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TRANSVERSE-LONGITUDINAL COUPLING FOR HARMONIC GENERATION AND BUNCH LENGTH MANIPULATION

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Abstract

A general harmonic generation and bunch length manipulation scheme using transverse-longitudinal coupling is presented. The method makes use of the freedom in projecting the three beam eigen-emittances into different physical dimensions. A realization of this coupling lattice, a PEHG variant, is given as an example. For the purpose of harmonic generation and bunch compression, this method is advantageous when the transverse emittance is small. The combination with sawtooth waveform modulation is proposed to boost the bunching further. Transverse-longitudinal coupling in storage rings are briefly discussed.

INTRODUCTION

Harmonic generation methods have the advantages to improve the longitudinal coherence of FEL radiation and shorten the radiator length while bunch length manipulations are useful in some applications of light sources and colliders. Some of the principles they implement apply to both. In this paper we present a general mechanism of harmonic generation and bunch length manipulation by using transverse-longitudinal coupling. The scheme makes use of the freedom in projecting the three beam eigen-emittances into different physical dimensions though their value can not be changed in a linear symplectic lattice [1].

HARMONIC GENERATION

HGHG

A representative harmonic generation scenario is HGHG [2]. Bunching factor formula of HGHG is

$$b_{n,\mathrm{HGHG}} = J_n(nDA\sigma_\delta)e^{-n^2D^2\sigma_\delta^2/2} \tag{1}$$

where $A = \frac{\Delta\gamma_m}{\sigma_\delta}$ is the modulation depth, n is the harmonic number, $D = 2\pi R_{56}/\lambda_m$ with R_{56} the dispersive strength of the chicane, and J_n is the Bessel function of the order n.

Bessel function term in Eq. 1 originates from the *sine* waveform modulation while the Gaussian term from energy spread. So naturally there are two main directions to enhance the bunching factor of high harmonics: one is to broaden the linear zone of modulation waveform for example the sawtooth modulation proposed in [3,4] to eliminate the J_n term and the other is to mitigate the influence of energy spread to make the Gaussian term decrease slower. One example of the second approach is PEHG [5, 6] and the mechanism it invokes is transverse-longitudinal coupling. It can be anticipated that even better bunching can be obtained by combining both approaches.

PEHG

The realization of a PEHG unit can be divided into four key functions:

- a dispersion generation ($R_{16} = \eta_x$) device for example a dogleg to correlate x and δ;
- a laser modulator to introduce energy chirp ($R_{65} = h$) to correlate δ and z;
- a dispersive section (R_{56}) for longitudinal phase space rotation to correlate z and δ;
- a phase-merging unit ($R_{51} = m$) to correlate z and x.

The physical principle of PEHG is easy to understand by observing the longitudinal phase space evolution, which is shown in the lower part of Fig. 1. In the phase space plots of Fig. 1, different colors of the particles means different transverse positions which reflects the transverse-longitudinal coupling introduced by the horizontal dispersion η_x.

Now we simplify the analysis by treating harmonic generation as a bunch compression of particles close to the zero-crossing phase. We linearize the *sine* modulation and the longitudinal coordinate z of a particle after traversing the whole unit is then

$$z_f = mx+(1+hR_{56})\eta_x x'+(1+hR_{56})z+(R_{56}+m\eta_x)\delta \tag{2}$$

For HGHG, $\eta_x = m = 0$, the optimized bunch compression condition is (note the optimized condition for *sine* waveform microbunching is slightly different)

$$1 + hR_{56} = 0 \tag{3}$$

and the final bunch length is

$$\sigma_{z,\mathrm{HGHG}} = |R_{56}|\sigma_\delta = \frac{\sigma_\delta}{|h|} \tag{4}$$

While for PEHG, it becomes

$$\begin{aligned} 1 + hR_{56} &= 0 \\ R_{56} + m\eta_x &= 0 \end{aligned} \tag{5}$$

And the final bunch length is

$$\sigma_{z,\mathrm{PEHG}} = |m|\sigma_x = \frac{\sigma_x}{|h\eta_x|} \tag{6}$$

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Similarly, the bunching factor formula of PEHG is

$$b_{n,\mathrm{PEHG}} = J_n(nDA\sigma_\delta)e^{-n^2D^2(\sigma_x/\eta_x)^2/2} \tag{7}$$

It can be seen from the comparison between Eq. 4 and 6 there is an "effective energy spread" $\frac{\sigma_x}{|\eta_x|}$ for PEHG and PEHG is in favored for harmonic generation or bunch compression when the transverse emittance is small. There could be many variants of PEHG for example using $\eta_{x',y,y'}$ rather than η_x, advantageous in different specific cases and the final bunching factor formulas change correspondingly, but the spirit is the same as analyzed here. For example in [7], an ingenious PEHG variant is invented to generate very high harmonics bunching by making use of the fact that the vertical emittance is much smaller than the horizontal one in usual planar rings.

A PEHG Variant

To better illustrate the mechanism, we divide PEHG into four functions. However there is some flexibility in the order of them. Also note the functions can be realized one by one or two or three of them simultaneously. For example, in the original publication of PEHG [5], a transverse gradient undulator (TGU) is used to realize the energy chirp and phase-merging at the same time. Here we propose a PEHG variant to shift the phase-merging function downwards to the bunch compression section by introducing transverse gradient in the chicane dipoles or inserting quadrupoles inside the chicane. Only normal undulator and other elements are needed in the proposed variant which should make it easier to implement and tune than the original PEHG. The layout and longitudinal phase space evolution of the proposed PEHG variant is shown in Fig. 1.

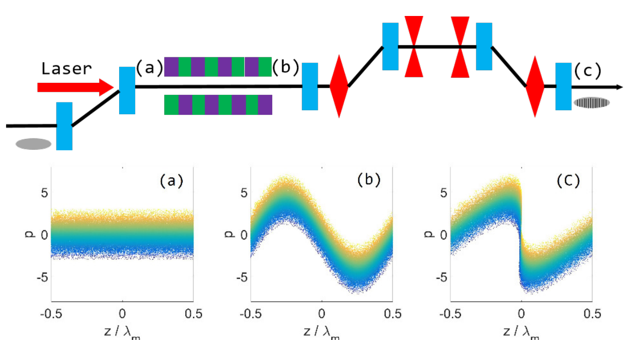

Figure 1: Schematic layout and longitudinal phase space evolution of proposed PEHG variant.

SAWTOOTH WAVEFORM MODULATION

As explained in the above section, the other natural approach to boost bunching is to broaden the linear zone of the modulation wave. An extreme case is replacing the *sine* by sawtooth waveform modulation [3, 4] in which the linearization applies to all the particles and there will be no Bessel function term J_n in the bunching factor formula. Figure 2 and 3 are some simulations of HGHG and PEHG, using *sine* and sawtooth modulation separately and the parameters choice in the simulation are given in Table 1. The modulation strength are kept the same in both waveforms so the R_{56} needed in sawtooth modulation is larger than that in *sine* modulation. This is why the Gaussian term of bunching factor formula in sawtooth decays faster, which can be seen in Fig. 3. An ideal model as sawtooth waveform is, it is easy to approximate a sawtooth waveform by the first two or three harmonics.

Table 1: Parameters Choice in Simulations

Parameter	Value
Energy spread σ_δ	1×10^{-4}
Modulation depth A	5
Emittance ϵ_x	10 pm
Beta function at PEHG entrance β_x	1.6 m
Dispersion of dogleg η_x	1 m
Effective energy spread in PEHG $\frac{\sqrt{\epsilon_x\beta_x}}{\lvert\eta_x\rvert}$	4×10^{-6}

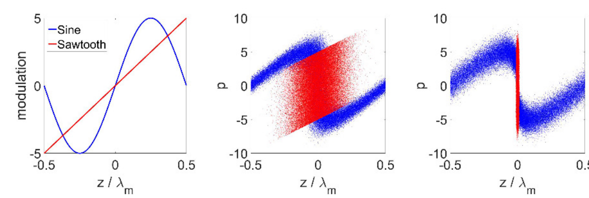

Figure 2: Sine waveform and sawtooth waveform (left) and longitudinal phase space after traversing HGHG (middle) and PEHG (right) unit.

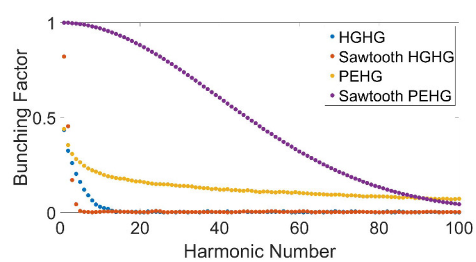

Figure 3: Bunching factors of *sine* and sawtooth modulation.

TRANSVERSE-LONGITUDINAL COUPLING IN STORAGE RINGS

The spirit introduced in the harmonic generation section above by using transverse-longitudinal coupling can also be used for bunch compression in linac. Now we extend the analysis from linac to storage ring.

Planner Uncoupled Lattice

We start from a planar uncoupled ring and ignore the vertical dimension. The state vector is $X = (x, x', z, \delta)^T$. Betatron coordinates, defined by $X_\beta = \mathcal{B}X$, will be used

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to parametrize the one turn map first [8]. The dispersion matrix $\mathcal{B}$ is given by

$$\mathcal{B} = \begin{pmatrix} 1 & 0 & 0 & -\eta_x \\ 0 & 1 & 0 & -\eta_x' \\ \eta_x' & -\eta_x & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{8}$$

Then the linear one turn map for the ring at s is of the form

$$M_{\text{uncoupled}} = \begin{pmatrix} M_x & 0 \\ 0 & M_z \end{pmatrix} \tag{9}$$

Following Courant and Snyder [9], the second moment matrix of betatron coordinate X_β is thus

$$\Sigma_\beta = \begin{pmatrix} \epsilon_x\beta_x & -\epsilon_x\alpha_x & 0 & 0 \\ -\epsilon_x\alpha_x & \epsilon_x\gamma_x & 0 & 0 \\ 0 & 0 & \epsilon_z\beta_z & -\epsilon_z\alpha_z \\ 0 & 0 & -\epsilon_z\alpha_z & \epsilon_z\gamma_z \end{pmatrix} \tag{10}$$

The second moment matrix of physical coordinate X is

$$\Sigma = \mathcal{B}^{-1}\Sigma_\beta\left(\mathcal{B}^{-1}\right)^T \tag{11}$$

We can then get the bunch length from the Σ matrix

$$\begin{aligned} \sigma_z^2 &= \Sigma_{55} \\ &= \epsilon_z\beta_z + \epsilon_x(\gamma_x\eta_x^2 + 2\alpha_x\eta_x\eta_x' + \beta_x\eta_x'^2) \\ &= \sigma_{z_\beta}^2 + \epsilon_x\mathcal{H}_x \end{aligned} \tag{12}$$

There are two contributions for bunch length, one from longitudinal emittance and the other coupled from transverse emittance through $\mathcal{H}_x$ (dispersion). Several authors have also derived Eq. 12 by different means [10–12]. It can be seen for a planner uncoupled lattice, we can play with the longitudinal beta function and horizontal chromatic function $\mathcal{H}_x$ to manipulate the bunch length. For example using $\mathcal{H}_x$ to lengthen the bunch could be helpful in suppressing coherent synchrotron radiation (CSR) and intrabeam scattering (IBS) in low alpha rings.

In many cases a shorter bunch is preferred for example in SSMB [13, 14] while for a planner uncoupled ring the transverse emittance can only cause the bunch length to grow and there is some limitation (mainly from the energy spread) in tuning the longitudinal beta function. But if we relieve our thinking from planner uncoupled to general coupled lattice, then there will be much more possibilities. 6D phase space manipulations can be invoked to tailor the beam for specific parameter for example the bunch length.

General Coupled Lattice

For a general coupled lattice, the Twiss function language of Courant-Snyder is not convenient anymore. 6D phase space vector $X = (x, x', y, y', z, \delta)^T$ is used and the 6×6 general coupled matrices are our starting point. The SLIM formalism [15] is applied to do the analysis.

From SLIM we know

$$\Sigma_{ij} = \langle X_i X_j\rangle = 2\sum_{k=I,II,III}\epsilon_k \mathrm{Re}(E_{ki}E_{kj}^*) \tag{13}$$

in which ϵ_k are the eigen-emittances and E_k are the eigenvectors of the linear one turn map M. Then

$$\sigma_z^2 = \Sigma_{55} = 2\sum_{k=I,II,III}\epsilon_k|E_{k5}|^2 \tag{14}$$

Eq. 14 means the three eigen-emittances contribute independently to σ_z^2 and the weight is determined by the 5^{th} component of eigenvector E_k. It can be seen from Eq. 12 for a planner uncoupled lattice, the weights are the longitudinal beta function β_z and the square root of horizontal chromatic function $\mathcal{H}_x$ separately.

The power of transverse-longitudinal coupling lattice lies in the fact that the coupling transport matrices can manipulate the eigenvectors which means manipulating the projection weights of the three eigen-emittances into different physical dimensions. For example in PEHG, $E_{III,5} = 0$ at the end of the unit which means the contribution from longitudinal emittance on bunch length is totally eliminated.

It can be anticipated there will be many interesting beam dynamics issues when we move to 3D general coupled lattice. One point needs emphasis is that storage ring is different from linac as it is a multi-pass device, we need to make sure the desired distribution is indeed the eigen-distribution of the ring. Study on this topic is ongoing and will be reported elsewhere in the future.

SUMMARY

In this paper, we present a general harmonic generation and bunch length manipulation scheme by using transverse-longitudinal coupling lattice. The physical reason behind this method is the freedom of projecting three beam eigen-emittances into different physical dimensions. Transverse-longitudinal coupling in storage rings are also briefly discussed. More ideas can be invented following this thinking.

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