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SPIN RESPONSE FUNCTION FOR SPIN TRANSPARENCY MODE OF RHIC *

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Abstract

In the Spin Transparency (ST) mode of RHIC, the axes of its Siberian snakes are parallel. The spin tune in the ST mode is zero and the spin motion becomes degenerate: any spin direction repeats every particle turn. In contrast, the lattice of a conventional collider determines a unique stable periodic spin direction, so that the collider operates in the Preferred Spin (PS) mode. Contributions of perturbing magnetic fields to the spin resonance strengths in the PS mode are usually calculated using the spin response function. However, in that form, it is not applicable in the ST mode. This paper presents a response function formalism expanded for the ST mode of operation of conventional colliders with two identical Siberian snakes in the highly-relativistic limit. We present calculations of the spin response function for RHIC in the ST mode.

INTRODUCTION

An experimental test of a new polarization control mode, the Spin Transparency (ST) mode, is planned in RHIC [1]. RHIC is a collider with two helical Syberian snakes. Its spin tune is determined by the angle φ between the snake axes and the stable polarization is vertical in the collider's arcs:

$$\nu = \varphi/\pi, \qquad \vec{n}_{arc} = \pm\vec{e}_y \,.$$

Thus, the spin tune equals one half (RHIC's regular PS mode of operation) if the angle between the snake axes is $\pi/2$. To convert RHIC to the ST mode, when the spin tune is zero, the angle between the snake axes must be set to zero, i.e. the snakes must be identical. From the spin dynamics point of view, RHIC then becomes equivalent to a figure-8 collider [2]. While geometrically obviously still different, the two kinds of rings have identical topologies of the spin motion. The two snakes located opposite to each other in a circular ring divide the ring into two 180° arcs. Due to the action of the snakes, the spin sees opposite fields in the two arcs in exactly the same way as it happens in a figure-8 ring. JINR (Dubna, Russia) develops the NICA collider project with two solenoidal snakes set in the ST mode [3].

The ST mode of a ring removes the integral effect of the whole ring lattice on the spin. In an ideal case, any spin direction on the closed design orbit of a spin transparent ring is periodic. In a realistic case, the spin dynamics is governed by lattice imperfections such as dipole roll and quadrupole misalignments. The spin effect of such imperfections is typically small. However, it breaks the degeneracy of a perfectly spin transparent ring. To regain control of the spin motion, one must introduce relatively weak magnetic elements into the ring whose spin effect is still relatively small but is much greater than the spin effect of the imperfections. To determine the required magnetic field integrals of the spin control elements, one must be able to accurately estimate the expected effect of imperfections, which can be characterized by an average spin field, or a zero-integer spin resonance strength (an absolute value of the average spin field).

The resonance strength can be most conveniently calculated using a periodic spin response function [4, 5]. The response function is a spin Green's function describing the effect of a δ-function-like magnetic field perturbation on the spin at a certain location in the ring. It takes into account the spin effect of the whole ring in response to a local field perturbation. The response function describes the spin effect in the linear approximation and can be calculated using the ring's linear optics. The total effect of the ring imperfections in terms of the spin resonance strength can be obtained by integrating the spin effects of all imperfections around the ring using the response function. This can be done in a statistical sense [6]. A generalized spin response function has earlier been derived for a ring with a preferred spin orientation [7]. However, that formalism has been developed under the assumption of a non-resonant case with an existing unique stable spin direction and is not directly applicable to a resonant case without a preferred spin direction such as the ST mode. In this paper, we present spin response formalism for RHIC with two identical Siberian snakes at opposite locations.

ST MODE SPIN REFERENCE FRAME

We introduce a system of spin unit vectors $\vec{e}_i^{\,s}$ to describe the spin motion. The spin reference frame is periodic in the lab frame and threfore in the accelerator frame because any spin direction in the ST mode repeats itself every particle turn. The choice of the spin unit vectors is arbitrary: for example, it is convenient to align them with the accelerator unit vectors $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ at the observation point. The spin unit vectors $\vec{e}_i^{\,s}$ are then oriented in the detector along the radial, vertical, and longitudinal directions, respectively. In the spin reference frame, changes in the spin components

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can only be caused by perturbing fields arising either due to deviation of the particle motion from the design orbit (associated with the beam emittances) or due to construction and alignment errors of the ring's magnetic elements.

Spin unit vectors are defined in RHIC with its two Siberian snakes in the following way. Vector $\vec{e}_2^{\,s} = \zeta \vec{e}_y$ is vertical in the arcs and changes sign $\zeta = \pm 1$ when passing through a snake from one arc into the other. The two other basis vectors $\vec{e}_1^{\,s}$ and $\vec{e}_3^{\,s}$ lie in the ring's plane making many turns in the arcs

$$\vec{e}_1^{\,s} + i\vec{e}_3^{\,s} = e^{i\alpha(1-\zeta)} e^{i\Psi} (\vec{e}_x + i\zeta \vec{e}_z),$$

where $\Psi = \gamma G \int_0^z K_y \zeta \, dz$ is the spin phase, which is a periodic function of the ring $\Psi(z) = \Psi(z+L)$, L is the orbit length, and α is the angle the snake axes $\vec{m}$ make in the ring's plane with the radial direction $(m_x + i m_z = e^{i\alpha})$.

RESPONSE FUNCTIONS IN ST MODE

In the general case, effect of periodic perturbing fields ΔB_x,ΔB_y, and ΔB_z on the spin is described in the ST mode by three vector response functions, radial $\vec{F}_x$, vertical $\vec{F}_y$, and longitudinal $\vec{F}_z$, which are determined by the ring's lattice [8]:

$$\vec{\omega} = \frac{1}{2\pi} \int_0^L \left[\frac{\Delta B_x}{B\rho} \vec{F}_x + \frac{\Delta B_y}{B\rho} \vec{F}_y + \frac{\Delta B_z}{B\rho} \vec{F}_z \right] dz,$$

where $B\rho$ is the magnetic rigidity. In the spin reference frame, the spin motion represents rotation about the average spin field $\vec{\omega}$, whose magnitude is equal to the zero-integer spin resonance strength (ST-resonance strength): $\omega = |\vec{\omega}|$.

Let us consider the response functions of RHIC in the relativistic limit $\gamma G \gg 1$, which is valid for protons starting from the injection energy. The longitudinal response function components are determined only by the longitudinal components of the spin unit vectors $F_{zi} = (1+G)\vec{e}_z \vec{e}_i^{\,s}$ and do not grow with energy. To the contrary, the transverse response functions change proportionally to energy thus making the spin effect of transverse perturbing magnetic fields dominant at high energies.

RHIC has no design transverse coupling. Out analysis shows that only two radial response function components F_{x1} and F_{x3} are non-zero at high energies. Thus, the main spin effect in the ST mode in RHIC comes from radial perturbing magnetic field and the spin field lies in the ring's plane.

The radial response function is given by

$$F = F_{x1} + iF_{x3} = \frac{\gamma G}{2i} \left[f_y^* \int_{-\infty}^{z} f_y' \left(e^{i\alpha(1-\zeta)} \frac{d}{dz} e^{i\Psi} \right) dz - f_y \int_{-\infty}^{z} f_y^{*\prime} \left(e^{i\alpha(1-\zeta)} \frac{d}{dz} e^{i\Psi} \right) dz \right],$$

where f_y is the vertical Floquet function, which is expressed in terms of the vertical Twiss β function as

$$f_y = \sqrt{\beta_y} \exp\left(i \int_0^z \frac{dz}{\beta_y} \right).$$

Using the periodic property of the Floquet function $f_y(z+L) = e^{2\pi i \nu_y} f_y(z)$, where ν_y is the vertical betatron tune, integrals of the form

$$\int_{-\infty}^{z} f_y' \Phi(z)\, dz, \qquad \int_{-\infty}^{z} f_y^{*\prime} \Phi(z)\, dz,$$

with a periodic function $\Phi(z) = \Phi(z+L)$ can be reduced to integration over one turn

$$\int_{-\infty}^{z} f_y' \Phi(z)\, dz = \frac{\int_0^L f_y' \Phi(z)\, dz}{e^{2\pi i \nu_y} - 1} + \int_0^z f_y' \Phi(z)\, dz,$$

$$\int_{-\infty}^{z} f_y^{*\prime} \Phi(z)\, dz = \frac{\int_0^L f_y^{*\prime} \Phi(z)\, dz}{e^{-2\pi i \nu_y} - 1} + \int_0^z f_y^{*\prime} \Phi(z)\, dz.$$

We used RHIC's injection optics shown in Fig. 1 in the above equations to obtain RHIC's response function in the ST mode. The absolute value of the response function is plotted in Fig. 2 for RHIC's injection at $\gamma G = 45.5$.

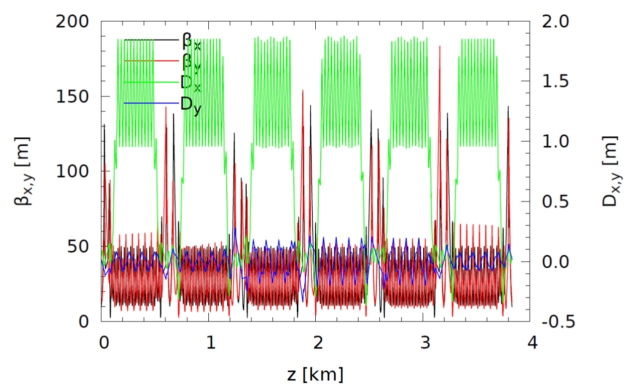

Figure 1: RHIC's injection optics.

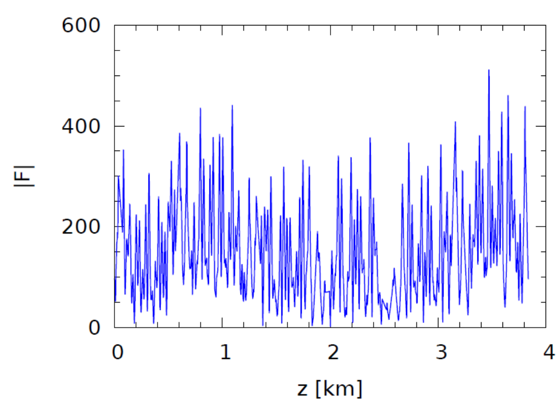

Figure 2: Absolute value of the response function in RHIC at $\gamma G = 45.5$.

ST-RESONANCE STRENGTH

We used the statistical model to evaluate the absolute value of the coherent part of the ST-resonance strength $|\omega_{coh}|$ for RHIC's injection lattice. This is the dominant component of the resonance strength. It assumes independent random

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errors and treats them in a statistical sense. It predicts a resonance strength of

$$|\omega_{\text{coh}}| = \sqrt{\frac{1}{4\pi^2(B\rho)^2} \sum_{\text{elements}} \overline{\Delta B_x^2} |F|^2 L_x^2},$$

where $(\overline{\Delta B_x^2})^{1/2}$ is the rms error of the radial magnetic field in an element, and L_x is the element's length. Radial error magnetic fields arise in a ring due to dipole roll

$$\overline{\Delta B_x^2} L_x^2 = \theta^2 \overline{\Delta\varphi^2} (B\rho)^2$$

and vertical quadrupole misalignments

$$\overline{\Delta B_x^2} L_x^2 = (k_1 L_x)^2 \overline{\Delta y^2} (B\rho)^2,$$

where θ is the dipole bending angle, $(\overline{\Delta\varphi^2})^{1/2}$ is the dipole's rms roll angle, $k_1 L_x$ is the integrated normalized quadrupole gradient, and $(\overline{\Delta y^2})^{1/2}$ is the rms vertical quadrupole misalignment. We chose $\overline{\Delta\varphi^2}$ and $\overline{\Delta y^2}$ that equally contribute to the rms vertical closed orbit excursion and together give an rms closed orbit excursion of 200 µm measured in RHIC. Figure 3 shows the most probable vertical orbit excursion given by

$$\sigma_y = \sqrt{\frac{\beta_y(z)}{8\sin^2(\pi\nu_y)} \sum_{\text{elements}} \frac{\overline{\Delta B_x^2} L_x^2}{(B\rho)^2} \beta_y(z_j)},$$

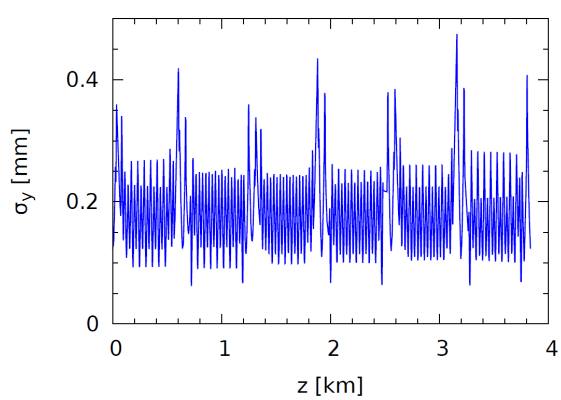

Figure 3: RHIC's most probable vertical orbit excursion around the ring with an rms value of 200 µm.

We used the obtained values of $\overline{\Delta\varphi^2}$ and $\overline{\Delta y^2}$ to calculate the spin resonance strength $|\omega_{\text{coh}}|$ as a function of energy. The result is shown in Fig. 4. Figure 4 indicates that, up to 100 GeV, the resonance strength does not significantly exceed a value of 0.01. This means that the spin control system should provide a spin tune value much greater than that.

CONCLUSION

Our earlier estimates were based on a spin tune of 0.05 [1], which can be easily set in RHIC using the existing Siberian snakes and spin rotators. The calculation in Fig. 4 confirms that a spin tune of 0.05 is sufficient for complete control of the proton polarization at least up to 100 GeV. Above 100 GeV, aside for a few interference peaks, the resonance strength stays at the level of about 0.01. The coherent part of the resonance strength does not cause depolarization. Therefore, it may still be possible to go to higher energies and control the polarization between the interference peaks with a small spin tune. This possibility requires further study.

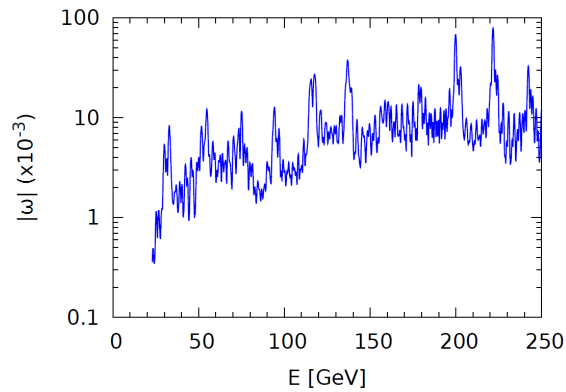

Figure 4: Coherent part of the ST-resonance strength in RHIC as a function of energy.

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