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GALACTIC AND GALACLIC: TWO VLASOV SOLVERS FOR THE TRANSVERSE AND LONGITUDINAL PLANES

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Abstract

GALACTIC and GALACLIC, two Vlasov solvers for the study, in the transverse and longitudinal plane respectively, of single-bunch coherent oscillation modes, were recently developed starting from the Vlasov equation and using a decomposition on the low-intensity eigenvectors, as proposed by Laclare and Garnier. The first Vlasov solver was used for instance to shed light on the destabilising effect of resistive transverse dampers and the second helped understanding the details of the mode-coupling behind some longitudinal microwave instabilities. Both theories are reviewed in detail, highlighting in particular the similarities and peculiarities of the two approaches.

INTRODUCTION

GALACTIC stands for GArnier-LAclare Coherent Transverse Instabilities Code, while GALACLIC stands for GArnier-LAclare Coherent Longitudinal Instabilities Code. The two approaches are very similar and solve the linearised Vlasov equation as discussed in Refs. [1] and [2]. In Ref. [1], Laclare obtained very elegantly an eigenvalue system to solve, but with the unknown frequency inside the matrix to be diagonalised and he proposed a procedure to solve it for the real part of the mode-frequency shifts only, as will be reviewed in the next section. However, the drawbacks of this method are on one hand that it does not allow to follow the individual modes and on the other hand it does not provide (at least not straightforwardly) the imaginary part of the mode-frequency shifts. The latter is very important to check whether the beam is unstable or not. Figure 1 depicts the case of a bunch interacting with a constant inductive impedance. From the real part of the mode-frequency shifts, one cannot say a priori if the bunch is unstable or not (as some modes cross each other) without looking at the imaginary part. This is why the analysis has been reviewed, starting from the proposition from Garnier and Laclare [2] to use a decomposition on the low-intensity eigenvectors to obtain an eigenvalue system with the unknown frequency outside the matrix to be diagonalised. The final results, which are close to but not exactly the same as obtained in Ref. [2], are compared to the results from Laclare [1].

LACLARE'S APPROACH [1]

In Ref. [1], Laclare presented a very nice formalism with a clear and similar procedure to treat both longitudinal and transverse planes, first for the low-intensity case (i.e. when the modes can be treated independently) and then for the general high-intensity case (when the modes cannot be treated independently). One starts with the single-particle motion, which is approximated by the one of a harmonic oscillator with the corresponding beam-induced electromagnetic forces in the longitudinal and transverse planes. Then, one looks at the spectrum of the single-particle signal, which is a line spectrum (around every harmonic of the revolution frequency, there is an infinite number of synchrotron satellites m) centred at 0 for the longitudinal plane and at the chromatic frequency for the transverse plane. A distribution of particles (particle density in phase space) is then considered and expressed as a sum of a stationary distribution and a perturbation. The beam-induced electromagnetic force can be expressed through the impedance, which is a complex function of frequency, for both longitudinal and transverse planes. In this respect, the case of the longitudinal plane is a bit more involved as one has first to study the effect of the impedance on the stationary distribution, which is called the Potential-Well Distortion (PWD): a new fixed point is then obtained, with a dependency on the bunch intensity of the synchronous phase, the incoherent frequency, the effective (total) voltage and the bunch length. Around the new fixed point, one writes the perturbation, which is coherent with respect to the satellite number m. Applying the Vlasov equation to first order, one ends up with an eigenvalue system to solve. The result is an infinite number of modes of oscillation mq, with m the azimuthal mode number and q the radial one. The latter is defined as $q = |m| + 2\,k$ (with k an integer between 0 and infinity): with this definition, q represents the number of nodes of the superimposed low-intensity standing-wave patterns, which is a usual observable in particle accelerators. Finally, for the high-intensity cases (both in longitudinal and transverse), the final eigenvalue systems are obtained by summing over all the modes m. They are given in Ref. [1] by Eq. (196) for the transverse plane and Eq. (124) for the longitudinal one, followed by the procedure to solve them. The results obtained from this approach are presented below in black in the different figures, as a function of a normalised parameter x, which is given by

$$x = \frac{\mathrm{Im}[\,Z_x(0)]\; e\, I_b}{4\,\pi\,\gamma\, m_0 c\, Q_{x0} B\, \omega_s} \tag{1}$$

for the transverse (e.g. horizontal) plane, and

$$x = \frac{\mathrm{Im}\left[\frac{Z_l(p)}{p}\right]_{p=0} 4\, I_b}{\pi^2\, B^3\, \hat{V}_T\, h \cos\phi_s} \tag{2}$$

for the longitudinal plane. Here, $Z_x(p)$ and $Z_l(p)\,/p$ are the horizontal (dipolar) and longitudinal impedances (at

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the bunch spectrum line p) respectively, e is the elementary charge, $I_b = N_b\, e\, f_0$ the bunch current (with N_b the number of charges and f_0 the revolution frequency), γ the relativistic mass factor, m_0 the rest mass, c the speed of light, Q_{x0} the horizontal tune, $B = f_0\, \tau_b$ the bunching factor with τ_b the full ($4\,\sigma$) bunch length, ω_s the angular synchrotron frequency, $\hat{V}_T$ the total (effective) peak voltage, h the harmonic number and ϕ_s the RF phase of the synchronous particle ($\cos\phi_s > 0$ below transition and $\cos\phi_s < 0$ above). It is important to note that in the longitudinal plane, B, $\hat{V}_T$ and ϕ_s depend on the bunch intensity due to the PWD.

GALACTIC

As proposed in Ref. [2], using a decomposition on the low-intensity eigenvectors, the eigenvalue system to solve can be written as in Ref. [3], where the destabilising effect of a resistive transverse damper was discussed. In the absence of a transverse damper, i.e. considering the case $F_{damper} = 0$ in Eq. (2) of Ref. [3], an equation similar to the one of Ref. [2] is obtained (but with a slightly different form). GALACTIC is compared to Laclare's approach [1] in Figs. 1 and 2, where a good agreement is obtained, for the two cases of a constant inductive impedance and a broad-band resonator respectively.

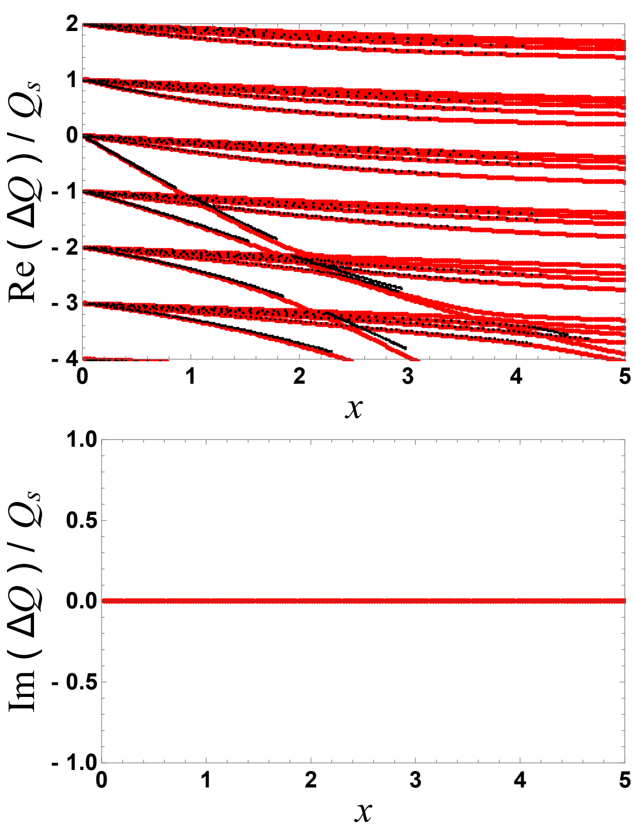

Figure 1: Comparison between GALACTIC (in red) and Laclare's approach [1] (in black) of the normalised mode-frequency shifts, in the case of a constant inductive impedance and for a "Water-Bag" (WB) longitudinal distribution [1]: (upper) real part and (lower) imaginary part (from GALACTIC only).

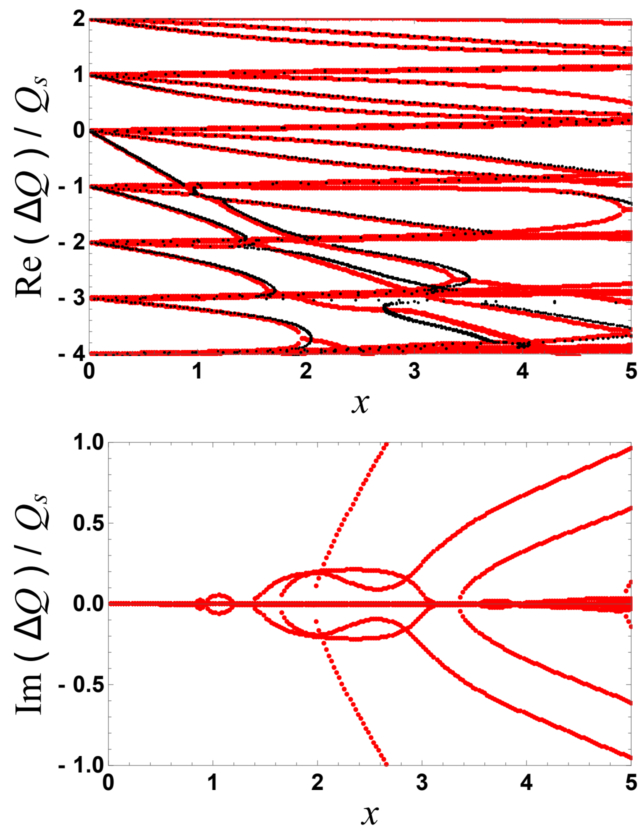

Figure 2: Comparison between GALACTIC (in red) and Laclare's approach [1] (in black) of the normalised mode-frequency shifts, in the case of a broad-band resonator impedance (with a quality factor of 1 and a resonance frequency f_r such that $f_r\, \tau_b = 2.8$) and for a WB longitudinal distribution [1]: (upper) real part and (lower) imaginary part (from GALACTIC only).

GALACLIC

Following the same approach as for GALACTIC, a similar eigensystem as in Ref. [3] is obtained in the longitudinal plane (without damper), with Qx0 = 0, the transverse impedance $Z_x(p)$ being replaced by the longitudinal impedance $Z_l(p)\,/\,p$ and replacing the x of Eq. (1) by the x of Eq. (2). As mentioned above, the additional complexity in the longitudinal plane is the PWD. To clearly see its effect, the two cases, without and with PWD, are discussed below.

Without Taking into Account PWD

This means that the eigensystem is solved and the results are plotted using Qs, i.e. the intensity-dependent synchrotron tune. GALACLIC is compared to Laclare's approach [1] in Fig. 3, where a good agreement is obtained, for the case of a broad-band resonator impedance above transition (a good agreement is also obtained for the case of a constant inductive impedance) [4].

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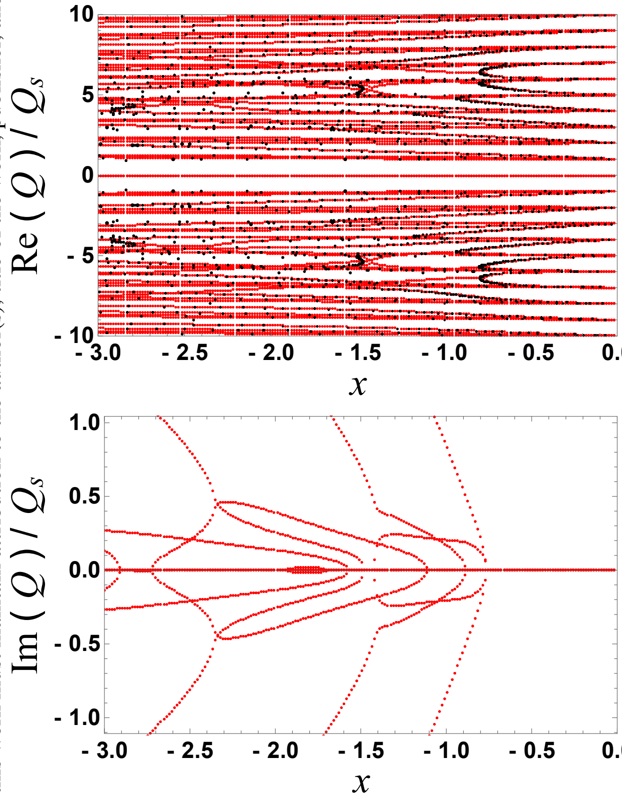

Figure 3: Comparison between GALACLIC (in red) and Laclare's approach [1] (in black) of the normalised mode-frequency shifts, in the case of a broad-band resonator impedance (with a quality factor of 1 and a resonance frequency f_r such that $f_r \tau_b = 2.8$), above transition, without taking into account the PWD and for a "Parabolic Amplitude Density" (PAD) longitudinal distribution [1]: (upper) real part and (lower) imaginary part (from GALACLIC only).

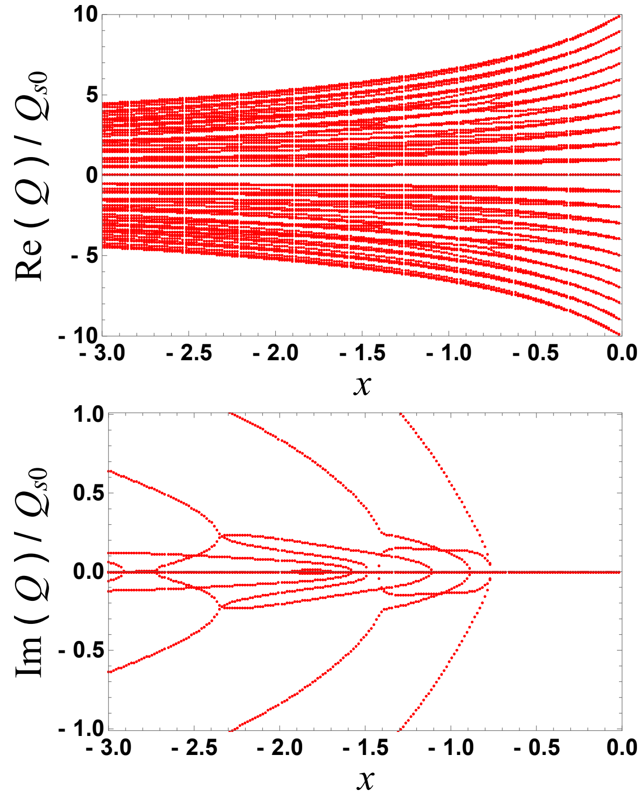

Figure 4: Normalised (to the low-intensity synchrotron tune) mode-frequency shifts from GALACLIC in the case of a broad-band resonator impedance (with a quality factor of 1 and a resonance frequency f_r such that $f_r \tau_b = 2.8$), above transition, taking into account the PWD and for a PAD longitudinal distribution: (upper) real part and (lower) imaginary part.

Taking into Account PWD

This means that the eigensystem is solved and the results are plotted using Q_{s0}, i.e. the low-intensity synchrotron tune, using Eq. (3)

$$\frac{Q}{Q_{s0}} = \frac{Q}{Q_s} \times F_{PWD} \quad \text{with} \quad F_{PWD} = \frac{Q_s}{Q_{s0}} = \frac{1}{\sqrt{1-\frac{4}{\pi}x}} \tag{3}$$

for the case of a PAD longitudinal distribution (and assuming here the simplified case where the shape of the distribution is preserved). The results from GALACLIC for the case of a broad-band resonator impedance above transition are shown in Fig. 4. The detailed comparison with macroparticle tracking simulations is discussed in Ref. [4]. It is worth noticing however that the same intensity threshold $x_{th} \approx -0.75$ is obtained for both cases without and with PWD.

CONCLUSION

The two Vlasov solvers GALACTIC, for the transverse plane, and GALACLIC, for the longitudinal plane, have been discussed and some benchmarks with Laclare's approach [1] have been presented. A good agreement has been reached for all the cases considered. A careful convergence study could be performed in the future to try and understand the small differences observed in some cases. Other successful benchmarks were also done in the past in the transverse plane [3,5] and a detailed comparison between GALACLIC and longitudinal macroparticle tracking simulations is discussed in Ref. [4], where a good agreement has also been reached.

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REFERENCES

[1] J.L. Laclare, "Bunched beam coherent instabilities", in *Proc. CAS - CERN Accelerator School: Accelerator Physics*, Oxford, UK, 16 - 27 September 1985, pp. 264-326 (CERN-1987-003-V-1).

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[2] J.P. Garnier, “Instabilités cohérentes dans les accélérateurs circulaires”, Ph.D. thesis, Grenoble, France, 1987.

[3] E. Métral et al., “Destabilising effect of the LHC transverse damper”, in *Proc. 9th Int. Particle Accelerator Conf. (IPAC'18)*, Vancouver, BC, Canada, Apr.-May 2018, pp. 3076-3079. doi:10.18429/JACoW-IPAC2018-THPAF048

[4] E. Métral and M. Migliorati, “Longitudinal Mode-Coupling Instability: GALACLIC Vlasov solver vs. macroparticle tracking simulations”, presented at the 10th Int. Particle Accelerator Conf. (IPAC'19), Melbourne, Australia, May 2019, paper MOPGW089, this conference.

[5] E. Métral, “Beam instabilities in circular particle accelerators”, CERN, Geneva, Switzerland, Rep. CERN-ACC-SLIDES-2018-0003, 2018, unpublished, https://cds.cern.ch/record/2652200/files/CERN-ACC-SLIDES-2018-0003.pdf