ISBN: 978-3-95450-208-0 doi:10.18429/JACoW-IPAC2019-THPGW087

TRANSVERSE JITTER TOLERANCE ISSUES FOR BEAM-DRIVEN PLASMA ACCELERATORS

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Abstract

Transverse jitter tolerances are considered for beam-driven plasma accelerators. A simple model for jitter transfer from the drive to witness beam is developed and concrete examples are studied for: high-brightness witness bunch injectors; high-energy boosters for FEL's; and future Linear Colliders. Compared with an existing PWFA driver facility ([1, 2]), the calculated tolerances are 18X – 170X tighter than achievable, even considering any upgrades with existing technology.

OVERVIEW

The electron beam-driven Plasma Wakefield Acceleration (PWFA) concept has been actively pursued in the past two decades with multi-GeV accelerating gradients demonstrated [1, 2]. Other test facilities are under construction and aim to demonstrate preservation of the accelerated beam quality [3, 4] and research practical applications such as FEL drivers. As the community moves towards progressing the PWFA concept into a viable engineering solution for a practical accelerator, it is timely to consider requirements for the required supporting infrastructure.

In this report, we consider the jitter requirements on the main particle beams (henceforth referred to as "witness" beams) used in future accelerators powered by PWFA acceleration cells and the contribution from the jitter of the "drive" beams used to form the PWFA acceleration plasma bubble. The drive beam is usually mismatched to the accelerated bunch in terms of its geometric emittance, frequently having order-of-magnitude larger emittance; this is true for laser-driven as well as beam-driven plasmas. The stability requirements of the drive beam are dictated by the phase-space of the higher-quality witness beam.

As specific examples, we consider high-brightness witness bunch injectors (HBI), high-energy "doubler" application for FEL's (EDA), and future Linear Colliders (LC). For each, we investigated the transverse tolerance requirements on the drive beam and the conventional accelerator component tolerances (RF, magnet, alignment etc.) necessary to meet these. The calculated tolerances are compared to an existing PWFA driver facility [2, 3] and are 18 to 170 times tighter than achievable, even considering any up-grades with existing technology. Designing an accelerator facility to deliver the required drive beam stability, then, represents a significant challenge, requiring considerable R&D breakthroughs on many subsystems to develop new technology and understand how these tolerances could eventually be met.

Typical requirements for a beam-driven PWFA application are to drive the plasma cell with a multi-nC electron bunch which is highly compressed (>10 kA peak current) and tightly focused (<<100 μm rms transverse size at plasma entrance). State-of-the-art high-brightness electron accelerators utilizing rf photo-injectors with conventional rf acceleration cavities and magnetic bunch compression systems can meet these requirements, but the achievable bunch emittances are necessarily in the multi μm-rad range. This should be compared with the nm-rad scale of required vertical emittance for the witness beam in a LC application. The acceleration channel seen by the witness bunch in the PWFA cells is formed by the drive beam and by design strongly focuses the witness beam within the plasma channel. It will therefore steer the witness beam according to any misalignment of the driver bunch. Given the large disparity between drive and witness bunch emittances, one would naively expect very tight tolerances on the allowable drive beam jitter.

To investigate the magnitude of the driver jitter tolerance challenge, we put forward a simple analytic model of jitter transfer between the drive and witness beams which we test using a particle tracking model. Using this jitter transfer model, we describe the jitter amplification of a physically realizable plasma cell and calculate the drive beam jitter tolerances implied by the witness bunch jitter requirements. Finally, we look at a currently under-construction accelerator facility (FACET-II) designed to deliver high-compression electron beams and compare the expected jitter performance with the calculated requirements.

WITNESS BUNCH JITTER REQUIREMENTS

The allowable jitter of the witness bunch as it is delivered to either the undulators of an FEL or the collision point of a collider are shown in Table 1 below. The required jitter tolerances for an FEL application are dependent on the design of the undulators, typically the beam jitter must be a small fraction of the beam size in the undulators for lasing to occur and a value of 0.1σ is used here for reference. The LC requirements are more complicated, and discussions can be found in the various LC design reports. The combined effect of *all* jitter sources (i.e. multiple PWFA stages) must sum to beneath the requirements stated below.

Table 1: Required Delivered Witness Beam Jitter to FEL Undulator Section or Collision IP

Application	*Horizontal Jitter Requirement / σ*	*Vertical Jitter Requirement / σ*
PWFA LC	Insensitive	0.3
HBI	0.1	0.1
EDA	0.1	0.1

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* Work supported by the Department of Energy under Contract Number: DE-AC02-76SF00515.

PWFA CELL JITTER TRANSFER MODEL

The PWFA acceleration cell is modeled in 3 sections: a central uniform-density gas section, and an input and output linear density ramp section to match the beam in/out of the accelerating section. The drive beam enters the PWFA cell with arbitrary position and angle and ionizes a plasma column through the cell. This plasma column has extremely large transverse focusing forces (>10^6 T/m). The witness beam enters the plasma column at the correct accelerating phase behind the drive beam and is strongly focused along the plasma channel defined by the drive beam, where it oscillates through many beta function wavelengths. The length of the plasma cell is assumed to be tightly matched to the correct length so that the phase advance of the witness bunch through the plasma channel is such that the witness bunch exits the channel with purely a positional offset.

The witness beam oscillates about the incoming drive beam trajectory. As the drive beam has considerably larger transverse emittance, this causes a "jitter amplification" in terms of comparing the jitter with respect to the relative beam sizes of the drive and witness bunches. This situation is depicted schematically in Fig. 1 below.

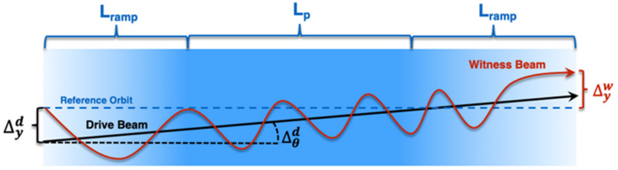

Figure 1: Jitter transfer model for a PWFA acceleration cell. The witness beam is focused along the plasma channel defined by the drive beam.

The beam dimensions within the plasma can be expressed in terms of a Twiss beta function expressed in terms of the plasma density and normalized beam energy:

$$\beta_{d,w} = \sqrt{2\gamma_{d,w}}\,\frac{c}{\omega_p}, \tag{1}$$

$$\omega_p = \left(\frac{e^2 n_p}{\varepsilon_0 m_e}\right) \approx 5.6E4.\sqrt{n_p}\,, \tag{2}$$

where, n_p is the plasma density (cubic cm).

The input and output density ramp sections are configured for a periodic match condition. The optimal ramp length and density profile is given by [5], and for a 2m central accelerating plasma cell length is shown in Fig. 2 (left) below which shows the ramp density profile, where the ramp length is given by:

$$l = \beta_{p0}\sqrt{\left[\frac{(N+1)\pi}{ln\beta_{goal}/\beta_i}\right]^2 + \frac{1}{4}}\,, \frac{L}{l} = \left(\frac{\beta_{goal}}{\beta_i} - 1\right) \tag{3}$$

where N=1,2,3…

Now, assuming equal input and output drive-beam beta functions β and a phase advance of ψ, the position and angle offsets of the witness beam ($\Delta_y^w, \Delta_\theta^w$) as it exits the plasma cell can be written in terms of the position and angle offset of the incoming drive beam (Δ_y^d and Δ_θ^d, respectively) and the total plasma cell length (L=$2L_{ramp} + L_p$):

$$\Delta_y^w = \Delta_y^d(1 - \cos\psi) + \Delta_\theta^d(L - \beta\sin\psi) \tag{4}$$

$$\Delta_\theta^w = (\Delta_y^d/\beta)\sin\psi + \Delta_\theta^d(1 - \cos\psi) \tag{5}$$

Now we can calculate the outgoing witness beam offset due to the drive beam, normalized by the rms witness beam size at the output of the plasma cell. To simplify the expression, we define M = β_{goal} / β_i (note that M is defined by the ramp and is the same for drive and witness beams), β_i is the plasma beta function, and $\varepsilon_y^d, \varepsilon_y^w$ are the drive and witness beam geometric emittance.

Finally, we introduce the factor N as the jitter factor for the drive beam, normalized by rms beam size, and assume that the jitter phase is random so $N^2 = \langle\Delta_y^{d^2}\rangle/\varepsilon_y^d\beta_d M = \langle\Delta_\theta^{d^2}\rangle\beta_d M/\varepsilon_y^d$. Further assuming that the plasma density is perfectly tuned so that there are 2nπ betatron oscillations, the expected outgoing witness beam jitter is:

$$\left(\frac{\Delta_y^w}{\sigma_y^w}\right)^2 = \frac{L^2N^2\{\varepsilon_y^d/\beta_d M\}}{\varepsilon_y^w\beta_w M} \;\; and \;\; \left(\frac{\Delta_\theta^w}{\sigma_\theta^w}\right)^2 = 0\,. \tag{6}$$

Note the strong dependence on M.

BEAM DYNAMICS SIMULATION

To verify the simple analytic model for drive beam to witness jitter transfer outlined above, we built a sliced tracking model using Lucretia [6]. A particle traversing each slice of the PWFA cell feels a radial electric field according to the plasma density in that slice (n_{e0}):

$$E_r = \frac{1}{2}\frac{en_{e0}}{\varepsilon_0}r \tag{7}$$

Each slice is modelled with a radial linear focusing force: $K_q = dE_r/dr\ 1/cB\rho$.

The PWFA configuration shown in Fig. 2 performs a X10 beta function ramp, with Twiss parameters through the PWFA cell shown in the right-hand plot.

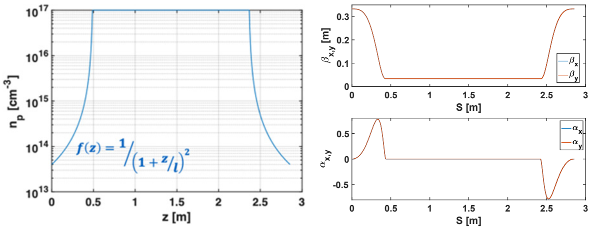

Figure 2: Plasma density profile and Twiss parameters for a 2m PWFA accelerating stage with an incoming/outgoing plasma density ramp according to [6].

Note that no acceleration of the witness beam is considered in this PWFA cell model. This is a reasonable simplification for higher witness beam energies (E_w>>10GeV, assuming ~10GV/m acceleration gradient). For lower energies, the beam dynamics are more complicated, but we assume the basic assumptions set out in the analytic expression hold for the sake of the following arguments.

PWFA LC Application Tracking Example

By offsetting the PWFA element in first position, and then angle, the beam dynamics of the witness bunch as it is tracked through the PWFA cell can be seen in Fig. 3 below. As shown, with the 2nπ phase-advance condition, we are only sensitive to incoming drive beam angles, which manifest as pure positional offsets of the witness beam. The beam parameters for this simulation are shown in Table 2.

Table 2: Beam Parameters for Tracking Simulation Example Shown in Fig. 3

Parameter	*Symbol*	*Value*
Witness energy	E_w	1.0 TeV
Witness emittance	$\gamma\varepsilon^w{}_{x,y}$	10, 0.035 µm-rad
PWFA flat-top length	L_p	2.0 m
β matching factor	M	10
Drive energy	E_d	25 GeV
Drive emittance	$\gamma\varepsilon^d{}_{x,y}$	5, 5 µm-rad

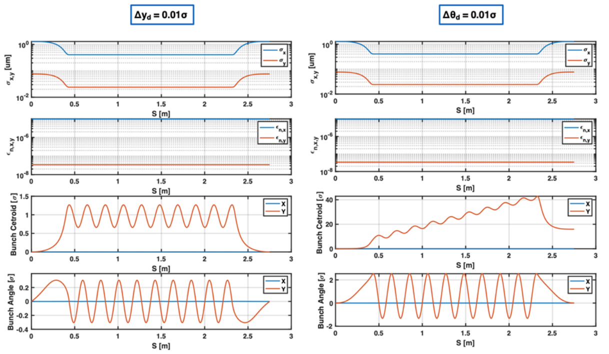

Figure 3: Particle tracking of a witness bunch through a PWFA cell generated with a drive beam with 1% beam size offset (left column) or 1% angular offset (right column of plots). From top to bottom, plots show: rms beam size, emittance, bunch centroid and bunch trajectory angle (each normalized by rms size/divergence).

PWFA LC JITTER TOLERANCE

Using the analytic jitter transfer model introduced above, we calculate the jitter tolerance requirements for the drive beam accelerator assuming an acceleration of 25 GeV per PWFA stage and an initial witness beam energy of 25 GeV. The drive beam energy is set at 25 GeV, with 5 µm-rad normalized transverse emittance. We only consider jitter in the vertical plane, where the witness beam normalized transverse emittance is 35 nm-rad [7]. Table 3 below shows the jitter requirements for the drive beam for a 1 TeV and a 3 TeV collider, assuming a 2m β matching length in each PWFA cell and L_p= 1 or 2m for the flat-top PWFA accelerating length.

Table 3: LC PWFA Jitter Tolerance

L_p	*N:* Jitter Tolerance / $\sigma_{y,y'}$	
	E_{CM}=1 TeV	E_{CM}=3 TeV
1.0 m	4.6E-4	1.1E-4
2.0 m	3.8E-4	9.0E-5

"HBI" APPLICATION JITTER TOLERANCE

Using the parameters shown in Table 4 below, the jitter requirements for a drive beam to produce an accelerated witness beam with <0.1σ jitter is given by *N*=0.02. For this application, there is in fact no incoming witness beam as the witness beam is born within the plasma channel. Here the part of the incoming witness beam is played by the source laser, for which the jitter requirements are not considered.

Table 4: HBI Application Parameters

Parameter	*Symbol*	*Value*
Witness energy	E_w	2.5 GeV
Witness emittance	$\gamma\varepsilon^w{}_{x,y}$	0.05 µm-rad
PWFA flat-top length	L_p	10 cm
β matching factor	M	900
Matching ramp length	L_{ramp}	1.0 m
Drive energy	E_d	10 GeV
Drive emittance	$\gamma\varepsilon^d{}_{x,y}$	5 µm-rad

"ENERGY DOUBLER" APPLICATION JITTER TOLERANCE

An example application considered here is the doubling of the LCLS-II HE beam energy from 8 to 16 GeV using a 1m PWFA acceleration section with a 1m β ramp (M=500). The assumed witness normalized emittance is 0.1 µm-rad and drive beam energy is 10 GeV with 5 µm-rad normalized emittance. Given these parameters, the drive beam jitter tolerance requirement is ***N*=0.008** to provide for a witness beam with <0.1σ jitter at the FEL undulator section.

CALCULATION OF EXPECTED DRIVE BEAM ACCELERATOR JITTER

The required relative beam jitter tolerance for the applications considered above range from 9E-5 to 0.02. To put this in context we consider a design for a real-world accelerator designed to generate electron pulses to drive a PWFA experimental program, FACET-II.

From this study, we conclude for a "warm" driver accelerator source the beam jitter in beam size units is 0.7 x 0.2σ for current technology (horizontal x vertical), and 0.4 x 0.015σ using upgraded components; the horizontal is more sensitive due to coherent synchrotron radiation in the bunch compressors. More detail can be found in [8].

CONCLUSIONS

We put forward here a simple analytic expression for the transfer of jitter from a drive beam to the accelerated witness beam in a PWFA acceleration cell. This was verified by using a separate particle tracking model. Using this jitter transfer model, we evaluated the jitter requirements for a PWFA LC and two FEL driver applications. The required jitter tolerance on the drive beam were found to be in the range 9E-5 to 0.02. Expected jitter values were studied for an injector facility based on FACET-II. This was found to be too noisy by a factor of 170 to drive the PWFA LC and by a factor of >18 times in the horizontal to drive an FEL application even considering upgraded driver accelerator systems within currently understood technology limits.

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