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DEVELOPMENT OF 6D PARTICLE TRACKING CODE FOR PARTICLE THERAPY SYSTEM

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Abstract

For achieving required specifications of a particle therapy system such as beam profile and beam current, it is important to tune system operation parameters to appropriate values before commissioning. We are developing 6d particle tracking code to analyze whole the through beam motion in a synchrotron from multiturn injection to the RF-knock out extraction for the precise tuning. The code includes effects of multipole magnetic fields and space charge effect. We report on the implementation of the code and discuss about the simulation results.

INTRODUCTION

Hitachi has developed particle therapy system [1,2,3]. The system consists of various components such as bending magnets, quadrupole magnets, sextupole magnets, RF acceleration cavities, and so on. It is important for achieving required specifications of a particle therapy system such as beam profile and beam current to tune the operation parameters of components to appropriate values before a beam commissioning. It leads to reduce the period of the commissioning. Therefore, we have designed the concept of a code for the precise tuning. The characteristic of the code is that it analyze whole the through beam motion in a synchrotron from the injection to the extraction in 6d (transverse and longitudinal) phase space. The code includes effects of multipole magnetic fields and space charge effect. Here, we describe the implementation of code and present the result of tracking in synchrotron with multipole magnetic field and beam acceleration.

IMPLEMENTATION OF CODE

6D Particle Tracking

Developed code is specialized for analyzing beam motion in a synchrotron. The code solves macro particle motion in transverse phase space and longitudinal phase space under operation patterns of components. Operation patterns consists of magnetic rigidity of bending magnet $B\rho$, time derivative of magnetic rigidity $\mathrm{d}B\rho/\mathrm{d}t$, frequency of RF acceleration f_{rf} and peak voltage of RF cavity V_0. Their values depend on time in the calculation t. Flowchart of the code is shown in Figure 1. When a tracking calculation begins, parameters such as time step Δt_n and synchronous phase ϕ_s are calculated based on operation patterns of a synchrotron at every turn. Δt_n and ϕ_s are calculated by Eq.(1) and Eq.(2) respectively. h is harmonic number and C is circumstance of synchrotron.

$$\Delta t_n = h/f_{\mathrm{rf}}. \tag{1}$$

$$\sin\phi_s = C/V_0 \frac{\mathrm{d}B\rho}{\mathrm{d}t} \tag{2}$$

Every time macro particles pass each optical component section (L represents total number of sections), routine of the transverse tracking and the space-charge effect are called. After calculation of transverse motion, longitudinal one is called. The space charge calculation routine is also called here to calculate the influence on longitudinal motion. Above calculation steps are repeated and turn number n is incremented until it reaches the number of total turns N_{turn}. In this paper, we discuss about the implementation and result of transverse and longitudinal tracking with multipole field and acceleration.

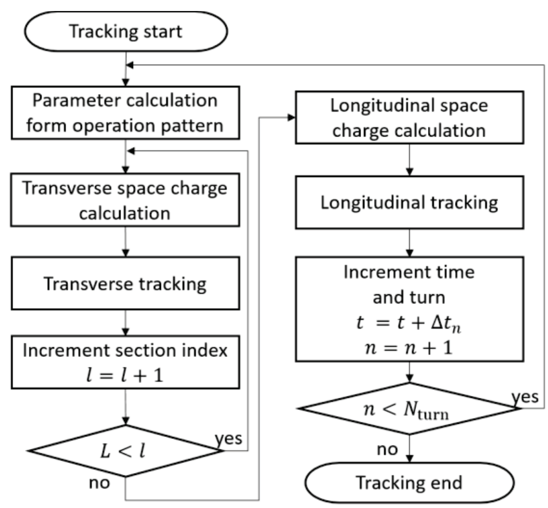

Figure 1: Flowchart of developed code.

Transverse Tracking

Transverse tracking routine mainly consist of two parts. One is linear tracking calculation by transfer matrix and applied to liner optics such as drift space and quadrupole magnets. Each macro particle motion in 6d phase space are update by transfer matrix (Eq.(3)). Transfer matrix M_l is 6×6 transfer matrix [4] at each section indexed by l.

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ s \\ \delta \end{pmatrix}_{\mathrm{new}} = M_l \begin{pmatrix} x \\ x' \\ y \\ y' \\ s \\ \delta \end{pmatrix}_{\mathrm{old}} . \tag{3}$$

Another is nonlinear tracking calculation by thin lens approximation and applied to multipole magnets and bending magnets that has nonlinear components in a fringing field. Nonlinear components are expressed by k_n. The definition of k_n is below.

$$k_n = -\frac{1}{B\rho}\frac{\partial^n B_y}{\partial x^n}. \tag{4}$$

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k_n is estimated based on magnetic field measurement or calculation. Conceptual view of evaluation k_n from magnetic field of a bending magnet is shown in Figure 2. First, magnetic field of is divided into some segments along s-axis. Second, vertical components of magnetic field B_y on a mid-plane at each segment are fitted by polynomial along x-axis. Fitted polynomial coefficients are equivalent to k_n. Finally, the field is transformed into multipole thin lenses and drift spaces.

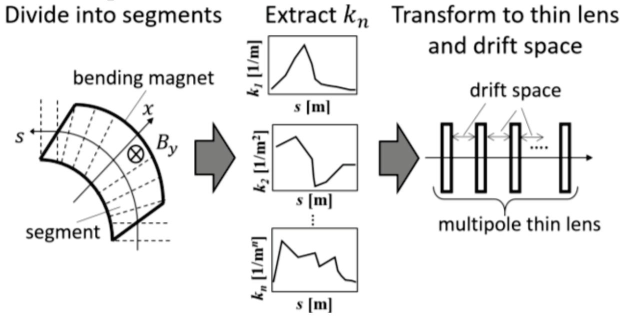


Figure 2: Conceptual view of evaluation multipole components from magnetic field of a bending magnet.

Influence on transverse motion is calculated in Eq.(5) via effective k-value $k_{1\,\mathrm{eff}}$ at multipole thin lens. ρ is bending radius of magnet, and $1/\rho = 0$ if the magnet is straight. $k_{1\,\mathrm{eff}}$ is expressed in Eq. (6). As a result, macro particles are kicked depend on their transverse position.

$$\begin{cases} x'_{\mathrm{new}} = \left(-\frac{1}{\rho} + k_{1\,\mathrm{eff}}\right) x_{\mathrm{old}} + x'_{\mathrm{old}} \\ y'_{\mathrm{new}} = -k_{1\,\mathrm{eff}} y_{\mathrm{old}} + y'_{\mathrm{old}} \end{cases}. \quad (5)$$

$$k_{1\,\mathrm{eff}} = k_1 + k_2 x + \frac{k_3}{2!}(x^2 - y^2) + \frac{k_4}{3!}(x^3 - 3xy^2) + \cdots. \quad (6)$$

Momentum along s-axis p_s is update at every section of synchrotron. On the other hand, total momentum p is preserved while transverse tracking.

$$\begin{cases} p_{\mathrm{new}} = p_{\mathrm{old}} \\ p_{s\,new} = \sqrt{p_{\mathrm{new}}^2 - {p_x}_{\mathrm{new}}^2 - {p_y}_{\mathrm{new}}^2} \end{cases}. \quad (7)$$

Longitudinal Tracking

Longitudinal tracking routine also consist of two parts. One is the part for synchrotron oscillation. Longitudinal distribution at every turn is available by solving canonical equation of motion [5]. We used Euler-symplectic integration method [6] to solve the differential equations (Eq.(8)). W and ϕ are canonical variables and $W = \beta_0 E_0 \delta / \omega_{\mathrm{rf}}$, $\phi = 2\pi h s / C$ respectively. E_0 and β_0 are total energy and ratio of velocity to light speed of reference particles respectively. q is charge of macro particle, and η is slippage factor.

$$\begin{cases} W_{n+1} = W_n + \frac{qV_0}{2\pi h}(\sin\phi_n - \sin\phi_s) \times \Delta t_n \\ \phi_{n+1} = \phi_n + \frac{\omega_{\mathrm{rf}}^2 \eta}{\beta_0^2 E_0} W_{n+1} \times \Delta t_n \end{cases}. \quad (8)$$

Another is the acceleration part. Particle acceleration by RF cavity is expressed in Eq.(9). Each particle gains the energy according to each phase against RF cavity.

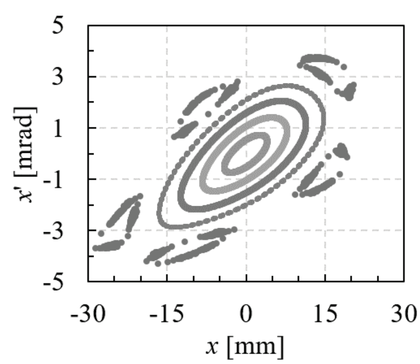
$$E_{n+1} = E_n + qV_0 \sin\phi_n. \quad (9)$$

SIMULATION AND RESULTS

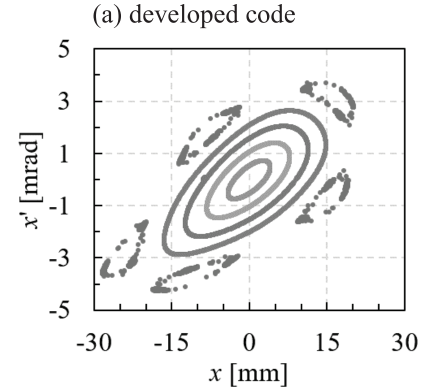
To confirm the validity of the code implementation, we simulated carbon ion (C^{6+}) beam motion in the designed synchrotron [7].

Result of Tracking in Multipole Magnetic Field

As a confirmation of transverse tracking result, we calculated macro particle motion in above synchrotron latice assuming that sextupole components is located in every bending magnet's center. Assumed integral of sextupole component $k_2 l$ equals 5.25 m^{-1}. Initial macro particle is located in (x, y) = (3 mm, 0 mm), (6 mm, 0 mm), (9 mm, 0 mm), (12 mm, 0 mm), (15 mm, 0 mm), (18 mm, 0 mm) against designed orbit. The result of tracking is shown in Figure 3 (a). For comparison, we calculated transverse orbit by MAD-X [8] in the same lattice and condition. The result is shown Figure 3 (b). The orbit of (x, y) = (15 mm, 0 mm) is omitted because the oscillation was unstable. From Figure 3, it turns out that these calculated orbits are agreed well including islands.



(a) developed code



(b) MAD-X

Figure 3: Tracking result in with sextupole fields. (a) : MAD-X, (b) : developed code.

For precise comparison, we calculated betatron tune for each orbit. The relative difference of horizontal tune between MAD-X and developed code is shown in Figure 4. Maximum relative difference is under 0.1% nevertheless

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betatron oscillation amplitude and effect of nonlinearity is large.

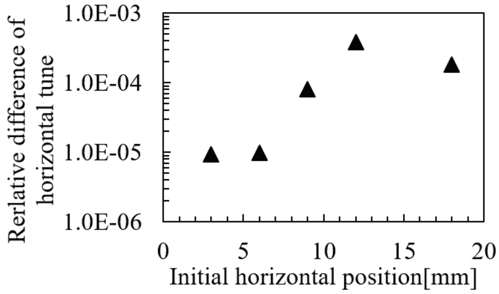

Figure 4: Relative difference of horizontal tune each oscillation amplitude.

Result of Beam Acceleration

Confirmation of beam acceleration is also conducted in the carbon synchrotron lattice and operation patterns. Acceleration scenario is that 4 MeV/u C^{6+} coasting beam is captured and accelerated to 430MeV/u. Number of macro particles is 1,000. Initial horizontal and vertical emittance is 3.9 and 1.4 π · mm · mrad respectively.

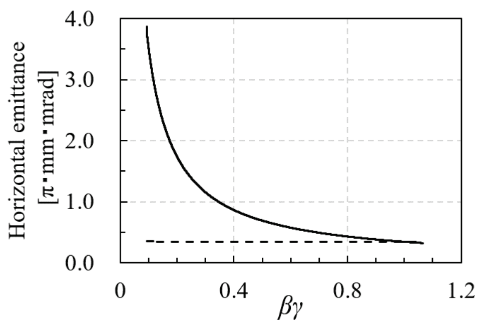

Figure 5: Adiabatic damping of horizontal emittance (solid line: horizontal emittance, broken line: normalized horizontal emittance).

Figure 5 shows the result of adiabatic damping of horizontal emittance during acceleration and that horizontal emittance decreases against $\beta\gamma$. On the other hand, normalized horizontal emittance (product of horizontal emittance and $\beta\gamma$) is found to be preserved.

CONCLUSION

We developed 6d tracking code in synchrotron for particle therapy system parameter tuning. Developed code can calculate beam acceleration in synchrotron with multipole magnetic field. Transverse tracking with multiploe field is calculated by thin lens approximation. Longituginal equation of motion is solved by Euler symplectic integration. The validity of transverse and longitudinal tracking result was confirmed from nonlinear betatron oscillation and adiabatic damping.

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