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STUDY OF FLUCTUATIONS IN UNDULATOR RADIATION IN THE IOTA RING AT FERMILAB*

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Abstract

We study turn-by-turn fluctuations in the number of emitted photons in an undulator, installed in the IOTA electron storage ring at Fermilab with an InGaAs PIN photodiode and an integrating circuit. Our study was motivated by the previous experiment [1]. We propose a theoretical model for the experimental data from [1] and in our own experiment we attempted to verify the model in an independent and more systematic way. Moreover, these fluctuations are an interesting subject for a study by itself, since they act as a seed for SASE in FELs. We improve the precision of the measurements from [1] by subtracting the average signal amplitude using a comb filter with a one-turn IOTA delay, and by using a special algorithm for noise subtraction. We obtain a reasonable agreement between our theoretical model and experiment. Along with repeating the experiment from [1], which was performed at a constant beam current, we also collect data for fluctuations in undulator light at different beam current values. Lastly, in our experiment we were able to see the transition from Poisson statistics to Super-Poisson statistics for undulator light, whereas in [1] only the latter statistics was observed.

INTRODUCTION

Reference [1] reports on the results of experimental studies of statistical properties of undulator and bending-magnet light in an electron storage ring at BNL. A silicon PIN photodiode combined with an amplifier and an integrator were used to obtain a signal (the number of photoelectrons n_e) proportional to the number of emitted photons per turn. Then, the average amplitude of this signal ($\bar{n}_e$) was varied by a set of neutral density (ND) filters, and the dependence of $\mathrm{var}(n_e)$ on $\bar{n}_e$ was studied. Experimental data from this experiment are plotted in Fig. 1. In this plot, the noise of the apparatus (shown by the red line) was subtracted. The authors concluded that for the bending-magnet radiation $\mathrm{var}(n_e) \propto \bar{n}_e$, and for the undulator radiation $\mathrm{var}(n_e) \propto \bar{n}_e^2$. However, only a qualitative explanation of the results was provided in [1]. Here, we present a theoretical model for the effect, that can predict the fluctuations very precisely, and then we repeat the BNL experiment in IOTA [2] with several major improvements in the setup.

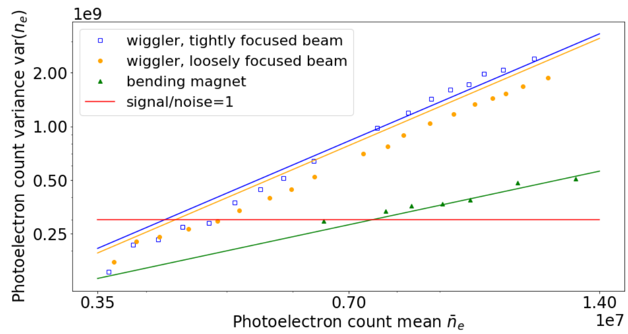

Figure 1: Experimental data from [1] and our model's predictions (solid lines), log-log plot.

THEORETICAL MODEL

It was shown in [3,4] that any classical current produces a radiation with Poisson statistics. Since a bunch of electrons in a bending magnet or in an undulator constitute a classical current (negligible electron recoil), one may argue that turn-by-turn statistics for both of these kinds of radiation in a storage ring is Poissonian, i.e., $\mathrm{var}(n_\gamma) = \bar{n}_\gamma$, where n_γ is a number of emitted photons. However, it is not correct, because every turn relative positions of the electrons in the bunch change and hence, every turn, it is a new classical current. That is, every turn the electrons interfere differently, producing different amounts of emitted power. These effects result in the following equation for variance of the number of emitted photons (n_γ)

$$\mathrm{var}(n_\gamma) = \bar{n}_\gamma + \alpha \bar{n}_\gamma^2, \quad (1)$$

where α depends on the kind of radiation (undulator, bending-magnet, etc.) and the bunch parameters. Although the Poisson contribution in Eq. (1) is related to the quantum nature of emitted light, the interference contribution (the second term) is purely classical [5]. The expression for α takes the form $\alpha = \Delta/\bar{n}_\gamma^2$, with $\bar{n}_\gamma = N_e \tilde{n}_\gamma$ and

$$\Delta = N_e (N_e - 1) \frac{\sqrt{\pi}}{\sigma_z} \int k^4 dk d\Omega_1 d\Omega_2 \times$$
$$I(k\boldsymbol{n}_1) I(k\boldsymbol{n}_2) e^{-k^2\sigma_x^2(\theta_{1x}-\theta_{2x})^2 - k^2\sigma_y^2(\theta_{1y}-\theta_{2y})^2}, \quad (2)$$

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where the direction unit vectors $\boldsymbol{n}_1$ and $\boldsymbol{n}_2$ are defined as $\boldsymbol{n}_1 \approx (\theta_{1x}, \theta_{1y}, 1)$, $\boldsymbol{n}_2 \approx (\theta_{2x}, \theta_{2y}, 1)$, N_e is the number of electrons in the bunch, k is the magnitude of the wave-vector, $d\Omega_1$ and $d\Omega_2$ are infinitesimal elements of solid angle, σ_x, σ_y, σ_z are transverse and longitudinal sizes of the bunch (Gaussian shapes are assumed), $I(\boldsymbol{k})$ is a quasi-classical spectral-angular photon number distribution for a single electron:

$$I(\boldsymbol{k}) = \frac{d\tilde{n}_\gamma}{d\boldsymbol{k}}, \qquad \tilde{n}_\gamma = \int d\boldsymbol{k} I(\boldsymbol{k}), \tag{3}$$

with $d\boldsymbol{k} = dk_x dk_y dk_z = k^2 dk d\Omega$, $\tilde{n}_\gamma$ is a number of photons emitted by a single electron. In the above derivations, we considered only one polarization, the beam divergence was assumed negligible, and $k\sigma_z \gg 1$. To get a better understanding of Eq. (2), note that for large σ_x and σ_y, when the integrand acts like $\delta(\theta_{1x} - \theta_{2x})\,\delta(\theta_{1y} - \theta_{2y})$, α takes a simple form:

$$\alpha \propto \frac{T_\gamma}{\sigma_x \sigma_y \sigma_z}, \tag{4}$$

where T_γ is the length of the electromagnetic pulse emitted by one electron. Equation (4) indicates a connection between α and the mode parameter M (see [6]). Since for undulator light $T_\gamma = N_u \lambda_{\rm rad}$ and for bending-magnet light $T_\gamma = \lambda_{\rm rad}$ (see [7]), it is clear that it is harder to see the square dependence of var(n_γ) for bending-magnet radiation, and it qualitatively explains the results of [1]. We also used the values of parameters from [1] and computed Eq. (2) numerically with expressions for $I(\boldsymbol{k})$ for wiggler radiation ($K_u > 1$) from [8]. Our model's predictions are plotted in Fig. 1 along with the experimental data. The agreement is good. However, the details of the experimental conditions are not available anymore. In particular, it is difficult to analyze the reasons for the small systematic discrepancy for the "loosely focused beam" data. These considerations, combined with the fact that the interference contribution to the fluctuations acts a seed for SASE in FELs [5,9], motivated us to carry out an independent experiment and to study the fluctuations in a more detailed and systematic way.

MEASUREMENTS

Our studies were performed in the IOTA ring and only concerned undulator radiation for now. Main parameters of the experiment are given in Table 1. We used an InGaAs PIN photodiode (Hamamatsu G11193-10R) connected to an op-amp based transimpedance amplifier with a regular RC low-pass filter ($R_{\rm f} = 10\,\mathrm{k\Omega}$, $C_{\rm f} = 2\,\mathrm{pF}$, $\tau = 20\,\mathrm{ns}$) in the feedback. Thus, the amplitude of the signal was proportional to the number of emitted photons. To considerably improve signal-to-noise ratio, we used a comb filter with a delay equal to exactly one revolution in IOTA (133 ns). That is, we used a signal splitter to obtain two copies of the initial signal from the photodetector, then one of the copies was delayed by one IOTA revolution, and then the two copies went to a hybrid, which yielded their sum and difference, which, in their turn, went to two channels of a Rohde&Schwarz RTO 1044 scope @20 GSa/s. We will refer to them as Σ- and Δ-channels, respectively. One experimental dataset constituted a scope waveform of about 11000 IOTA revolutions.

Table 1: Experiment Parameters

IOTA circumference	40 m (133 ns)
Beam energy	100 MeV
Max average current	3.5 mA
σ_x, σ_y @1.3 mA	670 µm, 54 µm
σ_z @1.3 mA	20 cm
Undulator parameter K	1.0
Undulator period	55 mm
Number of periods	10
First harmonic wavelength	1077 nm
Photodiode diameter	1 mm
Quantum efficiency @1077 nm	80 %

We will denote the signal from the photodetector averaged over many revolutions by $S(t)$ (where t is within one turn). Consider two consecutive revolutions with relative amplitude deviations δ_1 and δ_2, so that the signals going to the hybrid are given by $S_1 = (1+\delta_1)S(t)$ and $S_2 = (1+\delta_2)S(t)$, then the two outputs of the hybrid are $\Sigma = S_1 + S_2 \approx 2S(t)$ and $\Delta = S_1 - S_2 = (\delta_1 - \delta_2)S(t)$. Basically, our goal was to measure the turn-by-turn variance of the amplitude of the signal from the photodetector $A = (1+\delta)S(t_{\max})$, where δ is different for each revolution, and $t_{\max}$ corresponds to the maximum of $S(t)$. Since $\mathrm{var}(a-b) = \mathrm{var}(a) + \mathrm{var}(b)$ for independent a and b, it follows that $\mathrm{var}(\delta_1 - \delta_2) = 2\mathrm{var}(\delta)$. Hence, var($A$) can be found as $\mathrm{var}(A) = \mathrm{var}(\Delta(t_{\max}))/2$.

The above formula for var(A) can be used when noise is negligible. However, in the actual experiment with the undulator, in Δ-channel, signal/noise$\lessapprox 1$. Therefore, we had to develop a special algorithm to subtract noise. Its idea is that if we take the equation for Δ-channel at a fixed time t

$$\Delta(t) = (\delta_1 - \delta_2)S(t) + \text{noise}, \tag{5}$$

and take a variance over many IOTA turns, then we get

$$\mathrm{var}(\Delta(t)) = \mathrm{var}(\delta_1 - \delta_2)S^2(t) + \mathrm{var}(\text{noise}), \tag{6}$$

where variance of noise is just a constant. On top of this constant level there is a peak $\propto S^2(t)$, and one can deduce var(A) from the height of this peak.

To test the proposed algorithm and find its error we took measurements for a test light pulse source (laser diode @1064 nm modulated by a pulse generator) with almost identical to IOTA's time structure, but with larger fluctuations, determined by errors in the modulating pulse generator (the laser diode's own fluctuations are negligible [10,11]). The fluctuations were much larger than the noise in Δ-channel and could be reliably measured, they also remained constant with time. Then, we use a number of different ND filters to reduce signal amplitude. In this case, the fluctuations should decrease in proportion with the amplitude, since ND filters do not change relative fluctuations. Thus, we have a very

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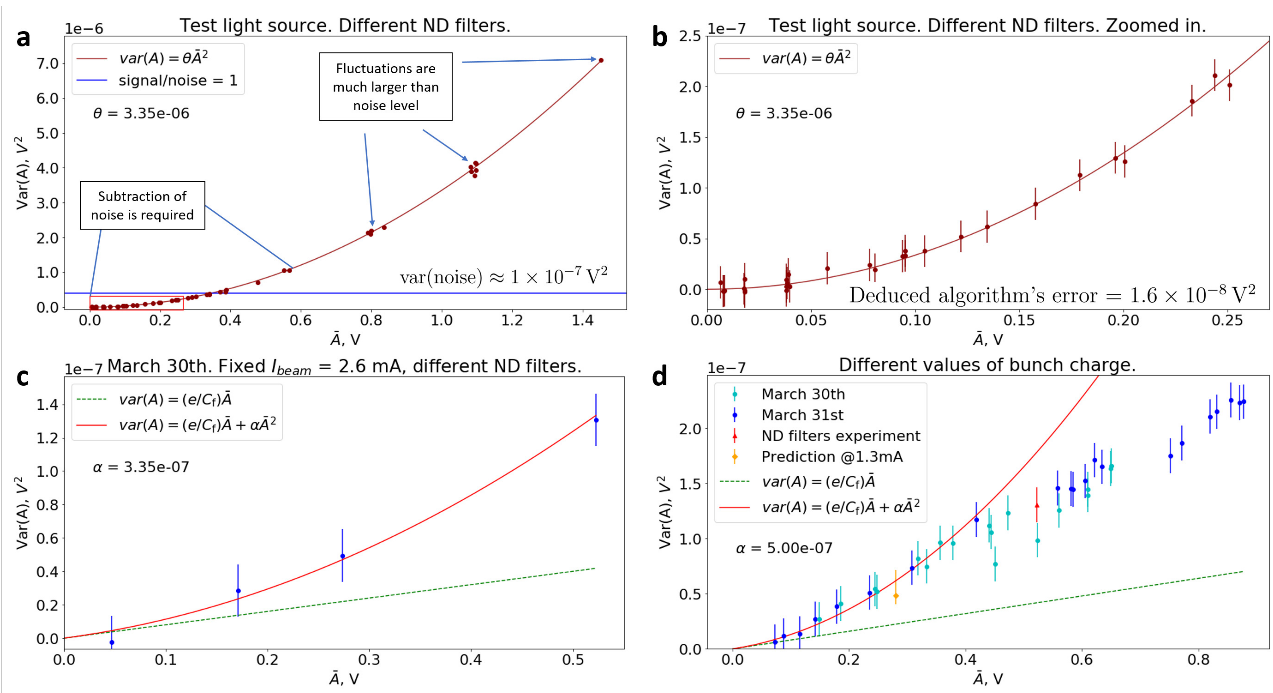

Figure 2: Results of measurements with the test light source (a,b) and with the undulator in IOTA (c,d).

accurate measurement of relative fluctuations at large amplitude, and a prediction for when we use ND filters to reduce the amplitude: $\mathrm{var}(A) = \theta \bar{A}^2$, where θ is the relative fluctuation measured at large amplitude. If fluctuations at small A (when signal/noise$\lessapprox 1$) measured by the algorithm with noise subtraction agree with the expected curve, then we can trust this algorithm. In Fig. 2a,b one can see that experimental points follow the expected curve very well. Figure 2b depicts the region of fluctuations that were measured in the actual experiment with the undulator in IOTA. Thus, three standard deviations from the expected curve in Fig. 2b were used as an error bar for the plots for the undulator radiation in IOTA in Fig. 2c ($\bar{A}$ varied by ND filters) and in Fig. 2d ($\bar{A}$ varied by changing bunch charge).

DISCUSSION

Our model states that the variance of the number of emitted photons complies with Eq. (1). In terms of the photodetector amplitude A, this equation takes the form $\mathrm{var}(A) = (e/C_\mathrm{f})\bar{A} + \alpha\bar{A}^2$, where e is the electron charge. The points in Fig. 2c lie well on a curve of this kind. However, in Fig. 2d the experimental points agree with the expected behavior only at small amplitudes (≤ 0.4 V) and start to deviate afterwards. This happens because the bunch size in IOTA increases with the beam current due to intrabeam scattering, as suggested by simulations. According to Eqs. (2) and (4), α decreases when the bunch size increases. Hence, it perfectly explains why in Fig. 2d experimental points with $\bar{A} > 0.4$ V lie below the red curve fitting the points for smaller $\bar{A}$.

For the beam current 1.3 mA we had camera images of bending-magnet light, from which we could extract beam emittances, and, hence, transverse bunch sizes at the undulator location. Also, we took measurements of the bunch length with a wall-current monitor in IOTA. These values are given in Table 1. Based on these numbers, our theoretical model predicted a point (orange diamond) in Fig. 2d for $\bar{A} = 0.28$ V ($I_\mathrm{beam} = 1.3$ mA), namely, $\mathrm{var}(A) = 4.8 \times 10^{-8}\ \mathrm{V}^2$. Since IOTA is still operating in a commissioning mode, there is some uncertainty in a number of parameters, and we estimate the error of σ_x, σ_y, σ_z to be about 20 %. This results in the confidence interval $(4.0 \times 10^{-8}\ \mathrm{V}^2, 7.2 \times 10^{-8}\ \mathrm{V}^2)$ for $\mathrm{var}(A)$. The solid red curves in Fig. 2c and d could correspond to, for example, $\sigma_x = 638\ \mu\mathrm{m}$, $\sigma_y = 51\ \mu\mathrm{m}$, $\sigma_z = 20$ cm and $\sigma_x = 549\ \mu\mathrm{m}$, $\sigma_y = 44\ \mu\mathrm{m}$, $\sigma_z = 20$ cm, respectively, i.e., our model predicts $\alpha = 3.35 \times 10^{-7}$ and $\alpha = 5.00 \times 10^{-7}$ for these bunch sizes.

In conclusion, we achieved a reasonable agreement between theory and experiment, and we expect that it will continue to improve as we get better in characterizing the beam in IOTA. We collected data for different values of bunch charge (see Fig. 2d), which was not done in [1]. We believe that the precision of our measurements was better than in [1] too, due to using the comb filter with one turn delay and the noise subtraction algorithm. Also, it is noteworthy that we worked with such beam parameters, that the Poisson contribution (dashed green lines in Fig. 2c,d) was comparable with the interference contribution to the fluctuations. Whereas in the BNL experiment, for undulator light, the Poisson contribution was negligible. The effect of fluctuations in undulator light may find an application in beam diagnostics. Since these fluctuations depend on the bunch size (e.g., see Eq. (4)), they can assist one in determining dimensions of the electron bunch. This technique can be especially useful for ultra small bunches, when other methods, e.g., camera images, cease to be reliable.

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REFERENCES

[1] M. C. Teich, T. Tanable, T. C. Marshall, and J. Galayda, "Statistical properties of wiggler and bending-magnet radiation from the Brookhaven vacuum-ultraviolet electron storage ring", *Phy. Rev. Lett.*, vol. 65, no. 27, p. 3393, Dec. 1990. doi:10.1103/PhysRevLett.65.3393

[2] S. Antipov *et al.*, "IOTA (Integrable Optics Test Accelerator): facility and experimental beam physics program", *J. Instrum.*, vol. 12, no. 3, p. T03002, Mar. 2017. doi:10.1088/1748-0221/12/03/T03002

[3] R. J. Glauber, "Some notes on multiple-boson processes", *Phy. Rev. Lett.*, vol. 84, no. 3, p. 395, Nov. 1951. doi:10.1103/PhysRev.84.395

[4] R. J. Glauber, "Coherent and Incoherent States of the Radiation Field", *Phys. Rev.*, vol. 131, no. 6, pp. 2766-2788, Sep. 1963. doi:10.1103/PhysRev.131.2766

[5] E. L. Saldin, E. V. Schneidmiller, and M. V. Yurkov, *The Physics of Free Electron Lasers*, Berlin, Germany: Springer-Verlag, 2000.

[6] K. J. Kim, Z. Huang, and R. Lindberg, *Synchrotron Radiation and Free-Electron Lasers*, Cambridge, United Kingdom: Cambridge University Press, 2017.

[7] K. J. Kim, "Characteristics of synchrotron radiation", in *Proc. 1988 US Particle Accelerator School (USPAS)*, Ithaca, NY, Aug. 1988. pp. 565-632. doi:10.1063/1.38046

[8] J. A. Clarke, "Undulator angular flux distribution", in *The Science and Technology of Undulators and Wigglers*, New York, NY, USA: Oxford University Press Inc., 2004, pp. 58–73.

[9] C. Pellegrini, A. Marinelli, and S. Reiche, "The physics of x-ray free-electron lasers", *Rev. Mod. Phys.*, vol. 88, no. 1, p. 015006, Mar. 2016. doi:10.1103/RevModPhys.88.015006

[10] P. R. Tapster, J. G. Rarity, and J. S. Satchell, "Generation of Sub-Poissonian Light by High-Efficiency Light-Emitting Diodes", *Europhys. Lett.*, vol. 4, no. 3, pp. 293-299, Aug. 1987.

[11] S. Machida and Y. Yamamoto, "Observation of sub-poissonian photoelectron statistics in a negative feedback semiconductor laser", *Opt. Commun.*, vol. 57, no. 4, pp. 290-296, Mar. 1986. 10.1016/0030-4018(86)90100-8