New features of beamstrahlung important for crab-waist e+e- colliders
or
"Short magnet" beamstrahlung at e+e- circular colliders

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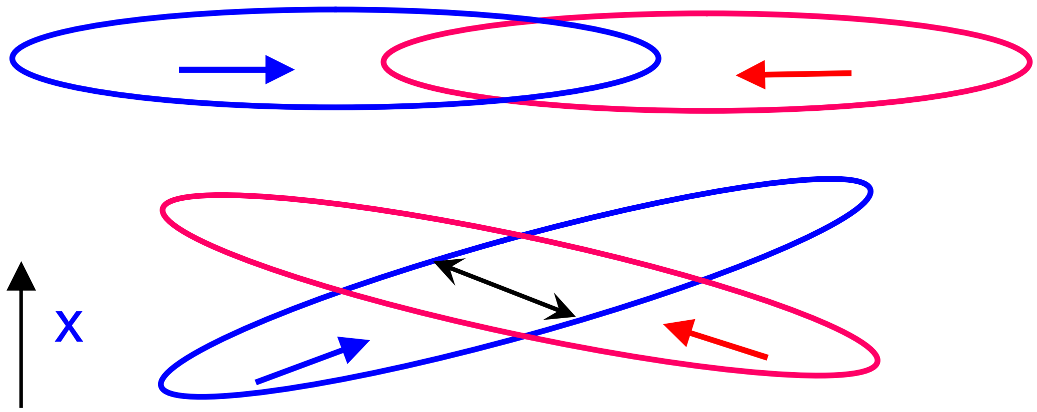
Introduction, beamstrahlung at e+e- colliders

Beamstrahlung is radiation in the field of opposing beam. It is a key effect at linear colliders which determine its parameters.

At circular e+e- colliders the beamstrhlung by now was negligible and did not influence collider performance. However, at future high energy circular e+e- colliders (Higgs factories) emission of high energy beamstra-hlung photons will determine the beam lifetime and thus attainable luminosity (V. Telnov, 2012), it leads also to the additional beam energy spread and bunch lengthening.

By now this effect was considered using well known formulas for synchrotron radiation. However, in the case of circular collider beamstra-hlung regime is intermediate between synchrotron radiation and a "short-magnet" radiation where spectrum is harder. Therefore more detailed consideration is required. This motivated my present talk.

Collision schemes



Head-on collision.
Rms collision length is $\sigma_z/2$

Crab-waist collision.
Rms collision length is $\sigma=\sigma_x/\theta$
The vertical $\beta_y \sim \sigma$

Top view

The maximum beam field (B=E+B)
For flat beams at z=0, x=0, $y>\sigma_y$

$$B = \frac{2eN}{\sigma_x \sigma_z}$$

For head-on collisions the tune shift ($\xi_y \le 0.1-0.15$) and the luminosity

$$\xi_y = \frac{N r_e \beta_y}{2\pi\gamma\sigma_x\sigma_y} \approx \frac{N r_e \sigma_z}{2\pi\gamma\sigma_x\sigma_y} \quad \text{for} \quad \beta_y \approx \sigma_z \qquad \mathcal{L} \approx \frac{N^2 f}{4\pi\sigma_x\sigma_y} \approx \frac{N f \gamma \xi_y}{2 r_e \sigma_z}$$

For the crab-waist scheme

$$\xi_y = \frac{N r_e \beta_y^2}{\pi\gamma\sigma_x\sigma_y\sigma_z} \quad \text{for} \quad \beta_y \approx \sigma_x/\theta \qquad \mathcal{L} \approx \frac{N^2 f}{2\pi\sigma_y\sigma_z\theta} \approx \frac{N^2 \beta_y f}{2\pi\sigma_x\sigma_y\sigma_z} \approx \frac{N f \gamma \xi_y}{2 r_e \beta_y}$$

In addition
at high energy $\frac{N}{\sigma_x\sigma_z} < 0.1\eta\frac{\alpha}{3\gamma r_e{}^2}$ - due to beamstrahlung (lifetime>30')

Synchrotron radiation

For classical SR (ω_c<<E) the intensity per unit length

$$\frac{dI}{dz\,d\omega}=\frac{\sqrt{3}}{2\pi}\frac{eBr_e}{c}F(y),\quad F(y)=y\int_y^{\infty}K_{5/3}(x)dx,\quad y=\frac{\omega}{\omega_c},\quad \omega_c=\frac{3}{2}\frac{\gamma^3 c}{\rho}=\frac{3}{2}\frac{\gamma^2 eB}{mc}$$

ω_c-critical energy

$$\mathrm{dI/d}\omega\propto(\omega R/c)^{1/3}\ \text{ for }\ \mathrm{y}\ll 1,\qquad \mathrm{dI/d}\omega\propto \mathrm{y}^{1/2}\mathrm{e}^{-y}\ \text{ for }\ \mathrm{y}\gg 1$$

The spectrum after crossing some magnet with a characteristic length σ and maximum field B_0

$$\frac{dI}{dy(\gamma^2 r_e^2 B_0^2\sigma)}=\frac{3\sqrt{3}}{4\pi\sigma}\int_{-\infty}^{\infty}\frac{B(z)}{B_0}\left(y\frac{B_0}{B(z)}\right)\int_{yB_0/B}^{\infty}K_{5/3}(x)dx\,dz$$

For example, for a Gaussian beam $B/B_0=\exp(-z^2/2\sigma^2)$

$$\frac{dI}{dy(\gamma^2 r_e^2 B_0^2\sigma)}=\frac{3\sqrt{3}}{2\sqrt{2}\pi}\int_0^1\frac{F(y/x)}{\sqrt{\ln(1/x)}}dx$$ - function of $y=\omega/\omega_c$ only

where $y=\omega/\omega_c$ and ω_c is related to B_0.

“Short magnet” radiation

The formation length of SR $l_{f,SR} \sim \rho/\gamma \quad \rightarrow \omega_c \sim c\gamma^3/\rho$ and $l_{f,SR} \sim (\omega_c/\omega)^{1/3}\rho/\gamma$ for $\omega/\omega_c<1$. If the deflection $\theta \sim \sigma/\rho < 1/\gamma$, then the radiation process can be considered as Compton scattering of electrons on virtual photons with $\lambda \sim \sigma$ ($\omega_0 \sim c/\sigma$) then emitted photons have $\omega_{sh} \sim \gamma^2\omega_0 \sim c\gamma^2/\sigma$ (“omega-short”). The ratio $\omega_{sh}/\omega_c \sim \rho/\gamma\sigma \sim 1/(\gamma\theta) > 1$. So, the “short magnet” radiation is harder than SR.

The spectrum is determined by the paramener $K=\rho/\gamma\sigma=eB_0\sigma/mc^2$.

K>>1 -SR, K<<1 -“short magnet” radiation. It is similar to the parameter K in wigglers or non-linear parameter is Compton scattering. All properties of radiation are determined by two parameters: ω_c and K. It is convenient to define $\omega_{sh}= 2\gamma^2c/\sigma$, then $\omega_{sh}/\omega_c=4/(3K)$. Below ω_{sh} is called “omega short”.

For K<<1 and $B=B_0\exp(-z^2/2\sigma^2)$ the spectrum was found by R. Coisson in 1979 (*2)

$$\frac{dI}{dx(\gamma^2 r_e^2 B_0^2 \sigma)} = 4\int_1^\infty (u^{-2} - 2u^{-3} + 2u^{-4})\, e^{-x^2u^2}\, du, \qquad x = \omega/\omega_{sh}, \qquad \omega_{sh} = \frac{2\gamma^2 c}{\sigma}$$

The spectrum is almost flat for x<1 and fall as $\exp(-x^2)$ for x>>1. In terms of SR variable $y=\omega/\omega_c$

$$\frac{dI}{dy(\gamma^2 r_e^2 B_0^2 \sigma)} = 3K\int_1^\infty (u^{-2} - 2u^{-3} + 2u^{-4})\, e^{-x^2u^2}\, du, \qquad x = \frac{3K}{4}y.$$

Remark:

The existence of “short magnet” radiation at some storage rings was discussed in many publications by V.Serbo et al. in 1992 and later, they called such radiation as a“coherent bremsstrahlung” - Compton scattering on virtual photons created by the whole bunch (while the usual bremsstrahlung is the Compton scattering of electrons on virtual photons accompanying individual charged particles). They derived formulas for K<<1 using QED method, but in fact their results are very similar to those obtained by R. Coisson in 1979 using classical electrodynamics.

Following Coisson I call this mode of radiation as “short magnet radiation”, although more precisely the type of radiation depends on the product of the magnitude of the field on the path length of the particle in the field.

General case, any field shape, any K

In crab-waist collider e+e-colliders K~1. Basic formulas for general case can be found in Classical Electrodynamics book by J. Jackson.

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\frac{d}{dt}\left[\frac{\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})}{1-\boldsymbol{\beta}\mathbf{n}}\right]e^{i\omega\left(t-\frac{\mathbf{nr}}{c}\right)}dt\right|^2$$

where **r**(t) is the electron trajectory in lab system. Formulas for numerical calculation

$$\frac{d^2 I_\sigma}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\frac{d}{dt}\left[\frac{-\beta_x(t)\sin\varphi+\beta_y(t)\cos\varphi}{1-\beta_x(t)\sin\theta\cos\varphi-\beta_y(t)\sin\theta\sin\varphi-\beta_z(t)\cos\theta}\right]e^{i\omega\left(t-\frac{\mathbf{nr}}{c}\right)}dt\right|^2$$

$$\frac{d^2 I_\pi}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty}\frac{d}{dt}\left[\frac{\beta_x(t)\cos\theta\cos\varphi+\beta_y(t)\cos\theta\sin\varphi-\beta_z(t)\sin\theta}{1-\beta_x(t)\sin\theta\cos\varphi-\beta_y(t)\sin\theta\sin\varphi-\beta_z(t)\cos\theta}\right]e^{i\omega\left(t-\frac{\mathbf{nr}}{c}\right)}dt\right|^2$$

After numerical integration over angles the resulting spectrum

$$\frac{dI}{dy(\gamma^2 r_e^2 B_0^2\sigma)} = F\left(\frac{\omega}{\omega_c}, K\right), \quad y=\omega/\omega_c,$$

or

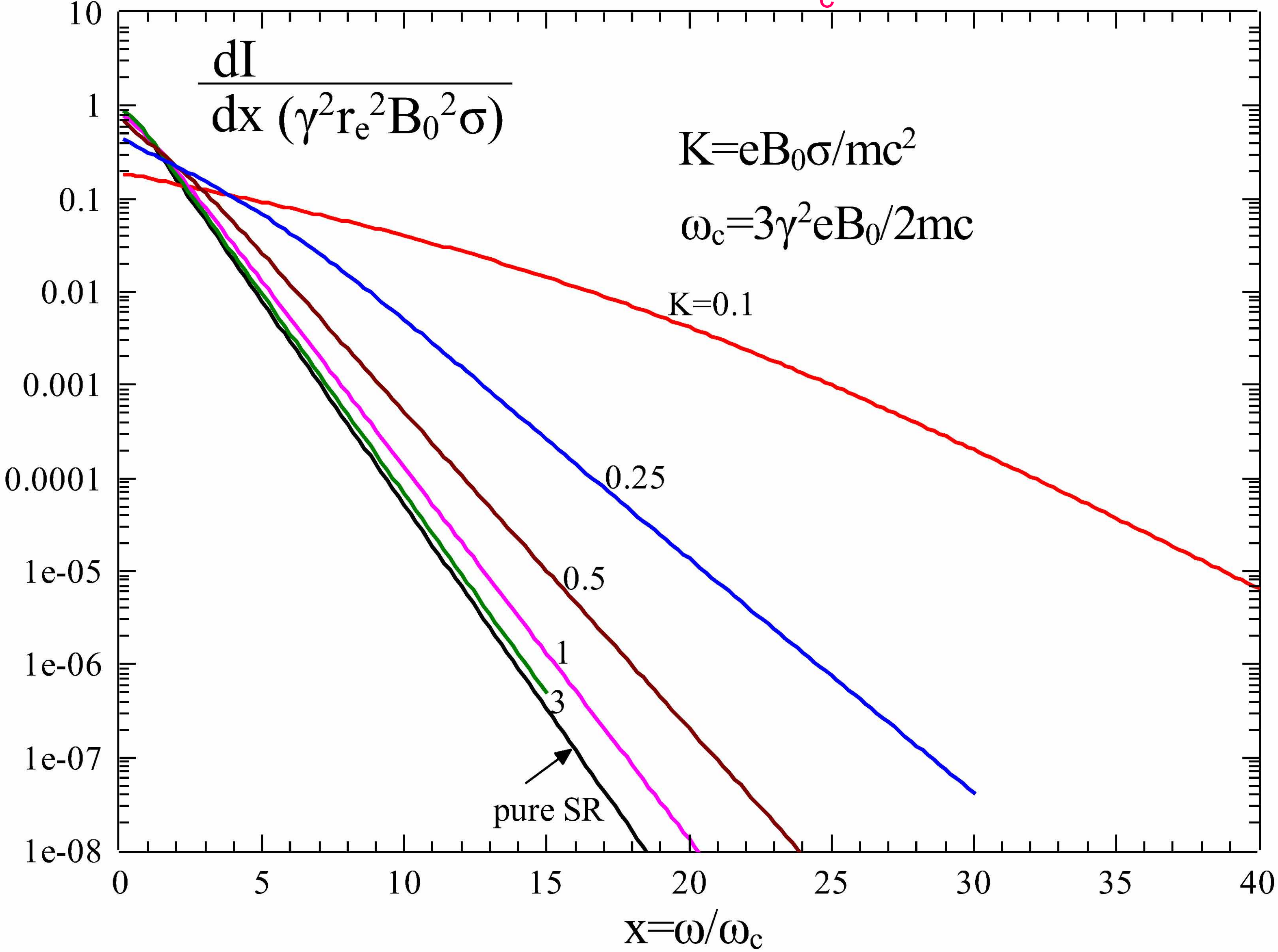
$$\frac{dI}{dy(\gamma^2 r_e^2 B_0^2\sigma)} = \frac{4}{3K}F\left(\frac{4y}{3K}, K\right), \quad y=\omega/\omega_{sh}.$$

Calculated spectra see below

Spectra for the field $B=B_0\exp(-z^2/2\sigma^2)$

(similar shape has the vertical beam field)

in terms of ω_c

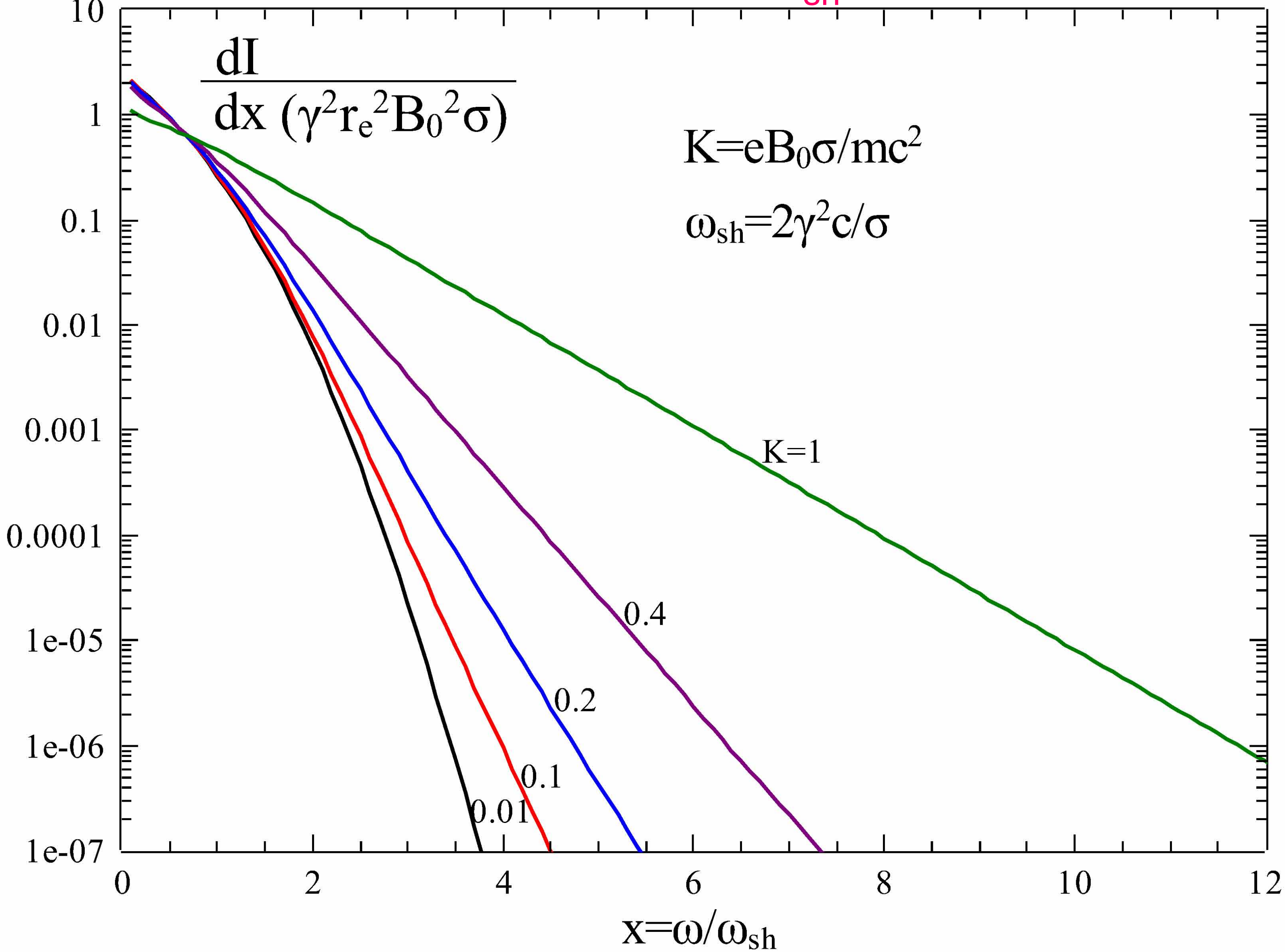


Curves tends to the SR spectrum for large K. The integral of spectra is the same for all curves.

Spectra for the field $B=B_0\exp(-z^2/2\sigma^2)$

(similar shape has the vertical beam field)

in terms of ω_{sh}



Curves tend to the "short magnet" spectrum for small K

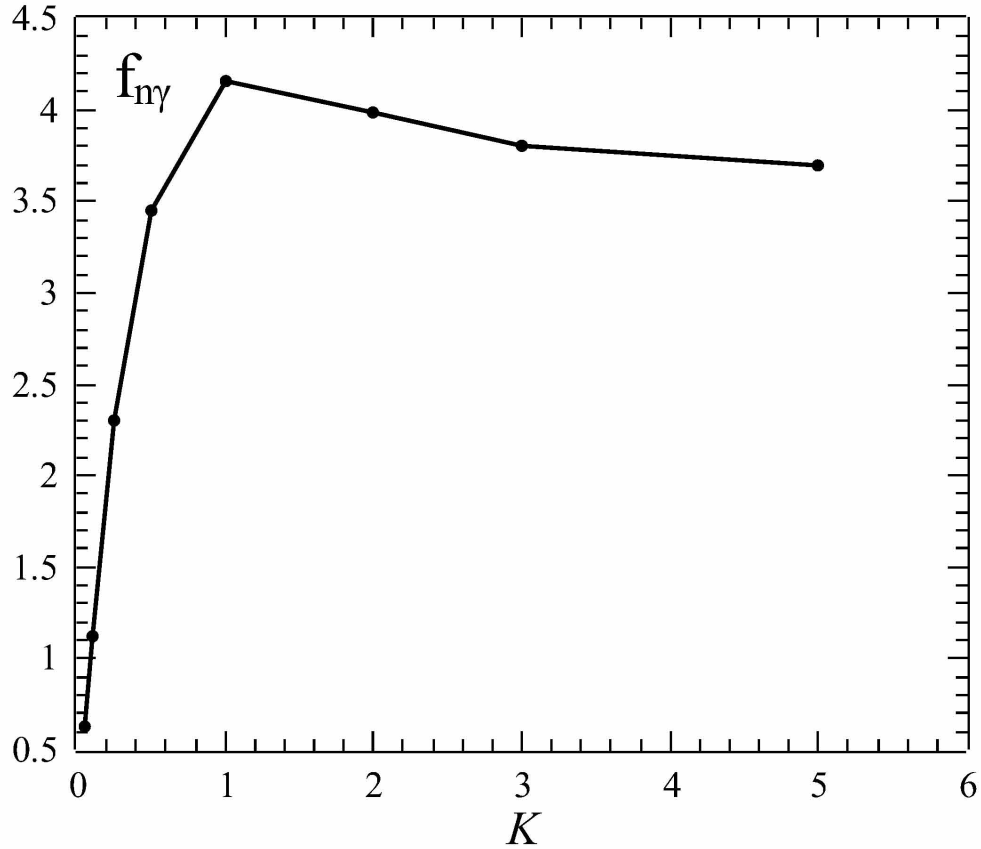
The number of emitted photons

For SR $N_\gamma = \dfrac{5\pi\alpha\gamma\theta}{\sqrt{3}}$ where $\alpha = e^2/\hbar c$, θ - total bending angle.

For the Gaussian field $\theta = \dfrac{eB_0\sqrt{2\pi}\sigma}{\gamma mc^2} \Rightarrow N_\gamma = 5\sqrt{\dfrac{\pi}{6}}\alpha K \approx 3.61\alpha K$

In general case $\mathrm{N}_\gamma = \int_0^\infty \frac{2}{3}\alpha K \frac{dy}{y} F(y,K) = \alpha K f_{n\gamma}(K)$

(here $F(y,K)$ – spectra obtained above)

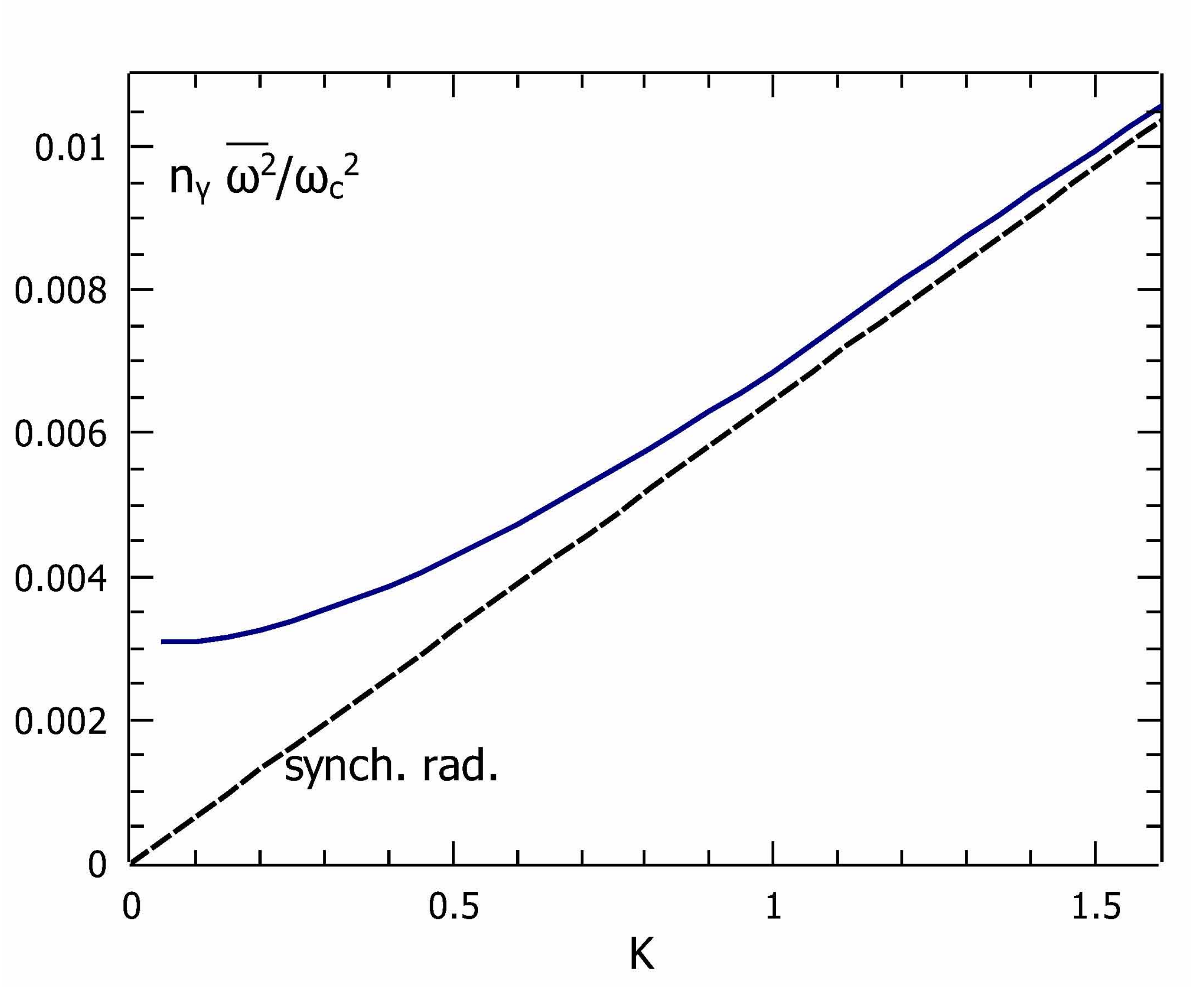


The number of photons decreases additionally with decreasing K because photons become harder.

(some bump may be due to accuracy of the calculation)

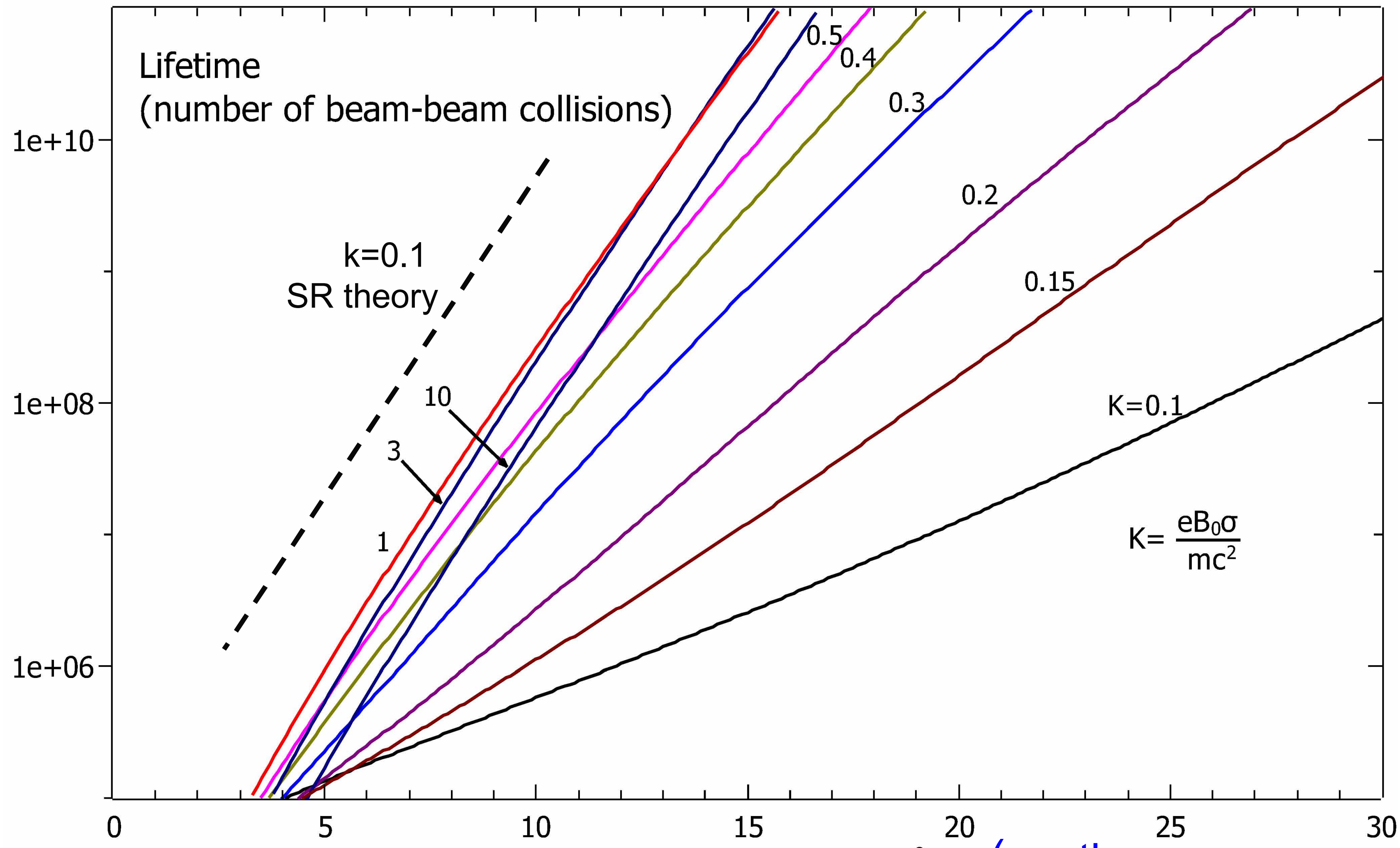
Contribution to the beam energy spread

Beamstrahlung leads to the increase of the energy spread (and following bunch lengthening). This contribution is proportional to $n_\gamma \left\langle \omega^2 \right\rangle$
One can see that for K<1 it is notably larger than give SR formulas



The beam lifetime with account of only Gaussian vertical field B_y.

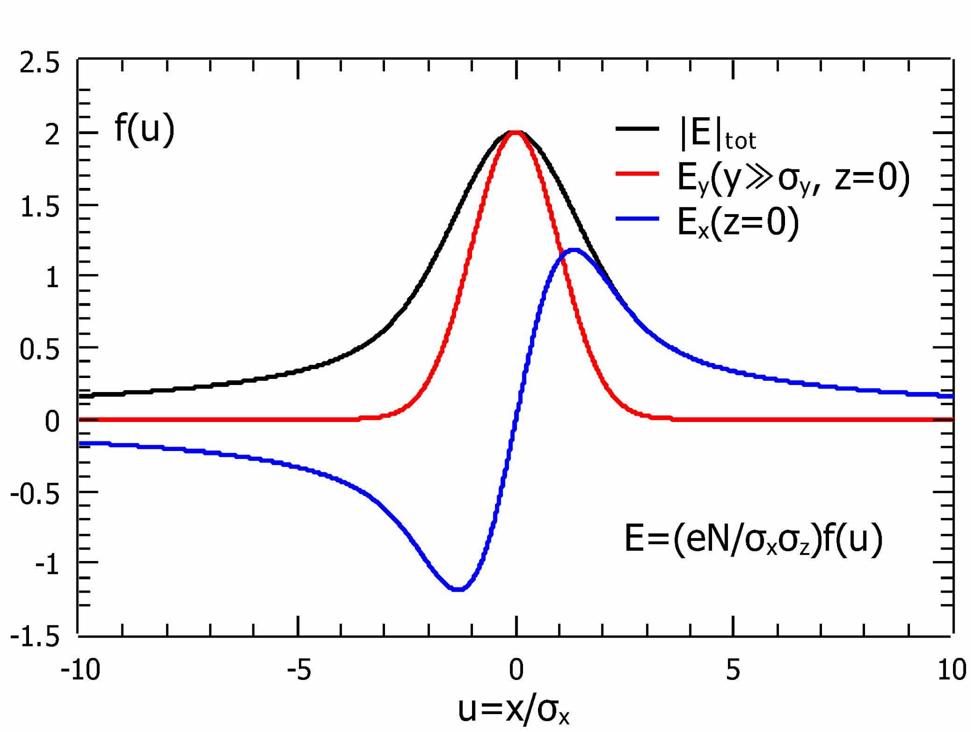
(variation of B_0 on y and z is taken into account)



$\eta E_0/\omega_c = \eta\alpha\sigma_x\sigma_z/N\gamma r_e^2$ (η – the energy acceptance)

One can see that beams with K<1 need larger energy acceptance than predicts SR theory, At large K curves shift to the right, that is because for the fixed B the spectrum is the same (SR) but the number of emitted photons is proportional to σ, therefore the lifetime is proportional to 1/K.

Beam fields B_y and B_x



One can consider that for flat beams B_x value depends only on x and z coordinate. The vertical field B_y depends both on x, y and z.

It is of interest that the energy loss in E_x field is larger than in E_y field by a factor of 2.65 (after averaging on y). However, the max. value has B_y, therefore it determines the beam lifetime for K>1.

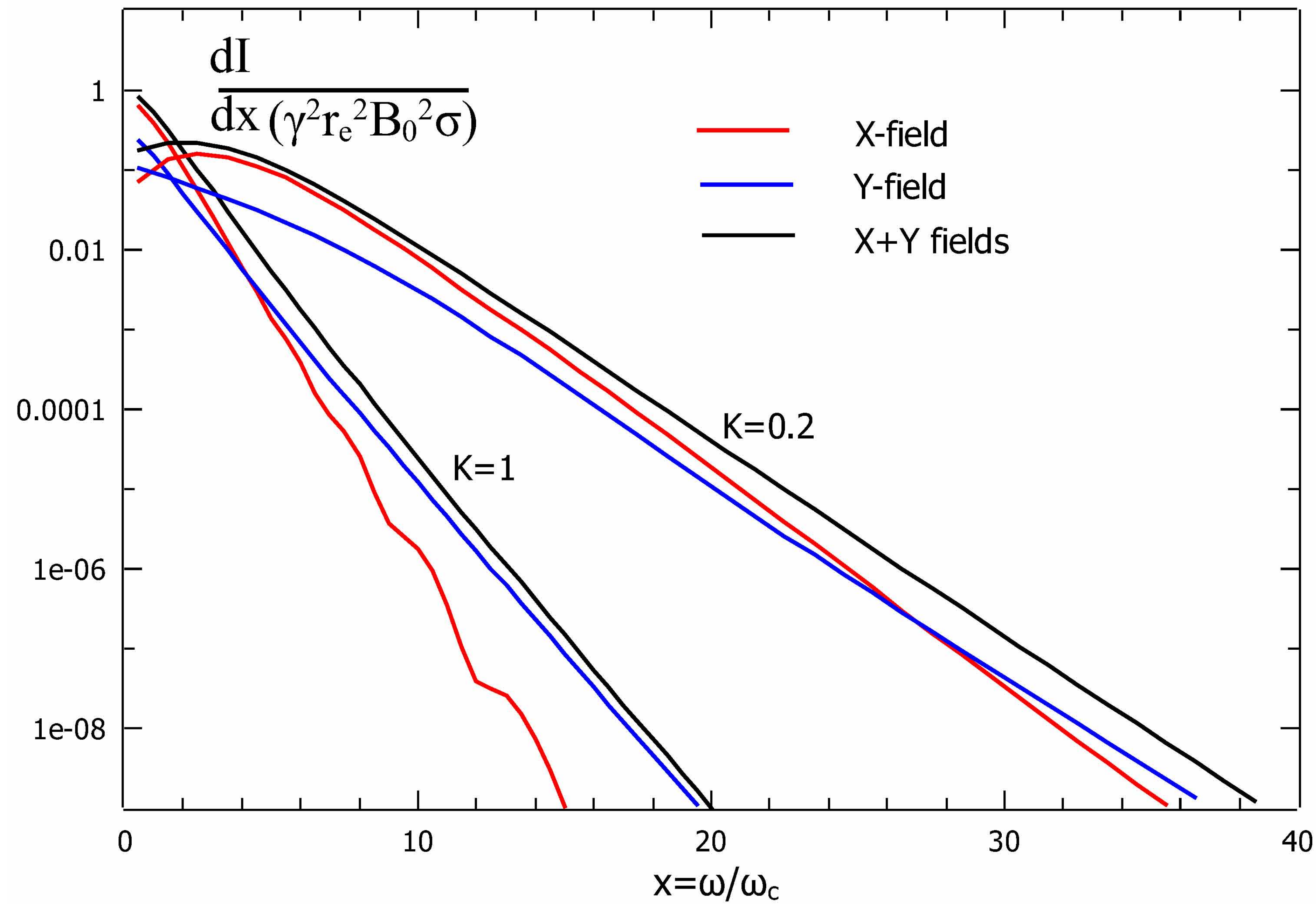
The maximum beam field at z=0, x=0 is $B_{max} \equiv B_0 = \dfrac{2eN}{\sigma_x \sigma_z} \quad \Rightarrow \quad K_0 = \dfrac{eB_0 \sigma_x}{\theta mc^2} = \dfrac{2r_e N}{\theta \sigma_z}$

Note, the decrease of σ_x (and $\sigma=\sigma_x/\theta$) by stronger focusing does not change K, because K is proportional to $B\sigma_x$=const. K depends only on $N/(\theta\sigma_z)$.

The dominant contribution to radiation gives the horizontal field which is about 1.7 times smaller and have similar effective σ. Also $\langle B_y^2 \rangle = B^2(y=0)/3$. Averaging of B^2 over z gives a $\langle B^2 \rangle = B^2(z=0)/\sqrt{3}$, therefore

$$K_{eff} = \sqrt{\langle K^2 \rangle} \approx K_0 / 1.7 / 3^{1/4} \approx 0.45 K_0$$

Spectra with account both B_x and B_y



In numerical calculations the B_x field was suppressed by factor $\exp(-x^2/8\sigma_x^2)$ in order to make it zero at $x=10\sigma_x$ (otherwise high harmonics appear dry to a sharp field edge).

The parameter K for head-on e+e- collliders

	$N/10^{10}$	σ_x,μm	K_{max}	K_{eff}
BEPC	20	890	0.6	0.27
BEPC-2	3.8	360	0.25	0.11
CESR-c	4.7	340	0.4	0.18
LEP	45	300	4.2	1.9
KEKB	4.7-6.4	120	1.1	0.5
PEP-2	5-8	157	0.9	0.4
VEPP-2000	16	125	3.5	1.7
VEPP-4	15	1000	0.4	0.18

$$K_{max}=Nr_e/\sigma_x$$

The parameter K for crab-waist collliders

$K_{max}=2N/(\sigma_z\theta)$

	$N/10^{10}$	E, GeV	σ_x,μm	σ_z,mm	θ,mrad	σ,mm	K_{max}	K_{eff}
FCC(Z)	17	45.6	6.3	12	30	0.2	2.6	1.17
FCC(H)	18	120	13	5.3	30	0.4	6.3	2.8
FCC(tt)	23	182	38	2.54	30	1.3	17	7.6
S-KEKB	6	7	11	5	83	0.13	0.88	0.4
	9.4	4	10	6	83	0.12	1.05	0.47
DAΦNE	3.2-2.1	0.5	260	20	50	1	0.15	0.07
S-c-тau	7	2	18	10	60	0.3	0.65	0.3

We see that the parameter K ~0.1-7.5 at storage rings, in average quite similar for head-on and crab-waist collision. For comparison at linear colliders ILC(500) and CLIC(500) K≈100.

What determine K value at ring colliders?

It is determined by the beam-beam parameter ξ_y (see page 4):

$$K \approx \frac{2\pi\gamma\sigma_y\xi_y}{\sigma_z} \quad \text{for head-on collisions}$$

$$K \approx \frac{2\pi\gamma\sigma_y\xi_y}{\beta_y} \quad \text{for crab-waist collisions}$$

In C-W scheme β_y is smaller than σ_z in head-on collisions, therefore in C-W case K should be larger, but it is compensated partially by smaller σ_y.

Due to presence of γ-factor one can expect smaller K for low energy colliders, that is confirmed by numbers in tables.

Low energy part of spectra, angles, formation length

For synchrotron radiation (K>>1) the formation length $l_{f,SR} \sim (\omega_c/\omega)^{1/3}\rho/\gamma$ and emission angles $\theta \sim (\omega_c/\omega)^{1/3}/\gamma$. It is not so for K<<1. The Gaussian beam field has a Fourier spectrum $B^2(\omega_0) \sim \exp(-\omega_0^2/\omega_{0,sh}^2)$, $\omega_{0,sh} = c/\sigma$. After Compton scattering on these virtual photons

$$\omega = \frac{2\gamma^2\omega_0}{1+\theta^2\gamma^2}$$

The formation length of radiation follows from $\omega(t - \mathbf{nr}/c) \sim 1$, which gives ($\cos\theta \sim 1-\theta^2/2$, $\beta \sim 1-1/\gamma^2$)

$$l_f = ct = \frac{c}{\omega(1-\beta\cos\theta)} \sim \frac{2\gamma^2 c}{\omega(1+\theta^2\gamma^2)} \sim \frac{c}{\omega_0}$$

So, the formation length is approximately equal the wavelength of Fourier components. The formation length for hard photons (for $\omega_{0,sh} = c/\sigma$) $l_f \sim \sigma$, while for lower ω_0 the formation length will be longer than the interaction length, i.e. all produced low energy photons with $\omega < \omega_{sh} = 2\gamma^2 c/\sigma$ will not create secondary backgrounds in collisions with particles from opposing beam.

Conclusions

- The beamstrahlung at e+e- circular colliders is characterized by intermediate values of parameters K~0.1-5, it is shown how to make calculations in this case.
- For FCC-ee(Z) one can expect small, about 5-10% increase of the energy spread compared to SR calculations .
- For FCC(H and tt) the beamstrahlung is very important and restricts the luminosity, but these cases can be rather well described by classical SR.
- For low energy colliders Super KEKB, c-tau and DAΦNE the beamstrahlung spectrum differ from SR substantially, but at these colliders the dominant contribution gives radiation in arcs, therefore beamstrahlung is not important for beam dynamics.
- Beamstrahlung photons participate in creation of secondary backgrounds (especially at linear colliders). At circular e+e- colliders such backgrounds will be suppressed due to long formation length of low energy photons (main backgrounds will be generated in collisions of virtual Weizsacker-Williams's photons).