# USING AN ENERGY SCAN TO DETERMINE THE TUNES AND ORBIT IN THE FIRST FFA GIRDER OF CBETA 

C. Gulliford, N. Banerjee, A. Bartnik, J. Crittenden, P. Quigley Cornell University, Ithaca, NY, USA<br>J. S. Berg, Brookhaven National Lab, Upton, New York, USA,

## Abstract

This work reports the results of performing a scan of the beam energy during the CBETA Fractional Arc Test (FAT) [1,2]. The CBETA machine is a multi-pass SRF ERL featuring a non-scaling FFA return loop. The FFA arc consists of identical doublets that are designed to have an energy acceptance from 42 to 150 MeV , with a betatron phase advance (i.e., tune) per cell and periodic orbit position that depends on energy. In the CBETA fractional arc test, we transported beam through 4 such cells (the first girder), with energies as high as 59 MeV . By creating betatron oscillations in the arc, we compute the phase advance per cell and periodic orbit position as a function of energy within that range.

## INTRODUCTION

The Cornell-BNL Energy recovery linac Test Accelerator (CBETA) [3], a 4-pass, 150 MeV ERL utilizing a Nonscaling Fixed Field Alternating-gradient (NS-FFA) permanent magnet return loop [4], is currently under design and construction at Cornell University through the joint collaboration of Brookhaven National Lab (BNL) and the Cornell Laboratory for Accelerator based Sciences and Education (CLASSE). The spring of 2018 saw the first major commissioning period for CBETA. Known as the Fractional Arc Test (FAT), this experiment bought together for the first time elements of all of the critical subsystems required for the CBETA project: the injector [5, 6], the Main Linac Cryomodule (MLC) [7, 8], the low energy (S1) splitter line which includes several new electromagnets, a path length adjustment mechanism, and a new BPM system, as well as a first prototype production permanent magnet girder [9] featuring 4 cells of the FFA return loop with its own corresponding vacuum system and BPM design. A schematic of the FAT layout is shown in Fig. 1.

## MEASUREMENTS

For energies ranging from 38 MeV to 59 MeV the beam was kicked using two linear combinations of the last two S1 splitter dipoles and the last two vertical correctors. These linear combinations were chosen to correspond to a betatron oscillation with a maximum amplitude at the FFA BPMs of 1 mm . The two linear combinations were chosen to give betatron oscillations that were $90^{\circ}$ apart in betatron phase. Each linear combination was multiplied by a factor which was scanned from -2 to 2 in unit steps. Only one setting vector was scanned at a time. For each setting, the beam
position on the four FFA BPMs were recorded. This procedure was automated and tested with the online CBETA Virtual Machine before use. While taking the final measured data, the BPM readings were sampled 10 times at 5 Hz , and the average value and standard deviation saved for offline analysis. A single second pause was used between magnet set-points to allow the beam to stabilize.

Our measurements at the $m^{\text {th }}$ BPM and the $n^{\text {th }}$ energy were fit in the least squares sense to the following functions:

$$
\begin{align*}
x_{m n}= & \left(s \cdot A_{x, n}^{(1)}+B_{x, n}^{(1)}\right) \cos \left(2 \pi m \cdot v_{x, n}+\phi_{x, n}^{(1)}\right) \\
& +\left(s \cdot A_{x, n}^{(2)}+B_{x, n}^{(2)}\right) \cos \left(2 \pi m \cdot v_{x, n}+\phi_{x, n}^{(2)}\right) \\
& +C_{x, m}+D_{m}  \tag{1}\\
y_{m n}= & \left(s \cdot A_{y, n}^{(1)}+B_{y, n}^{(1)}\right) \cos \left(2 \pi m \cdot v_{y, n} \cdot+\phi_{y, n}^{(1)}\right) \\
& +\left(s \cdot A_{y, n}^{(2)}+B_{y, n}^{(2)}\right) \cos \left(2 \pi m \cdot v_{y, n}+\phi_{y, n}^{(2)}\right) \\
& +C_{y, m} \tag{2}
\end{align*}
$$

$s_{x, y}^{(1,2)}$ are the scale factors of the kick, scanned from -2 to +2 in unit steps with only one of the four being nonzero. $A_{x, y, n}^{(1,2)}$ are the unit amplitudes of the two kicks used in each transverse plane at each energy; if our model is perfect they will be $1 \mathrm{~mm} . B_{x, y, n}^{(1,2)}$ is the amplitude of the betatron oscillation with no additional kicks; if the orbit found by hand were the periodic orbit, these would be zero. $C_{x, y, m}$ are the (energy independent) BPM offsets. $D_{n}$ is the (energy dependent) horizontal periodic orbit position; the vertical periodic orbit is known to be zero, so this term only appears in the horizontal function. $\phi_{x, y, n}^{(1,2)}$ are phase offsets of each betatron oscillation; $\phi_{x, y}^{(1)}$ and $\phi_{x, y}^{(2)}$ will differ by $\pi / 2$ if out model is perfect. Finally $v_{x, y, n}$ are the tunes per cell of the periodic orbit at energy $E_{n}$. The initial guess for $A_{x, y, n}^{(1,2)}$ coefficients


Figure 1: Schematic of the CBETA machine highlighting the components installed for Fractional Arc Test.
was set to 1 mm . Similarly, the initial guesses for $B_{x, y, n}^{(1,2)}$, $\phi_{x, y, n}^{(1,2)}$, and, $C_{x, y, m}$ were set to zero, and the initial horizontal periodic orbit positions $D_{m}$ guesses set to roughly 15 mm .

The phase advance per cell $v_{x, y}$ and horizontal periodic orbit terms $D_{n}$ are intrinsic to the arc design; they give a measure of how accurate our model of the FFA arc is. $C_{x, y, m}$ give an estimate of the BPM offsets. Note that in the horizontal plane, there is a redundancy between $C_{x, m}$ and $D_{n}$; one can add a given constant to all of the $C_{x, m}$ and subtract that same constant from the $D_{n}$. We adopt the convention that the average of the $C_{x, m}$ is zero. For full turn energy recovery operation, in particular for 150 MeV energy recovery where we pass through the arc with four different energies, having an estimate of the BPM offsets will be extremely helpful for orbit correction since they will help distinguish between orbit offsets caused by magnet errors and position reading errors arising from BPM offsets. Thus a similar energy scan and fit will be performed for the full ring to obtain an estimate of the BPM offsets using a similar method.

Figures 2 and 3 show example horizontal and vertical fits to the BPM data for the two applied kicks in each direction, respectively.

Fitting to this data was performed globally over all energies, resulting in the determination of the coefficients in Eqn. (1) and (2). The resulting horizontal and vertical phase advance per cell $v_{x, y}$ are shown in Fig. 4 along with the predicted values from particle tracking through 3D fieldmaps.

The agreement between the experimental results and simulation data is quite good. A close inspection of the simulation and measured curves indicates even better agreement is possible if one allows for a systematic scaling of all the measured energy values by 1.02 , indicating there may be an overall systematic discrepancy between the simulated quadrupole fields and/or the energy, and the measured data. The result of scaling the measured energies by 1.02 is shown in Fig. 5.


Figure 2: Horizontal BPM response on the BPMS in the fractional arc for each of the applied kicks at 42 MeV .

Table 1: Estimated BPM Offsets (mm)

| BPM | X Offset $\left(C_{x}\right)(\mathrm{mm})$ | Y Offset $\left(C_{y}\right)(\mathrm{mm})$ |
| :---: | :---: | :---: |
| IFABPM01 | $-0.40 \pm 0.02$ | $0.70 \pm 0.04$ |
| IFABPM02 | $-0.43 \pm 0.02$ | $0.34 \pm 0.04$ |
| IFABPM03 | $0.28 \pm 0.03$ | $0.45 \pm 0.04$ |
| IFABPM04 | $0.54 \pm 0.02$ | $-0.28 \pm 0.04$ |

In addition to the phase advance per cell, the fitting method also determines the periodic orbit position, shown in Fig. 6 along with the simulated values. Here again we see a systematic difference between the measured and predicted data.

In particular, the horizontal periodic orbit position shown has an average systematic error with respect to the theoretical prediction for the orbit position of roughly 1.5 mm . This could arise from a nonzero average in the BPM offsets (though it is unlikely to be this large), non-linearity in the BPM response, a systematic difference between the modeled and as-built magnets, and possibly other effects.

Finally, the fitting method in principle also gives an estimate of the energy independent BPM offsets. The values returned by the fitting routine are shown in Table 1. Unfortunately, we must report that subsequent simulations using the CBETA Virtual Machine online model have shown that the fitting procedure here is not robust enough to correctly determine the BPM offsets when their are both BPM and permanent magnet quadrupole offsets present in the lattice. As this is most certainly the case for the FAT measurements, we can not make any quantitative conclusions about the BPM offsets presented here.

## CONCLUSION

A method for determining the phase advance per cell and horizontal periodic orbit position in the FFA permanent magnet return loop of the CBETA machine as been developed and tested during the CBETA Fractional Arc Test. Data


Figure 3: Vertical BPM response on the BPMS in the fractional arc for each of the applied kicks at 42 MeV .


Figure 4: Comparison of the horizontal (blue) and vertical (red) phase advance per cell as a function of the beam energy with 3D field map tracking (dashed).


Figure 5: Comparison of the horizontal (blue) and vertical (red) phase advance per cell as a function of the beam energy with 3D field map tracking (dashed) with the measured energy values scaled by 1.02 .


Figure 6: Comparison of the measured and simulated horizontal periodic orbit as a function of energy.
was taken for energies ranging from $38-59 \mathrm{MeV}$. Measurements of the betatron phase advance per cell via induced betatron oscillations around the periodic orbit show very
good agreement with theoretical predictions from tracking particles through 3D fieldmaps. Closer inspection of the the data suggests an overall energy scale factor of 1.02 may improve the agreement between the measured and simulated phase advance per cell as a function of beam energy. This alight difference could arise from an unknown systematic discrepancy between the 3D field maps for the permanent magnets and the fields in the magnets themselves or from an error in our energy calibration. Testing of the algorithm described here indicated improvements are necessary to accurately estimate the energy independent BPM offsets. Work is currently underway to address this issue by forcing the periodic orbit terms $D$ and the phase advance per cell to be smoothly varying functions of energy, thus reducing the number of required fit parameters. The updated procedure will be tested both with simulation and during CBETA beam commissioning measurements currently underway at this time.

## ACKNOWLEDGEMENTS

This work was funded by NYSERDA, the New York State Energy Research and Development Agency. This manuscript has been authored by employees of Brookhaven Science Associates, LLC under Contract No. DE-SC0012704 with the U.S. Department of Energy.

## REFERENCES

[1] C. Gulliford et al, "Beam Commissioning Results from the CBETA Fractional Arc Test", https://arxiv.org/abs/ 1902.03370
[2] C. Gulliford et al, "Results from the CBETA Fractional Arc Test", presented at the 10th Int. Particle Accelerator Conf. (IPAC'19), Melbourne, Australia, May 2019, paper MOPRB0771, this conference.
[3] G. Hoffstaetter et al, CBETA Design Report, Cornell-BNL ERL Test Accelerator, arXiv:1706.04245[physics.acc-ph]
[4] K. Halbach, "Design of permanent multipole magnets with oriented rare earth cobalt material", Nuclear Instruments and Methods 169, 1 (1980).
[5] B. Dunham, et al, "Record high-average current from a high-brightness photoinjector", Applied Physics Letters 102, 034105 (2013).
[6] C. Gulliford et al, "Demonstration of low emittance in the Cornell energy recovery linac injector prototype", Phys. Rev. ST Accel. Beams 16, 073401 (2013)
[7] R. Eichhorn et al, "The Cornell Main Linac Cryomodule: A Full Scale, High Q Accelerator Module for cw Application", Physics Procedia 67, 785-790 (2015).
[8] F. Furuta et al, "Performance of the Cornell Main Linac Prototype Cryomodule for the CBETA Project", Proc. of the North American Particle Accelerator Conference, October 2016, Chicago, IL, USA, paper MOPOB60 pp. 204-207.
[9] S. Brooks, "Magnet and Lattice Specifications for the CBETA First Girder", Tech. Rep. CBETA-001 (Brookhaven National Lab, 2016), https://www.classe.cornell.edu/CBETA PM/notes/CBETA001.pdf

