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STRAIGHTNESS CORRECTION OF BALLISTIC TRAJECTORIES

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Abstract

We describe procedure for straightness correction of ballistic trajectories in the presence of unknown stray magnetostatic field and BPM offsets. We also discuss applicability of this method to the beam based alignment of the European XFEL undulators.

INTRODUCTION

In the framework of this paper, as a beam-based alignment (BBA) method we understand any procedure which uses as input beam trajectory measurements and allows to find offsets of the beam position monitors (BPMs) with respect to some straight line of, in general, unknown orientation in regard to the laboratory coordinate system. Such methods are important in many areas of accelerator applications, and our particular interest is connected with the problem of alignment of the European XFEL undulators.

BBA method, which tries to recover unknown BPM and quadrupole offsets by fitting beam orbits measured for various beam momenta to the known optical model of the beamline, has been established at the LCLS and later on was also used at the European XFEL and at the PAL-XFEL facilities [1–3]. The core of this method is an attempt to solve the ill-conditioned inverse problem and, therefore, in order to overcome its high sensitivity to the imperfections of the optical model and measurement errors, and nevertheless have reasonably accurate results, iterations and some regularization procedure are typically required.

Other popular method discussed in the last decades is the so-called dispersion free steering (DFS) algorithm, which was invented to deal with the emittance dilution due to chromatic effects [4]. Using DFS methodology one tries to minimize the difference between orbits measured for different beam energies but for the same transverse initial conditions by using available actuators (steerers, quadrupole movers and etc.). Unfortunately, even if the goal of the DFS algorithm will be fulfilled and the difference orbits will be corrected almost to zero, the straightness of the resulting trajectories can't be guaranteed. We mention DFS method only because, as we will see later on, it is closely related to the alignment approach suggested in this paper.

In the field free region the particle beam automatically follows a straight line, which is the basis of the so-called ballistic alignment method [5]. Unfortunately, at the European XFEL, even if we will not only turn off all undulator quadrupoles but also will degauss them, the uncontrollable ambient magnetic field remains still enough large and nonuniform to prevent direct usage of ballistic trajectories for the straight line definition purposes [6].

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In this article we present a new BBA method, which is based on the procedure of straightness correction of ballistic trajectories, and briefly discuss applicability of this approach to the alignment of the European XFEL undulators. Due to space limitation, we consider only the most basic questions and the more detailed description of the algorithm and, hopefully, practical alignment results will follow soon.

MOTION IN THE WEAK MAGNETOSTATIC FIELD

We describe transverse particle motion in the magnetostatic field using a (straight) Cartesian coordinate system with x, y and z being the horizontal, vertical and longitudinal direction, respectively. We assume that the longitudinal coordinate z can be introduced as an independent variable and use a set of variables $w = (x, q_x, y, q_y)^\top$ as transverse particle coordinates. In this set q_x and q_y are transverse mechanical monenta scaled with the value p_0, which is known as reference kinetic momentum and is defined via relations

$$p_0 \;=\; m_0\, c \,\sqrt{\gamma_0^2 \,-\, 1} \;=\; \beta_0\, \mathcal{E}_0 \,/\, c, \tag{1}$$

where m_0, $\mathcal{E}_0$, γ_0, and β_0 are the rest mass of the particle, its energy, its Lorentz factor, and its velocity in terms of the speed of light c, respectively.

In these variables, the equations describing the transverse motion of a particle in a static magnetic field take on the form

$$\frac{dx}{dz} = \frac{q_x}{q_z}, \qquad \frac{dq_x}{dz} = \frac{e}{p_0}\left(\frac{q_y}{q_z} B_z - B_y\right), \tag{2a}$$

$$\frac{dy}{dz} = \frac{q_y}{q_z}, \qquad \frac{dq_y}{dz} = \frac{e}{p_0}\left(B_x - \frac{q_x}{q_z} B_z\right), \tag{2b}$$

where

$$q_z \;=\; \sqrt{1 \,-\, q_x^2 \,-\, q_y^2}, \tag{3}$$

e is the particle charge, e/p_0 is the inverse particle stiffness, and $B = (B_x, B_y, B_z)^\top$ is the magnetic field.

Let us assume that all strong magnets in the beamline of interest are turned off (and even deguassed, if possible) and therefore the particle motion can be influenced only by uncontrolled stray magnetostatic fields and, possibly, by some weakly excited corrector magnets, which were used to keep the beam transmission with the main magnets switched off. Then the x and y components of the solution of equations (2) (i.e. ballistic trajectory) can be expressed in the form of the asymptotic series in powers of a field smallness

$$x(z) = x(0) \,+\, \frac{q_x(0)}{q_z(0)} \cdot z + \sum_{k=1}^{\infty} \left(\frac{e}{p_0}\right)^k \cdot X_k\,[z,\, w(0)], \tag{4a}$$

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$$y(z) = y(0) + \frac{q_y(0)}{q_z(0)} \cdot z + \sum_{k=1}^{\infty} \left(\frac{e}{p_0}\right)^k \cdot Y_k\left[z, w(0)\right], \quad (4b)$$

which are, in fact, expansions in powers of inverse particle stiffness and that is of principal importance.

GENERALIZED DIFFERENCE ORBITS FOR BALLISTIC TRAJECTORIES

Difference orbits are very useful tools in practical accelerator operations, especially in the presence of unknown BPM and magnet offsets. For example, the actual beam optics can be measured by using difference of two orbits having the same beam energy but different transverse initial conditions, and the beamline dispersion can be estimated by utilizing differences of trajectories measured for different beam energies but for the same transverse initial conditions.

The difference orbits, which we intend to introduce in this section, will be defined using particle trajectories taken for different beam energies but for the same transverse initial conditions. Thus, let us choose n different beam energies

$$\mathcal{E}_0(1) > \mathcal{E}_0(2) > \ldots > \mathcal{E}_0(n) \quad (5)$$

with the reference momenta

$$p_0(1) > p_0(2) > \ldots > p_0(n) \quad (6)$$

and let

$$[x_1(z), y_1(z)],\, [x_2(z), y_2(z)],\, \ldots,\, [x_n(z), y_n(z)] \quad (7)$$

be the corresponding x and y components of the solutions of equations (2) taken for the same (energy independent) initial conditions $w(0)$.

Let us now consider the system of n linear equations

$$c_1 + c_2 + \ldots + c_n = 0, \quad (8a)$$

$$\frac{c_1}{p_0^k(1)} + \frac{c_2}{p_0^k(2)} + \ldots + \frac{c_n}{p_0^k(n)} = \frac{1}{p_0^k(1)}, \quad (8b)$$

$$k = 1, 2, \ldots, n-1$$

with respect to the n unknowns $c_1, c_2, \ldots, c_n$.

The matrix of the system (8) is the Vandermonde matrix. It is non-degenerate due to condition (6), and therefore there exists unique solution for the unknowns c_m. For example, for $n = 2$

$$c_1 = -c_2 = -\frac{1}{u_2 - 1}, \quad (9)$$

and for $n = 3$

$$c_1 = -\frac{u_2 + u_3 - 1}{(u_2 - 1)\cdot(u_3 - 1)}, \quad (10a)$$

$$c_2 = \frac{u_3}{(u_2 - 1)\cdot(u_3 - u_2)}, \quad (10b)$$

$$c_3 = -\frac{u_2}{(u_3 - 1)\cdot(u_3 - u_2)}, \quad (10c)$$

where we have used the notation

$$u_m = \frac{p_0(1)}{p_0(m)} = \sqrt{\frac{\gamma_0^2(1) - 1}{\gamma_0^2(m) - 1}}. \quad (11)$$

As generalized difference orbits for ballistic trajectories we take the combinations $\tilde{x}_n(z)$ and $\tilde{y}_n(z)$ defined as follows

$$\tilde{x}_n(z) = c_1\, x_1(z) + c_2\, x_2(z) + \ldots + c_n\, x_n(z)$$

$$= \sum_{k=1}^{n-1} \left(\frac{e}{p_0(1)}\right)^k \cdot X_k\left[z, w(0)\right] + O\left[\left(\frac{e}{p_0(1)}\right)^n\right], \quad (12a)$$

$$\tilde{y}_n(z) = c_1\, y_1(z) + c_2\, y_2(z) + \ldots + c_n\, y_n(z)$$

$$= \sum_{k=1}^{n-1} \left(\frac{e}{p_0(1)}\right)^k \cdot Y_k\left[z, w(0)\right] + O\left[\left(\frac{e}{p_0(1)}\right)^n\right]. \quad (12b)$$

The usefulness of these generalized difference orbits is connected with the facts that in $\tilde{x}_n(z)$ and $\tilde{y}_n(z)$ the energy independent BPM offsets will be canceled during usage of the real BPM measurements and that they reproduce first $n - 1$ terms in the series expansions (4) taken for the highest reference momentum $p_0(1)$.

Note that if the generalized difference orbits will be calculated using BPM measurement data, then one will meet the problem of the BPM noise amplification, i.e. in $\tilde{x}_n(z)$ and $\tilde{y}_n(z)$ the rms BPM noise will be increased by a factor

$$\sqrt{c_1^2 + c_2^2 + \ldots + c_n^2}, \quad (13)$$

which grows rather quickly as n increases. This problem can be partially solved in the usual fashion by assuming that nonlinear terms in equations (2) are negligible and applying averaging over many trajectory measurements.

POSSIBILITIES FOR NONITERATIVE STRAIGHT LINE DEFINITION

Comparing (4) and (12) one sees that the differences

$$x_1(z) - \tilde{x}_n(z) = x(0) + \frac{q_x(0)}{q_z(0)} \cdot z + O\left[\left(\frac{e}{p_0(1)}\right)^n\right], \quad (14a)$$

$$y_1(z) - \tilde{y}_n(z) = y(0) + \frac{q_y(0)}{q_z(0)} \cdot z + O\left[\left(\frac{e}{p_0(1)}\right)^n\right] \quad (14b)$$

are the straight lines with the precision up to the n-th degree of the field smallness, and the accuracy of these formulas can be increased in practice if the field smallness could be reduced (in the integral sense) by proper setting of corrector magnets based, for example, on the ambient field measurements.

The main limiting factor for successful application of formulas (14) to the real alignment is mainly not the discussed above BPM noise amplification effect, but rather tight tolerances on the beam energy and BPM calibration, and on the ability to keep energy independent transverse initial conditions for the particle trajectories, which currently at the European XFEL can't be provided with the needed precision.

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ALGORITHM FOR ITERATIVE STRAIGHTNESS REFINEMENT

In this section we describe briefly an iterative straightness refinement procedure, which in the numerical simulations provides good results even with rather large errors in the beam energy and BPM calibration, and without ability to keep energy independent transverse initial conditions for the beam orbits.

Let us assume that the beamline under consideration consist of m cells bounded by $m+1$ BPMs with the longitudinal positions

$$z_0 < z_1 < \ldots < z_m, \tag{15}$$

and let L be the maximal cell length.

For an arbitrary function $\xi = \xi(z)$, the necessary and sufficient conditions to coincide with some straight line at the BPM locations can be written, for example, in the form

$$F_k(\xi) = 0, \quad k = 1, \ldots, m-1, \tag{16}$$

where

$$F_k(\xi) = L \cdot \left[\frac{\xi(z_{k+1}) - \xi(z_k)}{z_{k+1} - z_k} - \frac{\xi(z_k) - \xi(z_{k-1})}{z_k - z_{k-1}} \right]. \tag{17}$$

So, in order to improve straightness of ballistic trajectories and, in the same time, to avoid difficulties connected with the unknown BPM offsets, we suggest to use generalized difference orbits instead of ballistic trajectories themselves and iteratively minimize the function

$$\Psi(\tilde{x}_n, \tilde{y}_n) = \sum_{k=1}^{m-1} \left[F_k^2(\tilde{x}_n) + F_k^2(\tilde{y}_n) \right] \tag{18}$$

by using available in the beamline corrector magnets.

Due to space limitation, let us mention briefly in somewhat arbitrary order few points important for the practical realization of the suggested algorithm, and its detailed description will be published elsewhere.

Beam Transport without External Focusing

Without possibility to transport particles with external focusing switched off, there is no big sense in all our considerations. So, as a first step, we tested it at the European XFEL and have shown that the particle beam can be reliably transported through both its long undulators (SASE1-2) without quadrupole focusing at the energies of 10, 14, and 16 GeV.

Number of Different Energies

Extensive numerical simulations of the straightness refinement procedure in the presence of ambient fields of different amplitudes and spatial shapes indicate that usage of only two different beam energies separated by about 4 GeV is already sufficient in order to find lines of good straightness along the European XFEL undulators.

Trajectory Launch Conditions

Even if there exists a possibility to keep energy independent transverse initial conditions (for example, by upstream BPMs placed in the field shielded region), trajectories corresponding to the different beam energies will, in general, approach different straight lines during optimization process. It is not very convenient and may lead, for example, to transmission loss. So, it is better to minimize not the function (18), but to add to this function the term

$$\left[L \cdot \frac{\tilde{x}_n(z_1) - \tilde{x}_n(z_0)}{z_1 - z_0} \right]^2 + \left[L \cdot \frac{\tilde{y}_n(z_1) - \tilde{y}_n(z_0)}{z_1 - z_0} \right]^2. \tag{19}$$

One can show that with such addition our algorithm can be viewed as an analog of the DFS procedure applied to the generalized difference orbits, and that trajectories corresponding to the different energies will approach each other and therefore the same straight line during minimization.

It gives the following recipe how to deal with the launch conditions in the absence of upstream BPMs in the field shielded region. One fixes launch for the highest energy orbit by some upstream BPMs and follows it on all iterations. For any other trajectory, one searches at each iteration for the new launch, which minimizes rms difference of this orbit with the current highest energy orbit. Such procedure somewhat increases the number of iterations required for achieving good straightness, but works even in the absence of the possibility to keep energy independent initial conditions from the beginning.

Indirect Quality Indicator

As indirect quality indicator one can calculate at every iteration straightness of the generalized difference orbits.

Algorithm Output

As an approximation to to the straight line one either simply takes the highest energy trajectory or make use of the differences (14), and in the numerical simulations both approaches usually give compatible results.

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