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ENERGY LOSS OF AN ELECTRON BEAM WITH GAUSSIAN DENSITY PROFILE PROPAGATING IN A PASSIVE PLASMA BEAM DUMP

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Abstract

A semi-analytical 1D model is derived for the total energy loss of an electron beam with Gaussian density profile undergoing a passive plasma beam dump. The model is compared to a previous one, obtained for a half-sine longitudinal density profile. It is shown that both models agree if the beam density profiles are properly matched, and if their lengths are small in comparison to the length of wakefield decelerating phase. The beam energy obtained from both models is compared to 1D PIC simulation results.

INTRODUCTION

Under proper conditions, laser pulses or particle beams undergoing a quiescent plasma can drive intense wakefields. Laser-driven [1] and beam-driven [2] plasma accelerators are well known applications [3] for such wakefields, using them to achieve compact acceleration of charged particles [4, 5]. Passive and active (laser-driven) plasma beam dumps use the same physical phenomena of LPA and BPA to obtain compact beam deceleration [6–9]. Moreover, since this deceleration is achieved by interaction with collective fields rather than by particle scattering, the use of plasma beam dumps could reduce radiological hazards such as radioactivation induced by high-energy particles interacting with the beam dump structure.

In a previous work [7], an analytical model was proposed to evaluate the total energy loss of an electron beam with an arbitrary density profile, propagating in a passive or active plasma beam dump. Analytical expressions were obtained for a half-sine longitudinal and parabolic transverse beam density profile, and compared to *particle-in-cell* (PIC) simulations for both passive and active beam dump schemes.

In this work, a semi-analytical 1D model is derived for the total energy loss of an electron beam with Gaussian density profile in a passive plasma beam dump. Matching conditions are obtained to enable the use of previous analytical models [7], derived for beams with half-sine density profiles, to evaluate the total energy loss of beams with Gaussian profiles. The disagreement between both models is shown to be a function of the beam size with respect to the length of the wakefield decelerating phase. Results from both models are compared to a 1D PIC simulation.

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GENERAL MODEL

The model is developed under the following assumptions. A highly-relativistic electron beam ($\gamma \gg 1$) propagates in a plasma with initial velocity close to the speed of light in vacuum, $v_z \simeq c$. While the beam remains relativistic, it is stiff enough to maintain its shape along the propagation. The plasma is treated as a cold-fluid of electrons with density n_0, in a fixed ionic neutralizing background. The beam density profile is given by $n_b(\xi)$, where $\xi \equiv z - ct$ is the co-moving coordinate, $s \equiv ct$ is the propagation distance and, at $s = 0$, the beam is monoenergetic ($\gamma = \gamma_0$ for all its particles) and it has longitudinal momentum $p_z \simeq \gamma m_e c$, where m_e is the electron rest mass. From the linearised equation of motion for the plasma response and the Lorentz force, it follows that

$$d\gamma/ds \simeq -k_p\,(E_{zb}/E_0)\ , \tag{1}$$

where $k_p = \omega_p/c$ is the plasma wave number, $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, e is the electron charge, E_{zb} is the beam self-driven longitudinal plasma wakefield, and $E_0 = c\ m_e\omega_p/e$ is the cold non-relativistic wave breaking electric field. Integration of Eq. (1) over s leads to $\gamma(\xi, s) = \gamma_0 - k_p\, s\,[E_{zb}(\xi)/E_0]$. Defining the beam total energy as $U(s) = \int d\xi\,\gamma(\xi, s) n_b(\xi)/n_0$, and $U(s = 0) = U_0$, the following expression for the normalized beam total energy can be obtained,

$$\frac{U(s)}{U_0} = 1 - k_p\, s\,\frac{\int d\xi [E_{zb}(\xi)/E_0][n_b(\xi)/n_0]}{\gamma_0 \int d\xi\ n_b(\xi)/n_0}\ . \tag{2}$$

Eq. (2) is a simplified, one-dimensional version of the more general expression presented in [7], which can be used to calculate the evolution of the total energy loss for a given beam density profile $n_b(\xi)$, provided that the beam-driven wakefield $E_{zb}(\xi)$ is known. In the linear (up to the quasi-linear) regime, where $n_b/n_0 \lesssim 10$, and $E_{zb}/E_0 \ll 1$, the wakefield can be calculated by solving a Green's function [3],

$$\frac{E_{zb}(\xi)}{E_0} = -k_p \int_\infty^\xi d\xi' \cos\left[k_p(\xi - \xi')\right] n_b(\xi')/n_0 \tag{3}$$

GAUSSIAN DENSITY PROFILE

Longitudinal Wakefield

Assuming an electron beam with a Gaussian longitudinal density profile,

$$\frac{n_b(\xi)}{n_0} = \frac{n_b}{n_0}\ \exp\left(-\frac{\xi^2}{2\sigma_\xi^2}\right)\ ,\ \ n_b \equiv \frac{Q_{1d}}{\sqrt{2\pi} e \sigma_\xi}\ , \tag{4}$$

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where Q_{1d} is the "one-dimensional beam charge", adjusted to match the amplitude of the wakefield produced by a beam with real charge Q in a 3D (or 2D axis-symmetrical) simulation [10], the following expression can be obtained from Eq. (3),

$$\frac{E_{zb}(\xi)}{E_0} = \frac{\pi^{1/2}}{2^{3/2}}\left(\frac{n_b}{n_0}\right) k_p\sigma_\xi e^{-\frac{k_p}{2}\left(k_p\sigma_\xi^2+2i\xi\right)} \times \left[\operatorname{erfc}\left(\frac{\xi - ik_p\sigma_\xi^2}{\sqrt{2}\sigma_\xi}\right) + e^{2ik_p\xi}\operatorname{erfc}\left(\frac{\xi + ik_p\sigma_\xi^2}{\sqrt{2}\sigma_\xi}\right)\right], \tag{5}$$

where i is the complex number, and erfc is the complementary error function. It is worth noting that, by taking the limit of Eq. (5) for $\xi \to -\infty$, the amplitude of the sinusoidal wakefield left behind the beam can be obtained, and it is given by $E_{zb}^{max}/E_0 = \sqrt{2\pi}\,(n_b/n_0)\,k_p\sigma_\xi\,\exp\left(-k_p^2\sigma_\xi^2/2\right)$.

Matched Half-sine Beam Density Profile

If an equivalence between a Gaussian and a half-sine beam density profiles can be established, previous analytical models available for half-sine beams can be used to describe beams with Gaussian density profiles. In order to match the distinct beam density profiles, the following constraints are considered. First, since $E_{zb}/E_0 \propto n_b/n_0$, both profiles should have the same n_b/n_0. Second, the beam volumes (or areas, in the 1D case) should be the same. Then, a half-sine beam density profile, $n_b(\xi) = (n_b/n_0)\sin(\pi\xi/L)$, with $0 \leq \xi \leq L$ (where L is the beam full length), can be matched to the Gaussian profile from Eq. (4) as it follows:

$$\int_0^L d\xi\,\sin\left(\frac{\pi\xi}{L}\right) = \int_{-\infty}^{\infty} d\xi\,\exp\left(-\frac{\xi^2}{2\sigma_\xi^2}\right). \tag{6}$$

By solving Eq. (6), the matching condition $L = (\pi^{3/2}/\sqrt{2})\sigma_\xi$ is obtained. The same procedure can be applied to match a transverse parabolic profile, $n_b(r)/n_0 = (n_b/n_0)\left(1 - r^2/r_b^2\right)$, $0 \leq r \leq r_b$ (where r_b is the beam radius), and a Gaussian profile, $n_b(r)/n_0 = (n_b/n_0)\exp\left[-r^2/(2\sigma_r^2)\right]$. In this case, the matching condition is $r_b = 2\sigma_r$. Figure 1 shows a Gaussian beam and the respective matched half-sine density profile, as well as the longitudinal wakefields excited by both cases. This 1D case is equivalent to a 2D axis-symmetric Gaussian beam with $\sigma_\xi = 2\,\mu$m, $\sigma_r = 1.4\,\mu$m, and charge $Q \simeq 10$ pC, propagating in a plasma with density $n_0 \simeq 4.4 \times 10^{17}$ cm^{-3}. For this set of parameters, suitable, for example, for the EuPRAXIA [11], a wakefield with $E_{zb}^{max}/E_0 \simeq 0.09$ is excited. The matched half-sine density profile has $L \simeq 8\,\mu$m, and $k_pL \simeq 1$. Figure 1(a) shows the wakefield inside the beam, in the region $-3k_p\sigma_\xi \leq k_p\xi \leq 3k_p\sigma_\xi$, and Fig. 1(b) shows the sinusoidal wake left in the plasma behind the beam. In both cases, there is good agreement between the Gaussian beam wakefield (red, solid line), obtained from Eq. (5), and the matched half-sine beam wakefield (darker red, dotted

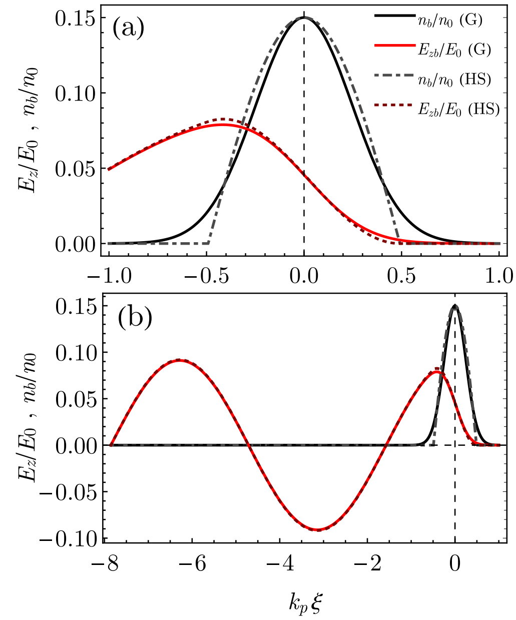

Figure 1: Gaussian and matched half-sine beam density profiles, with their respective wakefields showing good agreement. Panel (a) shows the region inside the beam, and panel (b) shows the sinusoidal wake behind the beam.

line), obtained from [7]

$$\frac{E_{zb}^{HS}(\xi)}{E_0} = \frac{\pi k_pL\,(n_b/n_0)}{\pi^2 - k_p^2L^2}\left[\cos\left[k_p(L-\xi)\right] + \cos\left(\frac{\pi\xi}{L}\right)\right]. \tag{7}$$

Beam Energy Loss

Eqs. (4) and (5) can be inserted in Eq. (2), which can be solved to obtain, $U(s)/U_0 = 1 - k_p\,s\,[dU/ds]_{\text{G}}$, where $[dU/ds]_{\text{G}}$ is the rate of total energy loss with respect to the propagation coordinate $s = ct$ for a Gaussian beam, given by the following expression:

$$\left[\frac{dU}{ds}\right]_{\text{G}} = \int_{-\infty}^{\infty} d\xi \frac{k_p(n_b/n_0)}{4\gamma_0}\, e^{-\left(\frac{k_p^2\sigma_\xi^4+\xi^2+2i\sigma_\xi^2k_p\xi}{2\sigma_\xi^2}\right)} \times \left[\operatorname{erfc}\left(\frac{\xi - ik_p\sigma_\xi^2}{\sqrt{2}\sigma_\xi}\right) + e^{2ik_p\xi}\operatorname{erfc}\left(\frac{\xi + ik_p\sigma_\xi^2}{\sqrt{2}\sigma_\xi}\right)\right]. \tag{8}$$

By solving Eq. (8) numerically, it is possible to evaluate the total energy loss of a Gaussian beam as it propagates in an uniform plasma. An expression analogous to Eq. (8) can be obtained from [7] for a half-sine beam, $U(s)/U_0 = 1 - k_p\,s\,[dU/ds]_{\text{HS}}$, where

$$\left[\frac{dU}{ds}\right]_{\text{HS}} = \frac{\pi^3 k_pL(n_b/n_0)\cos^2\left(k_pL/2\right)}{\gamma_0\left(\pi^2 - k_p^2L^2\right)^2}. \tag{9}$$

From Eq. (9), which can be solved analytically, one can see that the rate of energy loss is maximum when the normalized

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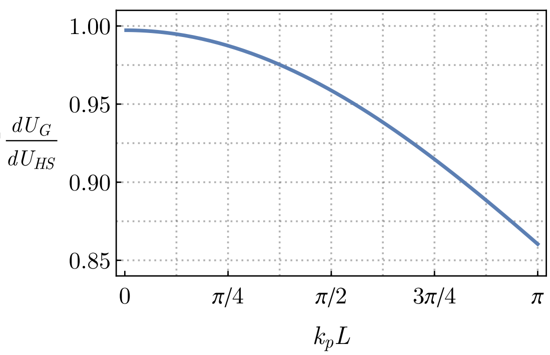

Figure 2: Ratio of total energy loss rates from the Gaussian and half-sine models, respectively. For short beams ($k_pL \ll \pi$), both models are equivalent ($dU_G/dU_{HS} \simeq 1$). As $k_p \rightarrow \pi$, the ratio decreases from the unity to $dU_G/dU_{HS} \simeq 0.86$.

beam length occupies the full decelerating phase, $k_pL \rightarrow \pi$. Figure 2 shows the ratio of both beam total energy loss rates, dU_G/dU_{HS}, given by Eqs. (8) and (9) respectively. For short beams, both models are equivalent ($dU_G/dU_{HS} \simeq 1$). As $k_pL \rightarrow \pi$, the ratio decreases to $dU_G/dU_{HS} \simeq 0.86$. This parameter can be used as a correction factor if the half-sine analytical model is adopted to describe a Gaussian beam.

PIC Simulation

A 1D *particle-in-cell* (PIC) simulation was performed with Epoch [12] to test the model for the total energy loss of a Gaussian beam. The Gaussian model is also compared to the previous one, derived for a half-sine beam. Beam and plasma parameters are the same previously used to plot the wakefields. Figure 3 shows the beam total energy loss as a function of the propagation distance, $U(s)/U_0$. PIC simulation (blue) shows a linear energy loss until $s \simeq 20$ cm. After this distance, the energy loss is compensated by re-acceleration of particles in the accelerating phase of the wakefield. Analytical estimates for the energy loss of the Gaussian (red, solid) and the half-sine (darker red, dotted) are shown. By calculating the maximum value of the wakefield inside the beam, E^*_{zb}, one can estimate the energy loss saturation distance, $s^* = \gamma_0/[k_p(E^{max}_{zb,in}/E_0)] \simeq 20$ cm, and the minimum energy achieved, $U(s^*)/U_0 = 1 - k_p[dU/ds]_G \simeq 0.44$. Further extraction in the passive beam dump scheme is possible by tailoring the plasma density profile [9].

CONCLUSION

A 1D semi-analytical model was derived for a Gaussian beam propagating in a passive plasma beam dump. By matching a Gaussian by a half-sine profile, analytical models available for half-sine beams [7] can be used to estimate the energy loss of Gaussian beams. Moreover, this model can be extended to a 2D axis-symmetrical geometry if the condition $r_b = 2\sigma_r$ is adopted in the parabolic transverse components of the previous model [7]. The rate of energy loss with respect to the beam propagation distance $s = ct$ of both Gaussian and equivalent half-sine beams were compared.

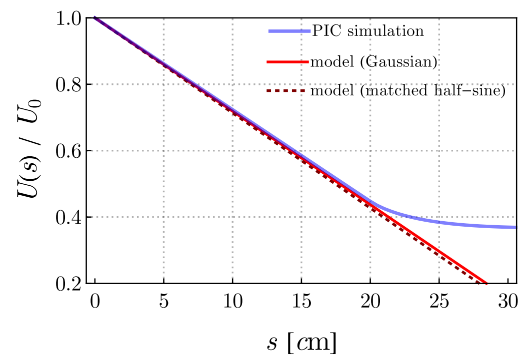

Figure 3: Total energy loss from PIC simulation (blue), Gaussian beam model (red) and half-sine beam model (darker red, dotted). Parameters are the same previously adopted.

The disagreement, which is small for $k_pL \ll \pi$, increases as $k_pL \rightarrow \pi$. As the beam gets wider, the Gaussian back tail eventually reaches the accelerating phase of the wakefield, attenuating the beam total energy loss for this density profile. Both models show good agreement with 1D PIC simulation results if $k_pL \ll \pi$.

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