# PROGRESS ON THE OPTICS MODELING OF BMI'S ION RAPID-CYCLING MEDICAL SYNCHROTRON AT BNL* 

François Méot ${ }^{\dagger}$, Piyush Nanubhai Joshi, Nicholaos Tsoupas, BNL, Upton, NY, USA Joseph Paul Lidestri, Best Medical International, Springfield, VA, USA

## Abstract

Optics studies are performed in support to the DC and AC magnetic field measurements, at BNL, on BMI's Ion RapidCycling Medical Synchrotron 60 degree 5-dipole girder.

## INTRODUCTION

BMI's iRCMS is a racetrack ion ring with top rigidity in the $6^{+}$T.m range, aimed at the acceleration of carbon and other ion beams for hadrontherapy (Fig. 1). DC magnetic field measurements have been performed recently, at BNL, on a prototype of the $60^{\circ} 5$-dipole sector, Fig. 2 (the $180^{\circ}$ arc is comprised of 3 sectors, spaced 114.5 m$)$. AC ( 15 Hz ) field measurements are in preparation [1].


Figure 1: iRCMS ion ring.


Figure 2: Geometry of the iRCMS BDH-BF-BD-BF-BDH $60^{\circ}$ sector OPERA field map, with angular extent $\Delta \theta_{F M}$. A reference arc is defined at $R=508.022 \mathrm{~cm} . \delta_{F}=\left(\Delta \theta_{F M}-\right.$ $60) / 2$ degree is an additional (entrance and exit) extent that accounts for BDH field fall-offs. $\delta_{E}=-\delta_{S}\left(=-\delta_{F}\right)$ are the angles that the reference orbit makes with the sector at respectively entrance into and exit from the field map.

[^0]It can be seen in Fig. 2 that the 5 combined function dipoles have a common geometrical wedge center, $\mathrm{R}=508.022 \mathrm{~cm}$ from an arc going through the geometrical center of the wedge dipoles (much in the manner of FFAG rings, where the wedge magnets have their center at the center of the ring). This has two consequences:
(i) the 5 dipoles are $\approx 8 \mathrm{~cm}$ distant from one another over the $60^{\circ}$ sector (the filling factor, magnetic length/arc length is less than 1 ), so that the reference orbit can not be at constant radius, it scallops around the reference $\mathrm{R}=508.022 \mathrm{~cm}$. Note: in the following, "reference orbit" stands for the periodic orbit that undergoes $60^{\circ}$ deviation across the sector and goes in and out at $\mathrm{R}=508.022 \mathrm{~cm}$ normal to the outer face of the BDH dipole.
(ii) the magnet faces are at a $7^{\circ}$ angle to one another, which means non-zero entrance and exit wedge angles,


Figure 3: Optical functions in iRCMS half-ring, in the MADX model. These quantities are taken as the design goals, in particular the $180^{\circ}$ arc is an achromat.

## PARAXIAL OPTICS

A simplified design model of the sector had been developed as a reference in the MADX code [2] and will be recalled. On the other hand, two different OPERA field maps of the $60^{\circ}$ sector are available and have been used recently to derive the optical properties and characterize the optics of the ring based on realist geometry and magnetic field of the sector. However, optics parameters are found out of specifications in latest field map which is derived from the STEP file thus corresponding to the sector as fabricated, and this is confirmed by the magnetic measurements [1] Finally, a realist model in a cylindrical coordinates system so allowing lifting ambiguities concerning the proper setting至 of the 5 dipoles to achieve the expected paraxial parameters produced using MADX.

## MADX Model

The model in MADX assumes the 5 combined function dipoles to be wedge magnets and uses SBEND with wedge $\stackrel{y}{0}$ angles zero. The $7^{\circ}$ angular distance between the dipoles Would require non-zero wedges though, however a marginal perturbation of the focal distance. Magnetic data are given in Table 1, lines 4-7 [2]. Resulting $180^{\circ}$ arc phase advances and ${ }_{\text {of }}$ dispersion are given in line 8 , to serve as a reference in the next 3 different simulations of the sector (note that ideally the arc should be an achromat, transport matrix identity, periodic dispersion zero).

| 츧 Table 1: Magnetic Parameters of iRCMS Sector Dipole . (Field and Gradient Values are for $\mathrm{B} \rho=6.347174 \mathrm{Tm}$ ). In . F the 'DIPOLE' model, A is the wedge magnet angle (ar $\Xi$ length is $R \times A$ ), index $=\mathrm{R} / \mathrm{B} \times \mathrm{dB} / \mathrm{dR}$ is the matched value fo recovering the same sector phase advance as in the MADX 플 model (line 8). In "WM" and "ET" field maps, magnet dat are computed (from the field map) assuming $\mathrm{R}=508.022 \mathrm{~cm}$ and same angles A as in line 9 , whereas field values as show $\pm$ (lines 13 and 16 , respectively) ensure $60^{\circ}$ deviation. |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | BD | BF | BDH |
| "Magnet Review" data: Dipole component |  |  |  |  |  |
|  | 1 Int b1 | Tm | 1.6467 | 1.6368 | 0.86677 |
|  | b1 (peak) | T | 1.30874 | 1.3085 | 1.3048 |
|  | $\mathrm{L}_{\text {eff }}$ | m | 1.2582 | 1.25 | 0.66 |
| Quadrupole component |  |  |  |  |  |
|  | 4 Int G | T | -11.451 | 11.4564 | -5.6677 |
|  | G (peak) | T/m | -9.5803 | 9.5768 | -9.5524 |
|  | $\mathrm{L}_{\text {eff }}$ |  | 1.1953 | 1.19 | 0.5933 |
| Index $=R \times($ Int $G) /($ Int 61$)$ |  |  |  |  |  |
|  | 7 |  | -35.327 | 35.558 | -33.2 |
|  | $8180^{\circ}$ arc $\mu_{x}, \mu_{y}, \eta: 1.988,1.553, \approx 0 \mathrm{~m}$ |  |  |  |  |
| Magnet data in DIPOLE, hard-edge case: |  |  |  |  |  |
|  | A | deg | 14 | 14 | 7.2 |
|  | $10 \quad \mathrm{~B}_{0}$ | T | 1.3667 | 1.3273 | 1.3354 |
|  | 11 Index |  | -36.304 | 36.149 | -33.07 |
| $12180^{\circ}$ arc $\mu_{x}, \mu_{y}, \eta: \approx 2 ., 1.578, \approx 0 \mathrm{~m}$ |  |  |  |  |  |
| Magnet data in WM field map: |  |  |  |  |  |
|  | $13 \mathrm{~B}_{\text {max }}$ | T | 1.322 | 1.322 | 1.322 |
|  | 14 Index |  | -35.234 | 35.552 | -33.030 |
|  | $15 \quad 180^{\circ}$ arc $\mu_{x}, \mu_{y}, \eta: 2,1.606$, |  |  |  |  |
|  | Magnet data in ET field map: |  |  |  |  |
|  | $16 \mathrm{~B}_{\text {max }}$ | T | 1.322 | 1.322 |  |
|  | 17 Index |  | -36.943 |  | -36.637 |
|  |  |  |  |  |  |

The optical functions are given in Fig. 3.

## Polar Coordinate Model

The 5-dipole sector is simulated in Zgoubi using a cylindrical frame (so-called 'DIPOLE' model in the code [3]),


Figure 4: iRCMS sector layout, from the STEP file geometry, as used in OPERA magnet computation.


Figure 5: Scalloping of the reference trajectory, along the 18.25 m long $180^{\circ}$ arc.


Figure 6: Optical functions from the 'DIPOLE' model. It can be observed that the $180^{\circ}$ arc comes out quasi achromatic as expected.
following strictly the geometry in Fig. 4. The magnetic field in DIPOLE writes

$$
\begin{equation*}
B(r)=B_{0}\left(1+N \frac{r-R}{R}\right) \tag{1}
\end{equation*}
$$

The arc length of any dipole is $\mathcal{L}=R \times A$ with $\mathrm{R}=508.022 \mathrm{~cm}$ and A the dipole sector angle, data listed in Table 1. The field integral in this model is the same as for the MADX model, effective field values and transverse indices are converted based on the arc length. A matching procedure is used to tweak the latter in order to recover the same 5-dipole sector focusing (same periodic betatron functions) as in the MADX model, and in particular an achromat $180^{\circ}$ arc; in addition a global scaling factor is applied to the

5 dipoles (the same for all) with the constraint of zero in and out reference orbit. That yields the adjusted index values shown line 11 in Table 1, consistent with (more accurate than) MADX ones, line 7 in Table 1, and a scaling factor very close to one (thus consistent with the expected coil current at the rigidity of concern).

The 5.7 mm amplitude scalloping of the reference orbit is shown in Fig. 5 The phase advances are reasonably close to the MADX model ones as expected from the sector rematching, compare lines 8 and 12 in Table 1.

## Design Field Map

Following the MADX studies an OPERA field map of the $60^{\circ}$ degree sector has been produced [4] (WM, in the following). The footprint is that of Fig. 4, field and indices computed from the map are given in Table 1, lines 13, 14. Arc focusing is given in line 15, Table 1, a perfect achromat, with vertical focusing $\mu_{y} \approx 1.6$ as expected. Optical functions are as expected, similar to Fig. 6. Figure 7 shows the orbit scalloping and the field across the magnet (i) along the $\mathrm{R}=508.022 \mathrm{~cm}$ arc and (ii) along the scalloping reference orbit. Referring to the MADX hypotheses above, the reference orbit should be at constant $\mathrm{R}=508.022 \mathrm{~cm}$, and the field should have the same maximum value across all 5 combined function dipoles (blue curve).

## Step File Field Map

This field map (ET in the following) is a subproduct of the fabrication process (from the STEP file). The footprint is as in Fig. 4, field and indices computed from the map are given in Table 1, lines 16, 17.

Arc focusing is given in line 18, Table 1, close to an achromat, vertical phase advance $\mu_{y} \approx 1.74$ about $12 \%$ stronger than expected, periodic dispersion 0.32 m .

This deserves improvements, which means possibly modifications on the magnet gaps, ways to achieve it, or other possible options, are under study.

## CONCLUSION

The OPERA field map of the $60^{\circ}$ sector under field measurements at BNL shows slight discrepancy with the theoretical expectations. Semi-analytical and field map based simulations discussed here have shown that this results from too strong a gradient ( $10 \%$ about) in the end BDH magnets of the sector. Strategies to overcome this - and to what extent this is necessary - are under study, including a (minor) re-machining of the magnet gap.


Figure 7: Top: $\approx 0.7 \mathrm{~cm}$ scalloping of the reference orbit, along the $60^{\circ}$ WM field map sector. Bottom: field along the $\mathrm{R}=508.022 \mathrm{~cm}$ arc (blue) and along the scalloping reference orbit (red).

## REFERENCES

[1] N. Tsoupas et al., "Electromagnetic Study and Measurements of the iRCMS cell", presented at the IPAC' 19, Melbourne, Australia, May 2019, paper MOPGW123, this conference.
[2] D. Trbojevic, "Magnet design review", Review Committee meeting, BNL (Aug. 2012), unpublished.
[3] Zgoubi Users' Guide, https://www.bnl.gov/isd/ documents/79375.pdf
[4] An OPERA field map due to W. Meng, BNL, 2012.


[^0]:    * Work supported by a TSA agreement between Best Medical International and Brookhaven Science Associates, LLC under Contract No. DE-AC0298CH10886 with the U.S. Department of Energy.
    † fmeot@bnl.gov

