# A CODE FOR MULTIBUNCHED BEAMS WITH WAKEFIELDS, GROUP VELOCITIES AND SPACE CHARGE 

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#### Abstract

Tracking multiparticle bunches with PIC codes is possible, but limited to very short distances. Using the modal description these codes provide, PARMTRACK can calculate detailed intra bunch dynamics, and bunch to bunch dynamics for travelling in relatively long beam lines. Macro-particle simulation allows for permanent redistribution in longitudinal as well as in transverse space, without any limiting approximations. Care is taken of group velocities associated with each frequency of the wakefield description. Applications are shown for the part of the CLIC two-beam test facility producing the 30 GHz power.


## Introduction

A Two Beam Linear Collider (CLIC) is studied at CERN for a main linac powered at 30 GHz by a drive linac. To support the study, a CLIC Test Facility has been developed over the last few years. At the entry of the energy transfer line of the CTF drive linac, the nominal parameters are now 13.4 nC per bunch of 0.6 mm rms length, for 48 bunches separated by 10 cm . Total energy dispersion per bunch is $7 \%$. Bunch energy is distributed on a parabola with dispersion also equal to $7 \%$, and an average at 62 MeV [1]. A new code was developed for calculating the effects of the wakes in the $30-\mathrm{GHz}$ power production part, built on the PARMELA code scheme [2], keeping the space charge calculations alive. First results obtained with PARMTRACK [3] for preliminary parameters of transfer structures and of the beam, led to a new design of the structures [4], with high damping of the transverse wake, and new plans for CTF2 such that the beam can be constrained in the apertures in the part which is studied, as demonstrated by PARMTRACK and now other codes [5, 6]. A version of the code has been worked out for parallel computing in the space charge and wakefields routines [7].

## Limits Due to Group Velocities

Group velocities are very important parameters for calculating wakefields and their effects [8, 9]. In the modal description of the delta wake potentials,

$$
\begin{aligned}
& \mathrm{w}_{\delta_{\mathrm{L}}}(\mathrm{t})=-2 \sum \mathrm{k}_{\mathrm{Ln}} \cos \left(\omega_{\mathrm{Ln}} \mathrm{t}\right) \\
& \mathrm{w}_{\delta_{\mathrm{T}}}(\mathrm{t})=2\left(\mathrm{c} / \mathrm{a}^{2}\right) \sum \mathrm{k}_{\mathrm{Tn}} \sin \left(\omega_{\mathrm{Tn}} \mathrm{t}\right) / \omega_{\mathrm{Tn}},
\end{aligned}
$$

where $t$ is the time between the passage of the leading and the test particle, $\omega_{\mathrm{Ln}}$ is the angular frequency for the longitudinal mode, $\mathrm{k}_{\mathrm{Ln}}$ the corresponding loss factor, and a the iris radius.

The wake on the test particle is obtained by summing over all particles having an effect on this particle. Attenuation is accounted for by multiplying by an exponential term.
If the group velocity $\mathrm{v}_{\mathrm{g}}$ for mode n is zero, all particles in front of the test particle contribute to the wake seen by it for this mode. If $\mathrm{v}_{\mathrm{g}}$ is not zero, it is represented by an angle in the longitudinal position versus time graph (Figs. 1 and 2). Test particle $M_{2}$ is influenced by particles in front, such as $M_{1}$, but the energy flowed from position $\mathrm{M}_{1}^{\prime}$ of $\mathrm{M}_{1}$. All particles influencing $\mathrm{M}_{2}$ at its position were on segment $\mathrm{M}_{2} \mathrm{M}_{1}^{\prime}$. The limit is given by the intersection of $\mathrm{M}_{2} \mathrm{M}_{1}^{\prime}$ with the structure entry face $\left(\mathrm{v}_{\mathrm{g}}>0\right)$ or with the structure exit face $\left(\mathrm{v}_{\mathrm{g}}<0\right)$. Designating the structure length by 1 ,

$$
\begin{array}{ll}
\mathrm{v}_{\mathrm{g}}>0: & 0<\mathrm{z}_{2}<\mathrm{v}_{\mathrm{g}}\left(\mathrm{z}_{1}-\mathrm{z}_{2}\right) /\left(\mathrm{c}-\mathrm{v}_{\mathrm{g}}\right) \\
\mathrm{v}_{\mathrm{g}}<0: & 1-\mathrm{v}_{\mathrm{g}}\left(\mathrm{z}_{1}-\mathrm{z}_{2}\right) /\left(\mathrm{c}+\left|\mathrm{v}_{\mathrm{g}}\right|\right)<\mathrm{z}_{2}<1
\end{array}
$$



Fig. 1. Forward wake


Fig. 2. Backward wake

## Frequencies and the Loss Factors

Code MAFIA has been used to calculate the wake potentials resulting from crossing through a small number of cells of the structures by a truncated Gaussian distribution of charge travelling along the structure axis or at 1 mm distance from it [10], in parallel with measurements [4]. These potentials are given as functions of distance $S$ to bunch head. One can try to get mode frequencies and loss factors by minimizing the differences between these results and those of the convolution of the delta wake potentials $\mathrm{w}(\mathrm{s})$ with the distribution. For longitudinal wake,

$$
\mathrm{W}(\mathrm{~S})=\int_{\mathrm{zi}}^{\mathrm{zx}} \mathrm{dz} \int_{\mathrm{si}}^{\mathrm{sx}} \mathrm{w}(\mathrm{~s}) \exp \left(-((\mathrm{s}-\mathrm{S}+\mathrm{n} \sigma) /(\sqrt{ } 2 \sigma))^{2}\right) \mathrm{ds},
$$

where $\mathrm{n} \sigma$ is the bunch half width, z is the distance of the test particle from the structure entry, $s$ is the distance to the
particle in the bunch. Limits are functions of S and $\mathrm{v}_{\mathrm{g}}$. The loss factor obtained for mode $\mathrm{n}=2$ is at least a factor of 100 smaller than for $\mathrm{n}=1$. A comparison of the input resulting from MAFIA with the wake reconstructed by the convolution is shown for 12 cells of a structure similar to the CTF ones. Transverse potential decreases because of $\mathrm{v}_{\mathrm{g}}$ (Fig. 3).


Fig. 3. Wake from MAFIA (dashed) and convolu tion (cont.).

## Main Features of the Calculating Procedures

Code PARMTRACK follows each of the particles of a beam through a given series of elements such as drifts, focusing devices, dipoles, accelerating cavities. For calculating wakefields, as for calculating space charge, it is necessary to know the particle distribution at as many time steps as required by the precision. After each step, the resulting changes in the momenta are superimposed on the momenta due to the other fields. For the space charge, the coordinates of all the particles at a given time are used. For the wakefields, the coordinates of the particles at the time they were sources of the energy flow are also important, as shown above. Coordinates are reconstructed from a series of tables corresponding to selected transverse planes, such as structure entry faces.

## Main Features of Beam Input Data

Accounting for multibunches and wakefields effects makes the calculations complex and limitation of the total number of particles, and therefore also of the number of particles per bunch, is necessary. The distribution is a truncated Gaussian one. A random choice in a Gaussian distribution would require too many samples. So, instead, positions of the particles are taken at centres of equally populated intervals of this Gaussian, giving as many particles as intervals. The series of positions (or phases) in a bunch is the series of increasing values thus obtained and normalized with the rms bunch length. Random permutations in the same series give, after having normalized with corresponding rms values, the initial coordinates $x, d x / d s, y$, dy/ds, $d E / E$ for each particle. Coupling between initial $\mathrm{x}, \mathrm{dx} / \mathrm{ds}$ or initial $\mathrm{y}, \mathrm{dy} / \mathrm{ds}$ is possible.

For each of these particles, three other particles are derived with same longitudinal position but inverse signs on pairs ( x , $\mathrm{dx} / \mathrm{ds}$ ) or ( $\mathrm{y}, \mathrm{dy} / \mathrm{ds}$ ), or both. The beam transverse symmetry imposed by the procedure suppresses a source of artificial wakefields.
By default, the distribution in successive bunches is the same as the one in the first bunch. A variation of the energy from bunch to bunch and a variation of the bunch centre transverse position can be specified.

## Application to CTF2 Drive Beam

A series of a maximum of 6 transfer structures, each of 0.6 m , is installed in the drive beam line after acceleration and compression devices. In longitudinal and in dipole modes, the loss factors are negligible for frequencies higher than the first one. Damping of the transverse mode is vital to restrain the beam in the apertures ( $\mathrm{r}=7.5 \mathrm{~mm}$ for structures, 12 mm for quadrupoles). The difference between the fundamental frequency and the dipole one is also important.

|  | longitudinal | transverse |
| :--- | :--- | :--- |
| frequency $(\mathrm{GHz})$ | 29.9855 | 29.9625 |
| $\mathrm{~V} / \mathrm{pC} / \mathrm{m}\left(/ \mathrm{m}^{2}\right)$ | 55.2 | $1.6310^{4}$ |
| $\mathrm{v}_{\mathrm{g}}$ | $\mathrm{c} / 2$ | $\mathrm{c} / 2$ |
| damping factor $(0.1 \mathrm{~m})$ | no | 0.5 |

By the effects of group velocity, structure length and bunch separation, steady state is reached after 12 bunches. The energy variation with z is shown (Fig. 4), with the position of the structures.


Fig. 4. Minimum, average, maximum energies in bunch 12.
Focusing is provided by triplets in intervals between structures, with identical forces $(0.904 \mathrm{~T}, 0.494 \mathrm{~T})$. The input emittance is $1000 \pi \mathrm{~mm}$ mrad in each plane. Transverse effects are caused by the following displacements:

|  | systematic | random |
| :--- | :---: | :---: |
| bunch to bunch shift $(\mu)$ | 100 | 50 |
| structures | 0 | 100 |
| quadrupoles | 0 | 20 |

The envelope is shown for a calculation made with 17 bunches and 84 particles per bunch (Fig. 5). Alphanumeric digits indicate to which bunch the particle at the limit is related.

Positions of the centres of each bunch cross over but with a similar variation with z (Fig. 6).
The correlation between the variation of the maxima of the envelopes and the variation of the central positions of the bunches may be used for displacing the triplets in a one to one correction scheme. The chosen BPMs allow averaging on the transverse position of the 5 successive bunches whose energies are close to the overall average. Results (Fig. 7) show the good transmission of the beam.


Fig. 5. Envelope in the Y plane (mm).


Fig. 6. Quasi-similar variation of bunch-centre trajectories.


Fig. 7. Envelopes in the Y plane after corrections.

In another study, the parameters were more severe: 21 nC for 1 mm rms bunch length, 43 MeV input energy, damping of 0.3 between bunches.

In this case, the lowest energy at the end is 20 MeV . Focusing was varied from ( $749 \mathrm{~T}, 441 \mathrm{~T}$ ) to ( $663 \mathrm{~T}, 390 \mathrm{~T}$ ) from triplets 1 to 6 , by more than required by the energy drop, but avoiding
over-focusing. The beam is lost on structure 5. With correction, the amplitude of the average position of the bunches is brought from $\pm 3 \mathrm{~mm}$ to $\pm 2 \mathrm{~mm}$ : the beam is lost in the middle of structure 6 (Fig. 8). More damping is needed if 6 structures are used.


Fig. 8 Envelopes in the Y plane after corrections, for 43 MeV .

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