ISBN: 978-3-95450-208-0 doi:10.18429/JACoW-IPAC2019-MOPGW006

LONG RANGE BEAM BEAM: TOWARDS FASTER COMPUTATIONS*

S.R. Koscielniak†
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3

Abstract

We outline some features of a program of study toward faster computation of the cumulative effect of a sequence of beam-beam interactions across the interaction region.

INTRODUCTION

The beam beam (BB) interaction between counter-rotating beams has a long history of study. The interactions may be long range (LR) or short range (SR) depending as the beams are separated or not (respectively). The interactions are impulsive, occurring each time bunches collide. Beam-beam limitations to the luminosity in proton-proton colliders has a long and relentless study for over four decades. Three machines have been the focus of these studies: the Tevatron and SCSC in the U.S. and the CERN LHC (and HL-LHC) in Europe.

The original intention was that this paper, with [1-3], would be the end goal in a concerted program to speed up the calculation of weak-strong beam-beam effects in the HL-LHC under the circumstances of unequal horizontal and vertical beam sigmas, asymmetric Interaction Region optics, and closed orbit distortions (COD).

PROGRAM OF STUDY

This program would calculate the short-range interaction at the central interaction point and the long-range (a.k.a parasitic) interactions of the separated beams in the interaction region; and then sums these with the appropriate betatron phase advances; and then finally perform the sum over two interaction regions using Lie-algebraic techniques throughout.

For particle tracking studies we need the electric field and transverse impulse as function of transverse displacements. Analytic tune shift studies need the potential as function of action-angle coordinates. Analytic resonance studies need the Fourier components (with respect to angle) of the potential.

Each calculation is long, complicated and repetitive. It was hoped that significant speed up of the calculation could be made by exploiting several aspects of the problem.

1. Using a simpler beam density and interaction potential
2. Making a complete separation of the short-range and long-range interaction.
3. Exploiting symmetries that exist in the optics (such as phase advances, and equality of the ratios of beam sizes and separations) downstream and upstream of the central IP to introduce cancellations or simple summations of the up and down-stream interactions.
4. Use of pre-computed expressions for particular amplitudes such as 6σ, is useful to resonance studies – but not particle tracking.

Item (1) is the main area of novelty, and has already many ramifications: field, potential, tune-shifts, Fourier components, etc., all have to be re-calculated. Contrastingly, item (3), exploiting symmetries, is already performed by other authors for many years.

Currently, some parts of this program are complete and others well-started but not complete; and some parts have been the subject of diversions; for example, how to deal correctly with the potential function for the closed orbit distortion that results from the long-range interaction. Further, the calibration and comparison of new and old results has proven much more demanding than expected.

BB INTERACTION POTENTIAL

Particle tracking relies on computation of the electromagnetic impulses. Theory relies on analysis of the adiabatic invariants in presence of the BB potential. The impulse and potential for a single BB interaction are both complicated functions, usually developed in high order series. In a single interaction region (IR), there may be several short range and many long range BB encounters. In a ring, there may be several IRs. Consequently, many impulses (or potentials), which are individually complicated, must be calculated and added together. Therefore, short expansion series with equivalent accuracy will be useful.

Traditionally, the beams are taken to have Gaussian transverse charge distributions. In this model, the witness particle (in the weak beam) is always inside the strong beam (provided they are within the same physical aperture). This has the advantage for particle tracking, that no test is needed for inside versus outside.

Houssais and later Bassetti & Erskine, took the transverse density to be $G=\mathrm{Exp}[-R^2/2]$ where $R^2 = [(x/\sigma_x)^2+(y/\sigma_y)^2]$ and where unequal r.m.s. are σx and σy. However, particle beams are not necessarily Gaussian unless synchrotron radiation is strong and the storage time is long. We take density in the quadratic form Q = [1-(R/b)^2]^N. With b ≥4σ and N ≥6 suitably chosen, this can approximate the Gaussian increasingly well, even for relatively small N>6. Let V(G) and V(Q) denote the potentials and E(G) and E(Q) denote the fields from the gaussian and quadratic density, respectively.

The power series expansions for field and potential are finite; and surprisingly accurate in the regime $b^2=(4+N)\sigma^2$. When we calculate the corresponding potential, for R>4sigma or larger, it differs little from the Gaussian case. This happens because the residual charge beyond radius

* TRIUMF receives funding via a contribution agreement with the National Research Council of Canada
† shane@triumf.ca

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>R is of order Exp[-R^2 /2] <<< 1 leaving only the Log[R] term, which is common to both potentials. In other words, at very large radius the only term that really counts in the field is 1/R; the E(G) and E(Q) both get this dominant term correct (indeed for R>b, it's the only term in E(Q)) ; but the series for E(G) goes on to use many terms to correct for the charge at R>b.

CONVERGENCE OF SERIES EXPANSION

Concerning the cost of evaluating a mathematical function, we take the following model. The function has an argument which is evaluated once, and then we compute a power series in the argument – a series which has some convergence property. Typically, the larger is the argument, the greater is the number of terms that must be summed to obtain a relative fractional accuracy.

In the case of the E(G), that series is in principle infinite whereas in the case of a power-law quadratic form E(Q) the series is always finite. How could it be that they (gaussian and quadratic) give a similar result? The quadratic makes the simplifying assumption that all the charge is within radius R=b, whereas the gaussian has to worry about the residual charge beyond this radius even though it becomes progressively and vanishingly small.

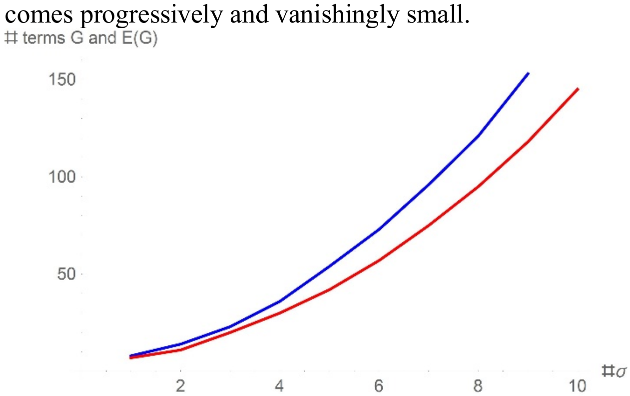

Figure 1: The number of terms are needed to achieve relative fractional accuracy better than 0.1% for G (shown blue) and E(G) (shown red) versus the number of standard deviations from the beam centre.

From the series expansion of E(G), it may be shown that the number of terms N_T to achieve excellent fractional accuracy at radius $R=N_\sigma \times \sigma$ is $N_T = -1 + (N_\sigma)^2$Exp[1]/2. This square law dependence is evident in Figure 1. The series for E(Q) converge more quickly; the number of terms is order 2^N where N =[N_σ^2-4]/2.

What is going on here? How come we can use so few terms? We do not try to construct a function that is good for all possible amplitudes.

We construct a function that is a very good approximation to the field and potential for all values less than a particular amplitude. Hence, we would have to construct different functions for different amplitudes. But we are lucky; at the outset that we are interested in particular amplitudes, such as 6 sigma or 9 sigma.

PARTITION OF LRBB

The mathematical form of the long-range interaction is such that it contains also the short range interaction; it may be considered as the short range part (that does not contain the beam separation) and a residual "true long-range part" that contain terms in powers of the beam separation. These two parts have different symmetry with respect to reversal of the sign of the displacements, and so will add up differently either side of the IP. Figures 2a and 2b and the bullet points below attempt to explain that conclusion.

- The lattice functions (β,γ) are symmetric about the IP.
- Both beams share the same lattice functions (β,γ)
- For a single beam, ellipse tilt α is antisymmetric about the IP
- The single-particle betatron phases flip by π across the IP
- The beam-beam force has the symmetry F(-x,y,-d) = F(x,y,d)
- Hence residual beam-beam forces do not cancel across the IP; instead they add.

These are important properties which result in simplifications.

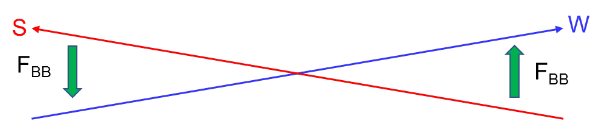

Figure 2a: If x does not flip sign across the IP, the F_{BB} cancels.

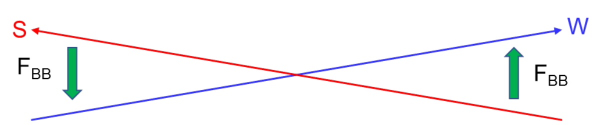

Figure 2b: But x flips sign across the IP, so the F_{BB} adds.

ASYMMETRIC INTERACTION REGION, UNEQUAL SIGMAS, AND COD

The weak-strong model subjects the test particle in the weak beam to transverse impulses from the strong beam. The LRBB interaction is a function of the quadratic form: [(x+Dx)^2/sx^2 + (y+Dy)^2/sy^2]. Here x,y = Sqrt[BetaWeak(s)_x,y]Cos[Phi(s)_x,y], sx,sy = Sqrt[BetaStrong(s)_x,y], and beam separations Dx,Dy propagate the reference orbits away from the interaction point (IP) where D_x,y=0 and there are crossing angles. Dx(s) is contributed to by the closed orbits of the weak and strong beams. To 1st order Dx = DxWeak(s)+DxStrong(s) propto Sqrt[BetaWeak]Cos[…] +Sqrt[BetaStrong]Cos[…] where the cosine terms propagate the IP conditions. Unlike x and y, Dx,y do not advance in phase each turn. To 2nd order, the constant terms in the LRBB impulse induce closed orbit distortions. These are compensated by impulses from magnetic elements up/down stream of the IP.

In many cases, the interaction region [IR] is symmetric about the IP; and so Beta[-s]=Beta[+s]. Moreover, for round beams Beta_x = Beta_y. This results in substantial simplifications. Much of the BB literature adopts those

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simplifying conditions. However, this is not so for IR1 and IR5 of the Large hadron Collider (LHC). In this case, beta_x not equal beta_y and beta(-s) not equal bet(+s) except in the drift between the final quadrupole triplets. Dx=0 in IP1 and Dy=0 in IP5. All of this must be included in the appropriate symmetries.

We sketch the method with an abbreviated IR lattice (See Figure 3) which we take to extend from just downstream of D1L to just upstream of D1R.

Make a sequence of Lie maps; see Figure 4. The strategy is to move all beam-beam kicks to the IP @ 4. Insert identity transforms, then perform similarity transforms. Start at C3 and work left. Start again at C5 and work right. The result, see Figure 5, is a single equivalent element at the IP book-ended with linear optics.

A($M14^{-1}$ z) means evaluate A1 in the coordinates of D4. The two sets of coordinates are linked by the Courant-Snyder transforms between those locations. Likewise A(M47 z) means evaluate A7 in the coordinates of D4.

The beam size and separation appear in the residual F_{BB} Because the beam size is the projection of the beam phase-space ellipse, and the beam ellipse is also transported by the Courant-Synder parameters, and because the beam separation (in the weak-strong model) is also transported by the same optics, we may hope that all the beam-beam kicks can be cast in a similar functional form that allows simpler (but far from trivial) summation over the beam-beam operators (A, B, C, D). It is possible (but far from guaranteed) that A, B, C, D commute.

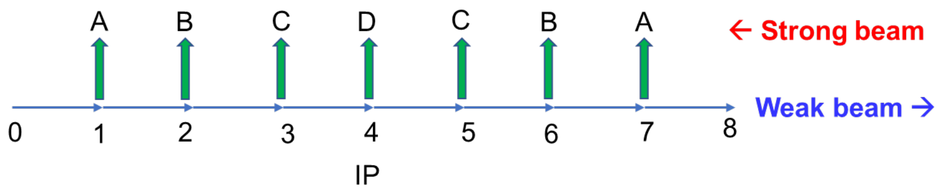

Figure 3: Schematic of the beam-beam interactions labelled A through D either side of the IP at parasitic interaction points labelled 1 through 8. In reality there are many more such long-range interactions as bunches pass one another.

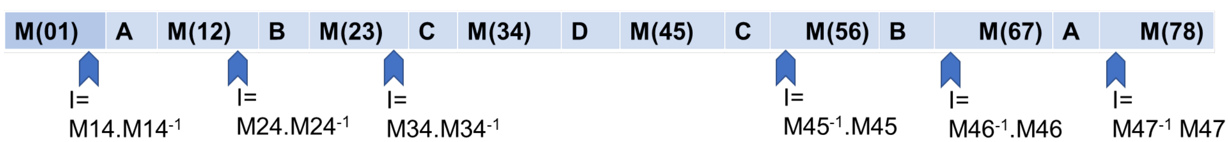

Figure 4: Sequence of identity transforms are inserted and then similarity transforms (for the beam optics) enable the successive beam-beam interactions (labelled A-D-A) to be moved from the locations 1-8 to the central interaction point. Here M(nm) are optics, and (A,B,C,…) are residual beam-beam elements.

M(04)	**A($M14^{-1}$ z)**	**B($M24^{-1}$ z)**	**C($M34^{-1}$ z)**	**D(Z)**	**C(M45 z)**	**B(M46 z)**	**A(M47 z)**	**M(48)**

Figure 5: Result of applying the Lie maps is to concentrate an effective single beam-beam interaction that is the Lie product (shown as blue cells) at the IP book-ended with linear optics (shown as the pink cells).

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