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DYNAMIC APERTURE LIMITATION IN e^+e^- COLLIDERS DUE TO SYNCHROTRON RADIATION IN QUADRUPOLES

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Abstract

In a lepton storage ring of very high energy (e.g. in the e^+e^- Higgs factory) synchrotron radiation from quadrupoles constrains transverse dynamic aperture even in the absence of any magnetic nonlinearities. This was observed in tracking for LEP [1] and the Future Circular e^+e^- Collider (FCC-ee) [2]. Synchrotron radiation in the quadrupoles modulates the particle energy at the double betatron frequency. Energy modulation varies transverse focusing strength at the same frequency and creates a parametric resonance of the betatron oscillations with unusual properties [3]. It occurs at arbitrary betatron frequency and the magnitude of the parameter modulation of the betatron oscillation depends on the oscillation amplitude. Equilibrium between the radiation damping and the resonant excitation gives the boundary of the stable motion. Here we continue comparison of tracking results with analytical calculations of the parametric resonance.

INTRODUCTION

Two future electron-positron colliders FCC-ee (CERN) [4] and CEPC (IHEP, China) [5] are now under development to carry experiments in the center-of-mass energy range from 90 GeV to 350 GeV. In these projects strong synchrotron radiation from final focus quadrupoles reduces dynamic aperture.

K. Oide demonstrated FCC-ee transverse dynamic aperture reduction due to radiation from quadrupoles [2] with SAD accelerator design code [6].

We crosschecked the simulation made by Oide using MAD-X PTC [7] and the homemade software TracKing [8] including SR from quadrupoles and found good agreement between all three codes. The reasons of particles loss in horizontal and vertical planes are different by nature [3].

Radiation from quadrupoles at large horizontal amplitude shifts the synchronous phase, induces large synchrotron oscillation, excites strong synchro-betatron resonances and, finally, moves the horizontal tune toward the integer resonance (due to the nonlinear chromatic and geometrical aberrations) according to the mechanism described by J. Jowett and K. Oide.

The energy loss from radiation in quadrupoles for the vertical plane is substantially smaller than for the horizontal plane and does not provide large displacement of the synchronous phase and synchrotron oscillation (Tables 1–3). Instead, we observed that increase of the vertical betatron oscillation amplitude modifies the vertical damping until, and at some threshold, the damping changes to rising and the particle gets lost. This new effect is a parametric resonance in oscillations with friction; radiation from quadrupoles modulates the particle energy at the double betatron frequency; therefore, quadrupole focusing strength also varies at the doubled betatron frequency creating the resonant condition. However, due to friction, resonance develops only if oscillation amplitude is larger than a certain value. The remarkable property of this resonance is that it occurs at any betatron tune (not exactly at half-integer) and hence can be labeled as "self-inducing parametric resonance".

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PARAMETERS VALUES AND OBSERVATIONS FROM TRACKING

For the FCC-ee lattice "FCCee_z_202_nosol_13.seq" at 45 GeV, Figure 1 shows dynamic aperture obtained by MADX PTC [7] tracking with synchrotron radiation from all magnetic elements and without, and obtained by homemade software (TracKing [8]) tracking with synchrotron radiation from dipoles only and with radiation from dipoles and quadrupoles. The observation point is interaction point (IP).

Inclusion of synchrotron radiation in quadrupoles into tracking software decreases dynamic aperture as follows:

- vertical direction from $R_y = 142\sigma_y$ to $R_y = 57\sigma_y$,
- horizontal direction from $R_x = 109\sigma_x$ to $R_x = 65\sigma_x$.

FCC-ee lattice has two IPs and Table 1 gives the parameters relevant to our study.

Table 2 lists total synchrotron radiation energy loss from different type of magnets (dipoles, final focus quadrupoles QFF, focusing and defocusing arc quadrupoles QF and QD). For particles with vertical amplitude energy loss in final focus (FF) quadrupoles dominates the loss in the arc quadrupoles. For particles with horizontal amplitude energy losses in FF and in the arc quadrupoles are comparable and significantly larger than for vertical amplitudes. Table 3 shows the shift of synchronous point and amplitude

Table 1: FCC-ee Lattice Parameters

E_0 [Gev]	45.6
tunes: $\nu_x/\nu_y/\nu_s$	269.14/267.22/0.0413
damping times: $\tau_x/\tau_y/\tau_\sigma$ [turns]	2600/2600/1300
IP: β_x/β_y [m]	0.15/0.001
$\varepsilon_x/\varepsilon_y$ [m]	$2.7\times10^{-10}/9.6\times10^{-13}$
IP: σ_x/σ_y [m]	$6.3\times10^{-6}/3.1\times10^{-8}$
σ_δ	3.8×10^{-4}

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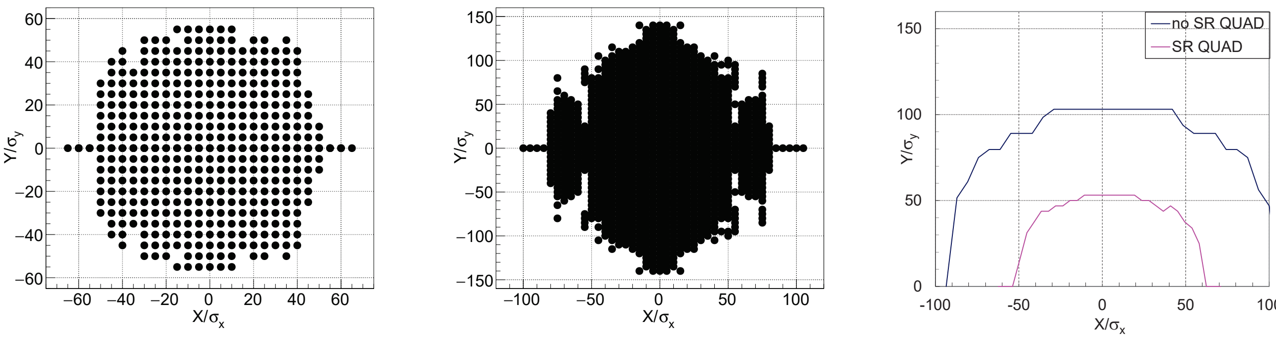

Figure 1: Dynamic aperture: left — tracking by MADX PTC with synchrotron radiation from all magnetic elements, center — tracking by MADX PTC without synchrotron radiation from all magnetic elements, right — tracking by homemade software with synchrotron radiation from quadrupoles (blue) and without (magenta).

Table 2: Total Energy Loss from Different Magnets

Type	N	$U(50\sigma_x)$, MeV	$U(50\sigma_y)$, MeV
Dipoles	2900	35.96	
QFF	8	12	2
QF	1470	4.1	3.7×10^{-3}
QD	1468	1.5	1.5×10^{-2}

Table 3: Synchronous Point and Amplitude of Synchrotron Oscillations

$\{X_0, Y_0\}$	$\{67\sigma_x, 0\}$	$\{0, 58\sigma_y\}$
$p_{\sigma,max}/\sigma_\delta$	4	0.29
$p_{\sigma,syn}/\sigma_\delta$	−2.5	−0.025
σ_{syn}/σ_s	3.1	0.29

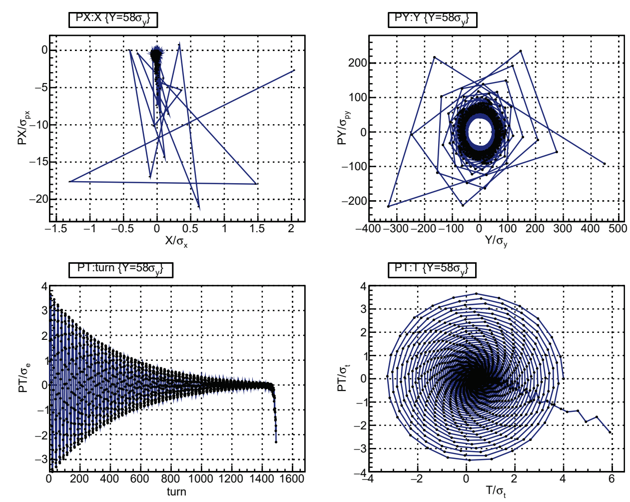

Figure 2: Phase and time trajectories of the first unstable particle with initial conditions $\{x = 0, y = 58\sigma_y, p_x = 0, p_y = 0, \sigma = 0, p_\sigma = 0\}$.

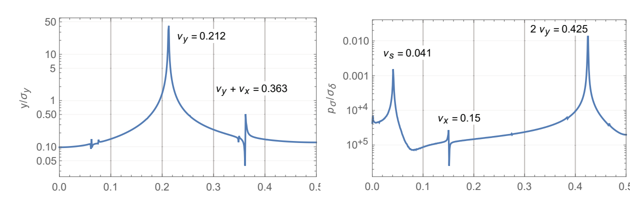

Figure 3: Spectrum of vertical (left) and longitudinal (right) motion from tracking corresponding to initial condition $y = 58\sigma_y$, and adjusted longitudinal initial conditions.

of synchrotron oscillations for different transverse initial conditions.

Figures 2 and 3 show phase and time trajectories and spectra of vertical and longitudinal motion of the first unstable (with accuracy to our step) particle with initial vertical coordinate $y = 58\sigma_y$ and remaining five coordinates are zero. The double frequency harmonic in longitudinal spectrum is obvious on the right plot of Figure 3, and the numerical value of the amplitude $p_\sigma = 2.4\times10^{-2}\sigma_\delta$ agrees with obtained from (2) $p_\sigma = 2.8\times10^{-2}\sigma_\delta$.

Figure 4 shows the change of envelope evolution for particles with initial vertical coordinate around the dynamic aperture boundary $y = \{50; 55; 57.5; 58\}\times\sigma_y$, horizontal coordinates are zero, longitudinal are chosen with respect to the new synchronous point. For the small initial amplitudes, vertical oscillations experience exponential damping, as expected, but with increase of the initial vertical amplitude and contribution of radiation power loss from quadrupoles, the envelope changes shape (left bottom plot on Fig. 4) until damping is replaced by excitation.

SOLUTION OF EQUATIONS OF MOTION

Equations of motion are obtained from expansion of the Hamiltonian [3] where radiation is included by hand with the term describing the change of the momenta. During solution of the equations we used the principal solution of the vertical motion equation [9]

$$\begin{aligned} y &= A_y f_y + A_y^* f_y^* \\ p_y &= A_y f_y' + A_y^* f_y^{*\prime}, \end{aligned} \tag{1}$$

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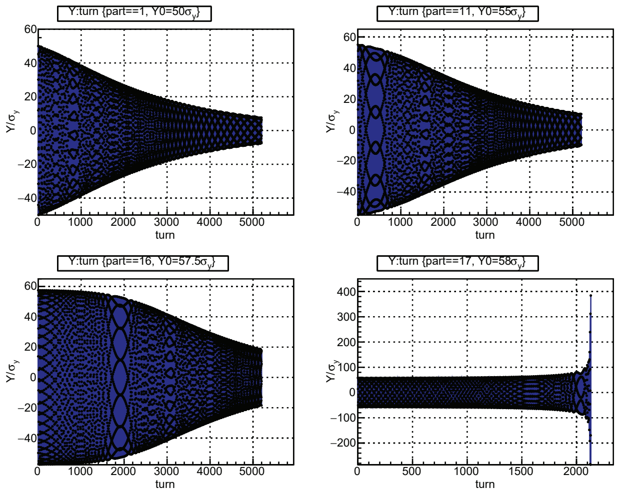

Figure 4: Time evolution of vertical oscillations for particles with initial vertical coordinate $y = \{50; 55; 57.5; 58\} \times \sigma_y$, horizontal coordinates are zero, longitudinal are adjusted for synchronous point.

where constant amplitude A_y depends on initial conditions, f_y is Floquet function.

Longitudinal equations of motion have a particular solution oscillating at fractional part of the doubled betatron frequency ν_y

$$p_\sigma = |c_n|\, J_y \cos\left(æ_y s + \chi_0\right) , \tag{2}$$

where J_y is vertical action, $æ_y = (2\nu_y + n)/R$, $n = -[2\nu_y]$ is the negative integer part of the double betatron tune, $c_n \approx i\Gamma F_{y,n}/æ_y$ and $\chi_0 = \arg(c_n)$, $F_{y,n}$ is a harmonic of $K_1^2\beta_y$, and $\Gamma = \left(C_\gamma/2\pi\right)\left(E_0^4/p_0c\right)$ is a constant related synchrotron radiation power.

Vertical equation of motions could be transformed to an equation of parametric oscillator with friction, where the second term depends on p_σ which contains terms oscillating at fractional double betatron frequency (2)

$$y'' - (K_1 - (K_1 - K_2\eta)p_\sigma)\, y + \Gamma\left(K_0^2 + K_1^2 y^2\right) y' = 0\,. \tag{3}$$

Introduction of modulus and argument of amplitude $A_y = a_y e^{i\varphi_y}$ transforms equation (3) into

$$a_y' = -a_y\,|B_1|\cos(\varphi_1) - a_y^3\,|B_2|\sin(-2\varphi_y + \varphi_2)\,, \tag{4}$$

$$\varphi_y' = -\,|B_1|\sin(\varphi_1) + a_y^2\,|B_2|\cos(-2\varphi_y + \varphi_2)\,, \tag{5}$$

Numerical solution of (4) and (5) shows that stability of the particle trajectory depends not only on the modulus of the initial amplitude but also on the initial betatron phase (Fig. 5). Direct tracking by PTC confirms this result (Fig. 6).

CONCLUSION

Dynamic aperture reduction in the vertical plane with inclusion of synchrotron radiation in quadrupoles in FCC-ee is due to parametric resonance with modulation amplitude

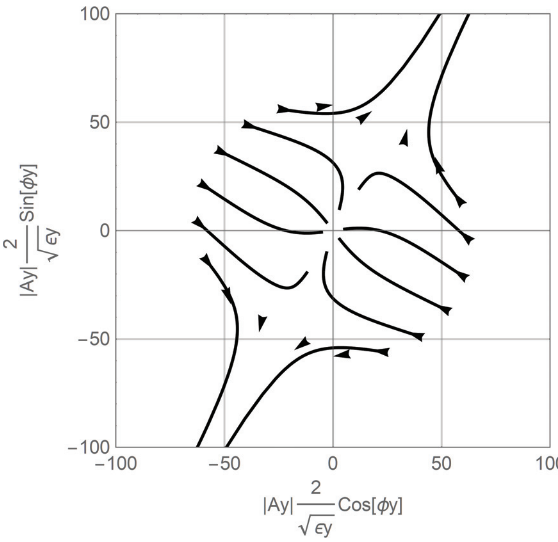

Figure 5: Evolution of the average particle trajectories, solution of equations (4) and (5) with the same initial amplitude and different initial phases. Initial amplitude corresponds to $y0(\varphi_y = 0) = 58\sigma_y$.

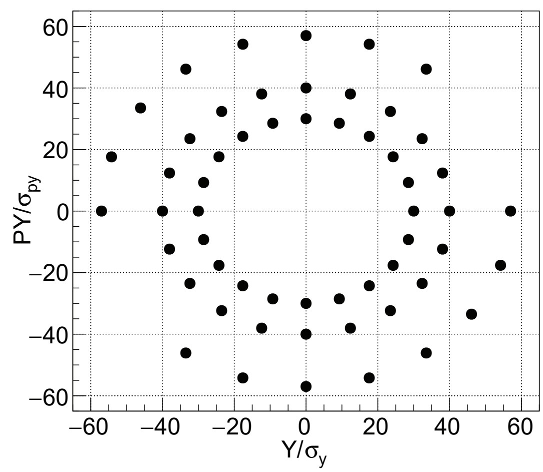

Figure 6: Vertical dynamic aperture from tracking in Y, PY plane.

dependent on the square of oscillation amplitude. Radiation from quadrupoles modulates the particle energy at the double betatron frequency; therefore, quadrupole focusing strength also varies at the doubled betatron frequency creating the resonant condition. However, due to friction, resonance develops only if oscillation amplitude is larger than a certain value. The remarkable property of this resonance is that it occurs at any betatron tune (not exactly at half-integer) and, hence, can be labeled as “self-inducing parametric resonance”. Our calculations give the border of dynamic aperture $R_y = 52.6\sigma_y$, which corresponds well to the tracking result $R_y = 57\sigma_y$.

Additional analysis of equations of motion confirms our explanation related to parametric resonance, even in particle trajectory stability dependence on initial betatron phase.

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