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RADIATION FROM A DIELECTRICALLY LOADED WAVEGUIDE WITH OPEN END*

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Abstract

We study electromagnetic field produced by a charged particle bunch exiting an open-ended circular waveguide with dielectric filling placed inside collinear vacuum waveguide of a larger radius. Based on the developed theory, we mainly investigate Cherenkov radiation (CR) generated penetrated vacuum regions of the structure due to the diffraction mechanism. We pay attention to the case of a train of short bunches resulting in high-order CR modes excitation. We also develop analytical procedure allowing performing the limiting process to the case of infinite radius of the outer waveguide.

INTRODUCTION

In recent years, an essential interest is observed in the area of contemporary sources of Terahertz (THz) radiation based on beam-driven waveguide structures loaded with dielectric. Despite of the fact that both ordinary vacuum THz devices are widely available and other mechanisms for THz sources are discussed [1], beam-driven sources are extremely attractive due to extraordinary THz radiation peak power [2]. In this report, we first present some results (based on corresponding rigorous solution) on generation of high-order CR modes in the "embedded" structure with open-ended dielectric-lined waveguide [3], as shown in Fig. 1, with the focus on the diffraction penetration of CR into vacuum regions of the structure. Second, we present current status of developing the analytical procedure for limiting process to the case of infinite radius of the outer waveguide ("opened" structure).

"EMBEDDED" STRUCTURE

According to the idea of beam-driven THz source, THz frequencies can be generated in mm- or sub-mm-sized waveguides by charged particle bunches with proper charge modulation, i.e. by bunch trains [4]. Figure 2 shows comparison of a typical single Gaussian bunch Fourier spectrum with the spectrum of a bunch train of 15 identical bunches with spacing $L > 2\sigma$. Here parameters are chosen so that the bunch train excites effectively the fifth CR mode. In the same manner, other CR frequencies can be generated. For example, Fig. 3 shows field distribution for the case of sub-mm inner waveguide (similar structures were used in [2]) with third CR mode (with frequency about 0.39 THz) generated. Behaviour of E_r component of CR in vacuum

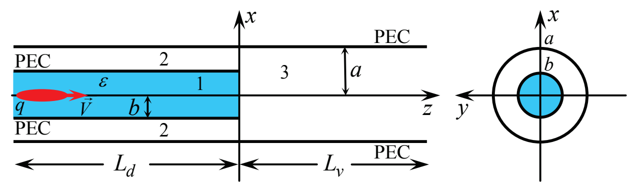

Figure 1: "Embedded" structure and notations, $V = \beta c$.

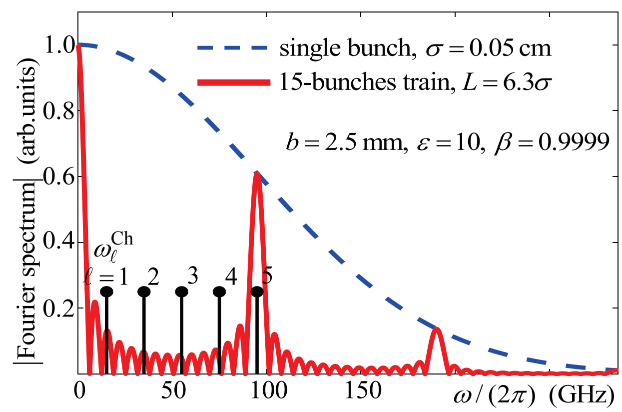

Figure 2: Typical Fourier spectrum of a single Gaussian bunch and that of a 15 bunches train with $L > 2\sigma$ spacing, black markers show CR frequencies.

regions of the structure (recall that CR is generated in the inner waveguide and penetrates vacuum sections by means of diffraction mechanism) is shown. Each thin (green) curve shows the E_r as a function of r at a given time moment t and given z. In total, each plot contains 151 curves covering the 1.5 ns time range with 0.01 ns interval. The highlighted solid (red) curve corresponds to the maximum field over the cross-section. As one can see, maximum field in coaxial region is always on the inner waveguide wall. In the wide waveguide, global field maximum is typically at the first or second local maximum.

The main purpose of Fig. 3 is to illustrate possibilities of the developed rigorous approach for investigation of field structure of high-frequency CR across the structure. It should be underlined that simulation time consumed by CST Particle Studio for similar structures but for the case of first CR mode generation typically takes about 20 hours at PC with Intel® Core i7 processor and 32 Gb memory. On the contrary, our MATLAB code based on analytical formulas shows approximately 20×60 times faster perfomance [3].

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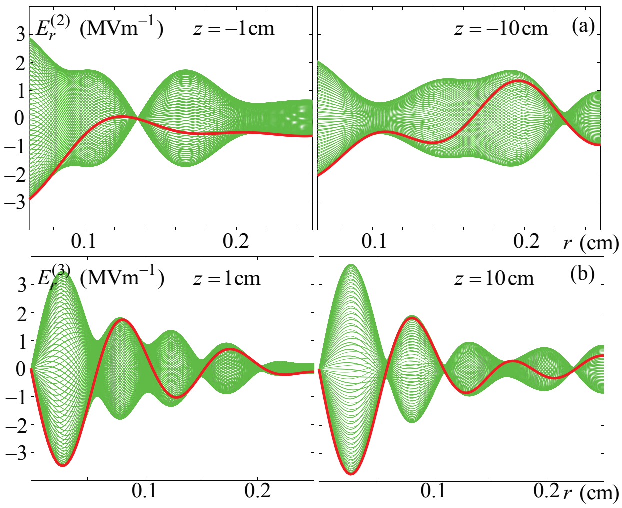

Figure 3: CR field (E_r component) at the third CR mode ($\approx$ 386 GHz), in the coaxial area (a) and in wide vacuum waveguide (b). Parameters: $b = 0.064$ cm, $a = 0.25$ cm, $\varepsilon = 3.8$, $q = 1$ nC, $\beta = 0.9999$, $\sigma = 0.012$ cm, $L = 6.45\sigma$.

Moreover, the case of a bunch train is difficult to simulate with CST PS. Therefore, the presented analytical approach can be considered as preferable method for analysis of CR field across the "embedded" structure.

"OPENED" STRUCTURE

We also deal with opened-ended dielectric-lined waveguide (inner waveguide of radius b in Fig. 1) in free space in accordance with [5]. For simplicity, we consider excitation by single l-th mode with coefficient $B^{(i)}$,

$$H_{\omega\phi}^{(i)} = B^{(i)} J_1\left(\frac{r j_{0l}}{b}\right) e^{-\kappa_{zl}^{(1)} z},$$

propagating from inside the inner waveguide. Reflected field in area (1) is presented over standart series over waveguide modes:

$$H_{\omega\phi}^{(1)} = \sum_{m=1}^{\infty} B_m J_1(r j_{0m}/b) e^{\kappa_{zm}^{(1)} z}, \tag{1}$$

where $\kappa_{zm}^{(1)} = \sqrt{j_{0m}^2 b^{-2} - \varepsilon k_0^2}$, $\mathrm{Re}\kappa_{zm}^{(1)} > 0$, $J_0(j_{0m}) = 0$, $\{B_m\}$ are unknown coefficients. Scattered field in "opened" vacuum areas:

$$H_{\omega\phi}^{(2)} = \int_0^{\infty} C(\xi') \exp(\Gamma z) Z(r, \xi') \xi' d\xi', \tag{2}$$

for $z < 0$, $b < r < \infty$and

$$H_{\omega\phi}^{(3)} = \int_0^{\infty} A(\xi) \exp(-\gamma z) J_1(r\xi) \xi d\xi, \tag{3}$$

for $z > 0$, $0 < r < \infty$, where $A(\xi)$, $C(\xi')$ are unknown functions, $\gamma(\xi) = \sqrt{\xi^2 - k_0^2}$, $\Gamma(\xi') = \sqrt{\xi'^2 - k_0^2}$, $\mathrm{Re}(\Gamma, \gamma) > 0$,

$$Z(r, \xi') = J_1(r\xi') - \frac{J_0(b\xi')}{N_0(b\xi')} N_1(r\xi').$$

We perform matching of tangential field components for $z = 0$, integrate these relations,

$$\int_0^{\infty} \cdot J_1\left(r j_{0p}/b\right) r dr \quad \text{for } 0 < r < b,$$
$$\int_b^{\infty} \cdot Z(r, \xi) r dr \quad \text{for } b < r < \infty,$$

and utilize tabular formulas for Bessel functions including the following:

$$\int_0^{\infty} J_1(r\xi') J_1(r\xi) r dr = \delta(\xi - \xi')/\xi,$$

$$\int_b^{\infty} N_1(r\xi) N_1(r\xi') r dr = \delta(\xi - \xi')/\xi +$$
$$+ \text{V.p.}\ \frac{b\left[\xi N_0(b\xi) N_1(b\xi') - \xi' N_1(b\xi) N_0(b\xi')\right]}{\gamma^2(\xi) - \Gamma^2(\xi')},$$

$$\int_{P_\xi} d\xi' W(\xi') \int_0^{\infty} dr r J_1(r\xi) N_1(r\xi') =$$
$$= \frac{2\xi}{\pi} \text{V.p.} \int_0^{\infty} d\xi' \frac{W(\xi')/\xi'}{\Gamma^2(\xi'^2) - \gamma^2(\xi)},$$

$$\int_{P_{\xi'}} d\xi W(\xi) \int_0^{\infty} dr r J_1(r\xi) N_1(r\xi') =$$
$$= \frac{2}{\pi\xi'} \text{V.p.} \int_0^{\infty} d\xi \frac{W(\xi)\xi}{\Gamma^2(\xi'^2) - \gamma^2(\xi)},$$

where $W(\xi)$ is arbitrary function, contour P_ξ coincides with positive ξ'-semiaxis excluding the point $\xi' = \xi$ which is bypassed by small semicircle or above either below (and similarly for $P_{\xi'}$). After a series of transformations, we obtain the following systems ($p = 1, 2, \ldots$):

$$\int_0^{\infty} d\xi \frac{\xi \tilde{A}(\xi) \left[\frac{\kappa_{zp}^{(1)}}{\varepsilon} - \gamma(\xi)\right]}{\left(\gamma_{zp}^{(1)}\right)^2 - \gamma^2(\xi)} = \frac{2\kappa_{zp}^{(1)} \tilde{B}_p}{\varepsilon}, \tag{4}$$

$$\int_0^{\infty} d\xi \frac{\xi \tilde{A}(\xi) \left[\frac{\kappa_{zp}^{(1)}}{\varepsilon} + \gamma(\xi)\right]}{\left(\gamma_{zp}^{(2)}\right)^2 - \gamma^2(\xi)} = \frac{\kappa_{zl}^{(1)} B^{(i)} b J_1(j_{0p})}{\varepsilon} \delta_{pl}, \tag{5}$$

$$\frac{2}{\pi\xi N_0(b\xi)} \text{V.p.} \int_0^{\infty} \frac{\xi' \tilde{A}(\xi') d\xi'}{\Gamma(\xi') - \gamma(\xi)} + 2\gamma(\xi) A(\xi) = 0, \tag{6}$$

$$\frac{-2}{\pi\xi N_0(b\xi)} \text{V.p.} \int_0^{\infty} \frac{\xi' \tilde{A}(\xi') d\xi'}{\Gamma(\xi') + \gamma(\xi)} =$$
$$= 2\gamma(\xi) C(\xi) \left(1 + J_0^2(b\xi) N_0^{-2}(b\xi)\right), \tag{7}$$

where $\tilde{B}_p = b B_p(\xi) J_1(j_{0p})/2$, $\tilde{A}(\xi) = \xi A(\xi) J_0(b\xi)$.

To solve these systems simultaneously, one should construct the "resolvent" function $f(w)$. To do this, it is convenient to start from corresponing "embedded" structure (Fig. 1), to construct corresponding "resolvent" function $\bar{f}(w)$ and to perform limiting procedure $a \to \infty$. For the "embedded" structure we have the following systems [6]:

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$$\sum_{m=1}^{\infty} \frac{\tilde{A}_m \left[\frac{\kappa_{zp}^{(1)}}{\varepsilon} - \gamma_{zm}^{(3)}\right]}{\left(\gamma_{zp}^{(1)}\right)^2 - \left(\gamma_{zm}^{(3)}\right)^2} = \frac{2\kappa_{zp}^{(1)}\tilde{B}_p}{\varepsilon}, \tag{8}$$

$$\sum_{m=1}^{\infty} \frac{\tilde{A}_m \left[\frac{\kappa_{zp}^{(1)}}{\varepsilon} - \gamma_{zm}^{(3)}\right]}{\left(\gamma_{zp}^{(1)}\right)^2 - \left(\gamma_{zm}^{(3)}\right)^2} = \frac{\kappa_{zp}^{(1)} B^{(i)} b J_1(j_{0p})}{\varepsilon}\delta_{pl}, \tag{9}$$

$$\sum_{m=1}^{\infty} \frac{\tilde{A}_m}{\gamma_{zm}^{(3)} - \gamma_{zn}^{(2)}} = 0, \tag{10}$$

$$\sum_{m=1}^{\infty} \frac{\tilde{A}_m}{\gamma_{zm}^{(3)} + \gamma_{zn}^{(2)}} = -2\gamma_{zn}^{(2)}\tilde{C}_n, \tag{11}$$

where $n = 0, 1, \ldots$, $\{\tilde{C}_m\}$ and $\{\tilde{A}_m\}$ are modal expansion coefficient in areas (2) and (3), correspondingly, $\gamma_{z0}^{(2)} = -ik_0$, $\gamma_{zm}^{(2)} = \sqrt{\chi_m^2 - k_0^2}$, $\mathrm{Re}\gamma_{zm}^{(2)} > 0$, $\gamma_{zm}^{(3)} = \sqrt{j_{0m}^2 a^{-2} - k_0^2}$, $\mathrm{Re}\gamma_{zm}^{(3)} > 0$, $J_0(b\chi_m)N_0(a\chi_m) - J_0(a\chi_m)N_0(b\chi_m) = 0$.

The solution of these systems can be obtained using modified residue calculus technique [5, 6] by constructing the "resolvent" function $\bar{f}(w)$:

$$\bar{f}(w) = \frac{2B^{(i)} b J_1(j_{0l})\gamma_{zl}^{(1)}\kappa_{zl}^{(1)}}{\left(\kappa_{zl}^{(1)} + \varepsilon\gamma_{zl}^{(1)}\right)\left[\bar{g}\left(\gamma_{zl}^{(1)}\right) + R_l \bar{g}\left(-\gamma_{zl}^{(1)}\right)\right]}\bar{g}(w),$$

$$\bar{g}(w) = \frac{w-\gamma_{zl}^{(1)}}{1-\frac{w}{\Gamma_l^{(1)}}} \frac{\prod_{n=1}^{\infty}\left[1 - \frac{w}{\gamma_{zn}^{(2)}}\right] e^{\frac{wd}{\pi n}} \prod_{s=1}^{\infty}\left[1 - \frac{w}{\Gamma_s^{(1)}}\right] e^{\frac{wb}{\pi s}}}{\prod_{m=1}^{\infty}\left[1 - \frac{w}{\gamma_{zm}^{(3)}}\right] e^{\frac{wa}{\pi m}} e^{\frac{w}{\pi}\left[b\ln\left(\frac{b}{d}\right) + a\ln\left(\frac{d}{a}\right)\right]}},$$

where $d = a - b$,

$$R_m = \left[\varepsilon\gamma_{zm}^{(1)} - \kappa_{zm}^{(1)}\right]\left[\varepsilon\gamma_{zm}^{(1)} + \kappa_{zm}^{(1)}\right]^{-1}.$$

For example, using this "resolvent" function one can easily obtain modal coefficients in the area (3), $\tilde{A}_m = \mathrm{Res}_{\gamma_{zm}^{(3)}}\bar{f}(w)$. Other coefficients are expressed by ever simpler formulas.

The first step of the limiting procedure is to present $\bar{f}(w)$ in specific form. To illustarte this, we put $\varepsilon = 1$ and get:

$$\bar{f}_0(w) = B^{(i)} b J_1(j_{0l})\gamma_{zl}^{(1)}\bar{g}_0(w),$$

$$\bar{g}_0(w) = \tilde{g}_0(w)G(w)\left[\tilde{g}_0\left(\gamma_{zl}^{(1)}\right)G\left(\gamma_{zl}^{(1)}\right)\right]^{-1},$$

$$\tilde{g}_0(w) = \frac{w-\gamma_{z0}^{(2)}}{1-\frac{w}{\gamma_{zl}^{(1)}}} \frac{\prod_{n=1}^{\infty}\left[1 - \frac{w}{\gamma_{zn}^{(2)}}\right] e^{\frac{wd}{\pi n}} \prod_{s=1}^{\infty}\left[1 - \frac{w}{\gamma_{zs}^{(1)}}\right] e^{\frac{wb}{\pi s}}}{\prod_{m=1}^{\infty}\left[1 - \frac{w}{\gamma_{zm}^{(3)}}\right] e^{\frac{wa}{\pi m}}},$$

$$G(w) = \exp\left[\frac{-w}{\pi}\left(b\ln\left(\frac{b}{d}\right) + a\ln\left(\frac{d}{a}\right)\right)\right].$$

We introduce the functions

$$\bar{R}_0(w) = \prod_{s=1}^{\infty}\frac{1 - w/\gamma_{zs}^{(3)}}{1 - w/\gamma_{zs}^{(2)}}\exp\left(\frac{w}{\gamma_{zs}^{(3)}} - \frac{w}{\gamma_{zs}^{(2)}}\right) \text{ and} \tag{12}$$

$$\begin{aligned}\bar{\chi}_0(w) &= \frac{w}{\pi}\left[b\ln\left(\frac{b}{d}\right) + a\ln\left(\frac{d}{a}\right)\right] + \\ &+ w\sum_{n=1}^{\infty}\left(\frac{b}{\pi n} - \frac{1}{\gamma_{zn}^{(3)}} + \frac{1}{\gamma_{zn}^{(2)}}\right),\end{aligned} \tag{13}$$

and obtain

$$\bar{g}_0(w) = \frac{\bar{R}_0\left(\gamma_{zl}^{(1)}\right)}{\bar{R}_0(w)}\exp\left[-\bar{\chi}_0(w) + \bar{\chi}_0\left(\gamma_{zl}^{(1)}\right)\right].$$

Presented functions allow convenient limiting process to the case $a \to \infty$. To do this, one should present (12) and (13) as integrals over certain closed contours in the complex plane, as shown in [5]. For example, after a series of manipulations, we get the following representation for $R_0(w) = \lim_{a\to\infty}\bar{R}_0(w)$ which corresponds to the vacuum case of "opened" structure:

$$R_0(w) = \exp\left\{\frac{2}{\pi^2}\int_0^{\infty}\frac{\left[\ln\left(1 - \frac{w}{\sqrt{\eta^2 - k_0^2}}\right) + \frac{w}{\sqrt{\eta^2 - k_0^2}}\right]d\eta}{\eta\left[J_0^2(b\eta) + N_0^2(b\eta)\right]}\right\}.$$

In a similar way we can obtain the limiting value χ_0 and solve "opened" problem for vacuum case. The next step is using a similar analysis of the more complicated function $\bar{f}(w)$. We hope to perform this task in the near future.

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