\pi)^3 \sigma_{v_{ex}}^2 \sigma_{v_{ey}}^2 \sigma_{v_{es}}^2} \int d^3u \quad I_{x,1} \cdot I_{y,0} \cdot I_{s,0} \left(\frac{u^2 - u_x^2}{u^3} \frac{1}{\sigma_{v_{ex}}^2} - \frac{u_x u_y}{u^3} \frac{1}{\sigma_{v_{ey}}^2} - \frac{u_x u_s}{u^3} \frac{1}{\sigma_{v_{es}}^2} \right) \tag{12}$$

where $I_{m,n} = I(\frac{1}{2\sigma_{v_{em}}^2}, \frac{1}{2\sigma_{v_{em}}^2}, u_m, n)$. However, this formula is not suitable if there is dispersion or non-zero α function in the lattice, because we assumed a Gaussian beam distribution in phase space without correlation in Eq. (9). That condition is only satisfied in the cooling section of the ring. Therefore, this formula can't give the IBS heating rate of a whole ring.

In order to check Eq. (12), we compared the IBS rates calculated by gas model and Bjorken-Mtingwa model, respectively. Instead of the whole ring, we just calculated the IBS heating rates at a certain position where $\beta = 300m$, $\alpha = 0$ and $D = 0$. The fast numerical method from S. Nagaitsev was used to calculate the B-M IBS rates [12]. Table 2 gives the results for the beam with different initial parameters at the energy of 150 MeV. It shows that the two models are very close, which demonstrates that the integrals in the gas model were done correctly.

Table 2: Comparason of IBS Heating Rate ($\tau_x/\tau_y/\tau_s$)

$\epsilon_{x/y}(nm)$	dp/p	B-M (s^{-1})	Gas (s^{-1})
2.0/2.0	7.5e-4	-0.8/-0.8/1.7	-1.1/-1.1/2.2
5.0/2.0	7.5e-4	-11.3/-12.3/14	-12.6/-15.3/17.6
1.0/2.0	7.5e-4	100/-26.8/-23.8	125/-32.6/-28
2.0/2.0	1.0e-4	-102/-102/11894	-122/-122/14365
2.0/2.0	1.0e-3	12.6/12.6/-14.6	15.5/15.5/-17.5

CONCLUSION

The electron heating effect by ions was investigated and two models were introduced to calculate the heating rate. Base on the simulation, we estimate that the heating effect by ions can reach to 25% of the IBS heating effect. This model was also applied to IBS calculation, and benchmarked with B-M IBS model.

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