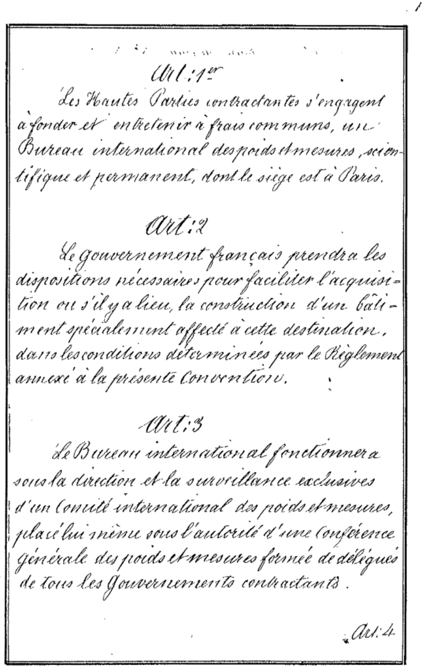
The Kilogram Redefined:
SI Units and Fundamental Constants

Alan Steele, NRC Canada

IPAC'18 Vancouver May 2, 2018

The Metre Convention: May 20, 1875



Art: 1er

Les Hautes Parties contractantes s'engagent à fonder et entretenir à frais communs, un Bureau international des poids et mesures, scientifique et permanent, dont le siège est à Paris.

Art: 2

Le gouvernement français prendra les dispositions nécessaires pour faciliter l'acquisition ou s'il y a lieu, la construction d'un bâtiment spécialement affecté à cette destination, dans les conditions déterminées par le Règlement annexé à la présente Convention.

Art: 3

Le Bureau international fonctionnera sous la direction et la surveillance exclusives d'un Comité international des poids et mesures, placé lui même sous l'autorité d'une Conférence générale des poids et mesures formée de délégués de tous les Gouvernements contractants.

Art: 4

- International Treaty that creates the BIPM and launches the idea of a *unified system of units* for measurement across the globe
- As of March 23, 2018, there are *59 Member States* and 42 Associate States and Economies

The Metre Convention: May 20, 1875

- The International Bureau of Weights and Measures is located in *Sèvres*, near Paris, France
- The Pavillon de Brateuil was already more than *200 years old* at the time of the signing

The Metre Convention: May 20, 1875

- The Pavillon de Brateuil was already more than *200 years old* at the time of the signing
- It had *suffered damage* during the Prussian War, but funds were set aside to refurbish it for use as the BIPM headquarters

The International Prototype of the kilogram

- The French *"kilogram of the Archives"* was a Pt cylinder that realized the litre-of-water definition chosen in *1795*

- The new PtIr *IPK* was produced in *1879*, compared to the kg des Archives in *1883*, and ratified as the defining artefact for the metric system at the 1st CGPM in *1889*

IPK: Restricted Access and Limited Use

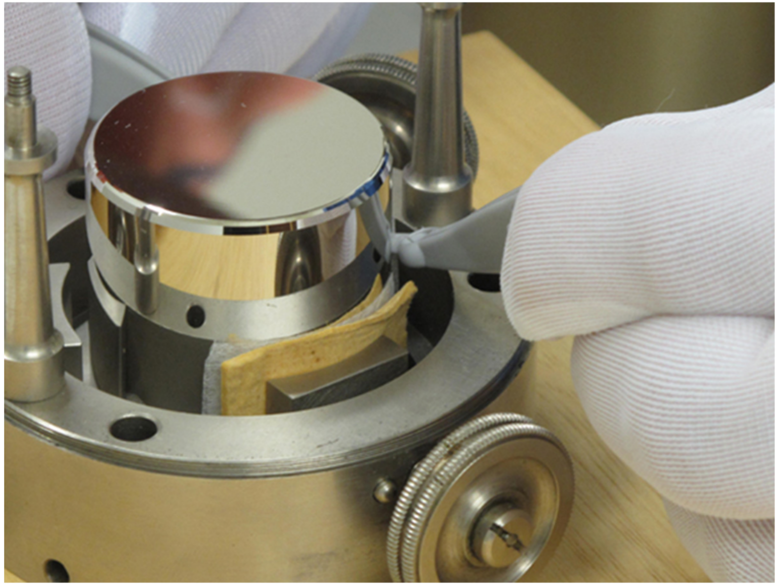
ART. 18.

Le directeur du Bureau n'aura accès au lieu de dépôt des prototypes internationaux du mètre et du kilogramme qu'en vertu d'une résolution du Comité et en présence de deux de ses membres.

Le lieu de dépôt des prototypes ne pourra s'ouvrir qu'au moyen de trois clefs, dont une sera en la possession du directeur des Archives de France, la seconde dans celle du président du Comité, et la troisième dans celle du directeur du Bureau.

Les étalons de la catégorie des prototypes nationaux serviront seuls aux travaux ordinaires de comparaisons du Bureau.

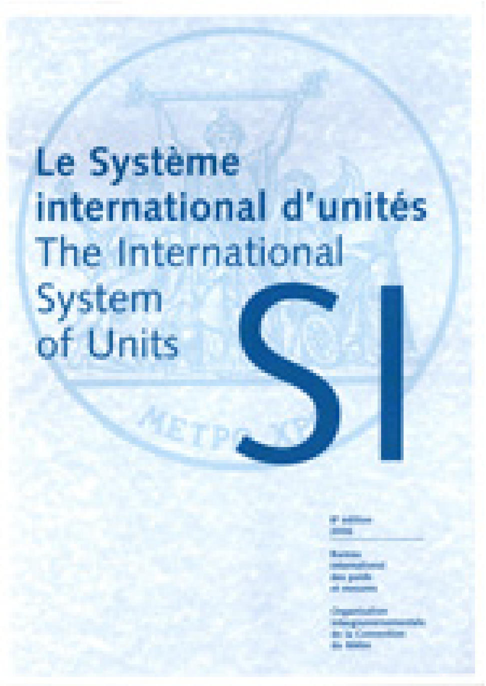
Mass and the International System of Units



- *The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.*

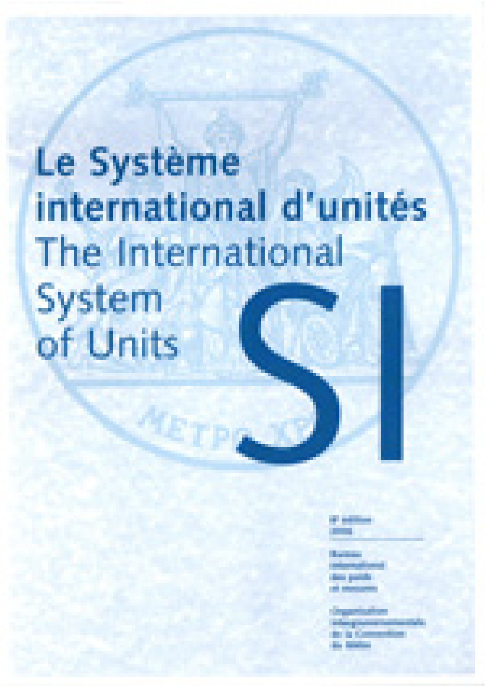
- Since 1883, there have been *only four* "periodic verification" measurements using the IPK

Base Units in the SI



- The *metre* is the *length* of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second.
- The *kilogram* is the unit of *mass*; it is equal to the mass of the international prototype of the kilogram.
- The *second* is the *duration* of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom.

Base Units in the SI



- The *ampere* is that constant *current* which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.
- The *kelvin*, unit of *thermodynamic temperature*, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.

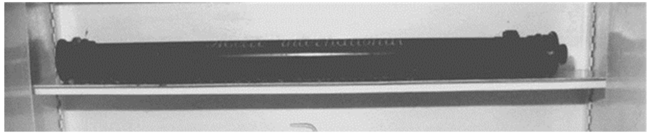
Base Units in the SI

- The *mole* is the *amount of substance* of a system which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon 12.
- The *candela* is the *luminous intensity*, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.

Defining the SI via Fundamental Constants

- The proposed structure of the International System of Units is quite different from the one we have now
- We will move away from defined Base Units (s, m, kg, A, K, mol, cd)
- We will shift to defined constants ($\Delta\nu_{Cs}$, c, h, e, k_B, N_A, K_{cd})
- We've done (some of) this before…

The International Prototype of the metre



- The French prototype *"metre of the Archives"* was a Pt bar that realized the anticipated meridian definition chosen in *1799*

- The new PtIr *International Prototype of the metre* was ratified as the defining artefact for the metric system at the 1st CGPM in *1889*

From Mechanical to Optical Length

- The optical definition of the metre was ratified by the 11th CGPM in *1960*: 1650763.73 wavelengths of the $2p^{10}$ and $5d^{5}$ transition of krypton-86

- At the 17th CGPM in *1983*, the numerical value of the speed of light was defined to be 299 792 458 metres per second *exactly*

The SI: Redefinition at CGPM in 2018

In essence, the change involves *exactly fixing the values* of *7 constants* that set the scale of the SI units :

- *c* – speed of light
- $\Delta\nu$ – ground state hyperfine splitting frequency of ^{133}Cs
- *h* – Planck constant — $E = h\upsilon$
- *e* – the elementary charge — *charge on a proton*
- *k* – Boltzmann constant — $E = kT$
- N_A – Avogadro constant — *number of entities in a mole*
- K_{cd} – luminous efficacy — *of* 540×10^{12} Hz *radiation*

The SI: Redefinition at CGPM in 2018

In essence, the change involves *exactly fixing the values* of *7 constants* that set the scale of the SI units :

c = 299 792 458 m/s

$\Delta\nu$ = 9 192 631 770 Hz

h = 6.626 070 15 × 10^{-34} J s

e = 1.602 176 634 × 10^{-19} C

k = 1.380 649 × 10^{-23} J/K

N_A = 6.022 140 76 × 10^{23} mol^{-1}

K_{cd} = 683 lm/W

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Short Communication

The CODATA 2017 values of h, e, k, and N_A for the revision of the SI

D B Newell[1], F Cabiati, J Fischer, K Fujii, S G Karshenboim, H S Margolis, E de Mirandés, P J Mohr, F Nez, K Pachucki, T J Quinn, B N Taylor, M Wang, B M Wood and Z Zhang

Committee on Data for Science and Technology (CODATA) Task Group on Fundamental Constants

E-mail: dnewell@nist.gov

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Abstract
Sufficient progress towards redefining the International System of Units (SI) in terms of exact values of fundamental constants has been achieved. Exact values of the Planck constant h, elementary charge e, Boltzmann constant k, and Avogadro constant N_A from the CODATA 2017 Special Adjustment of the Fundamental Constants are presented here. These values are recommended to the 26th General Conference on Weights and Measures to form the foundation of the revised SI.

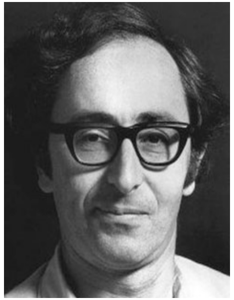
Keywords: international system of units, fundamental constants, SI redefinition

Why Haven't We Done This Already?

- Consistency and the uncertainty in *h* has been the limiting factor
- The mass community wants *15 ppb* or better …

1962
Josephson
voltage quantization

$$V = n\left(\frac{h}{2e}\right)f$$



1980
Von Klitzing
resistance quantization

$$R = n\left(\frac{h}{e^2}\right)$$

… but by 1990 the electrical community couldn't wait any longer!

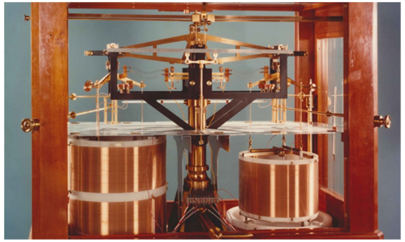
$$V = n\,\mathrm{K}_{\mathrm{J}90}f$$

$$R = n\,R_{K90}$$

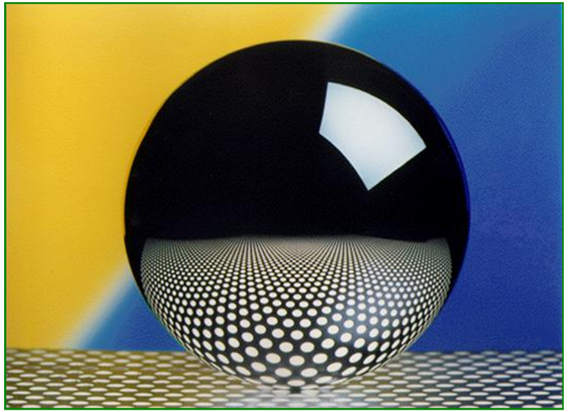
The kg and Measurement of h and N_A

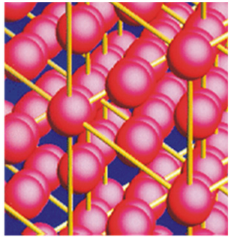
- What's involved in measuring the Planck constant, h, or the Avogadro number, N_A?

$$N_A h = \frac{M(e) c \alpha^2}{2 R_\infty}$$



Consider the Avogadro Constant

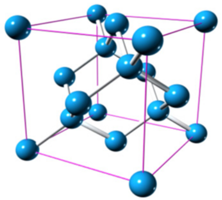




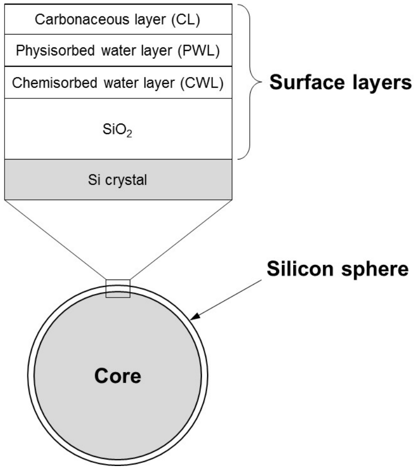
- To measure the number of entities in a mole, N_A, you could count the atoms in a known mass of a given crystal, say Si
- Silicon sphere measurements:
  - Volume
  - Mass
  - Atomic spacing
  - Molar mass

Silicon Sphere Measurement Approach

$$N_A = 8M/(\rho a^3)$$



$$m_{\text{sphere}} = m_{\text{core}} + m_{\text{SL}}$$



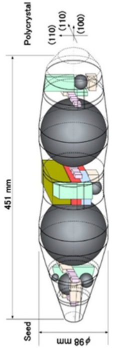
$$m_{\text{core}} = \frac{2hR_\infty}{c\alpha^2} \frac{\sum_i f_i A_r({}^i\text{Si})}{A_r(\text{e})} \frac{8V_{\text{core}}}{a^3} - m_{\text{deficit}}$$

- Electron rest mass: $\frac{2hR_\infty}{c\alpha^2}$
- Mass ratio: $\frac{\sum_i f_i A_r({}^i\text{Si})}{A_r(\text{e})}$ (Isotope ratio and atomic weights)
- Number of Si atoms: $\frac{8V_{\text{core}}}{a^3}$ (Volume and lattice spacing)
- Impurities, vacancies and surface layers: m_{deficit} (A lot of chemical and surface analysis)

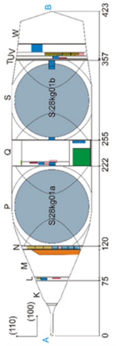
m_{core}: ‘weigh’ the sphere

Building a Perfect Sphere

IAC



PTB



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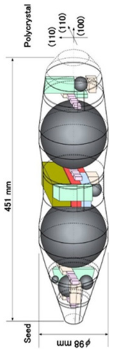

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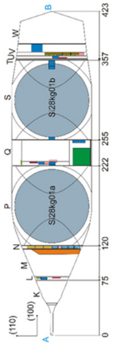

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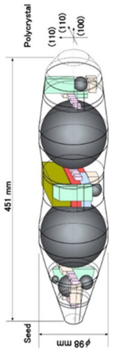

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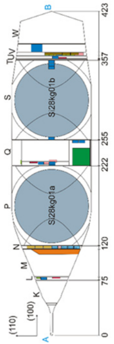

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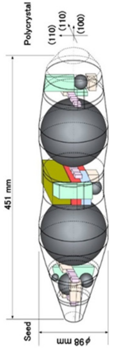

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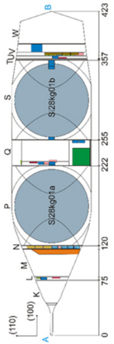

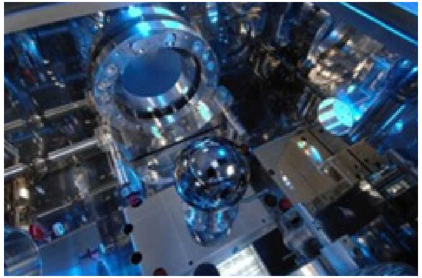

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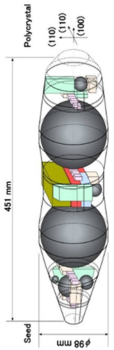

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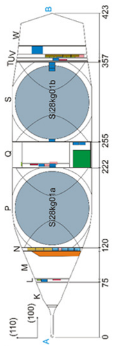
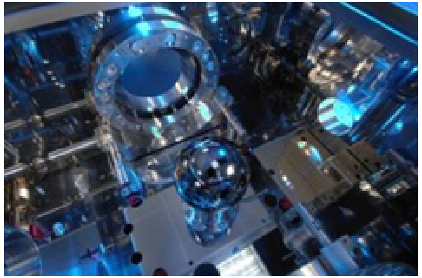
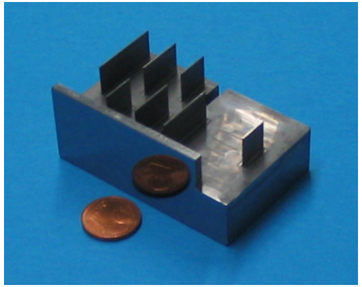
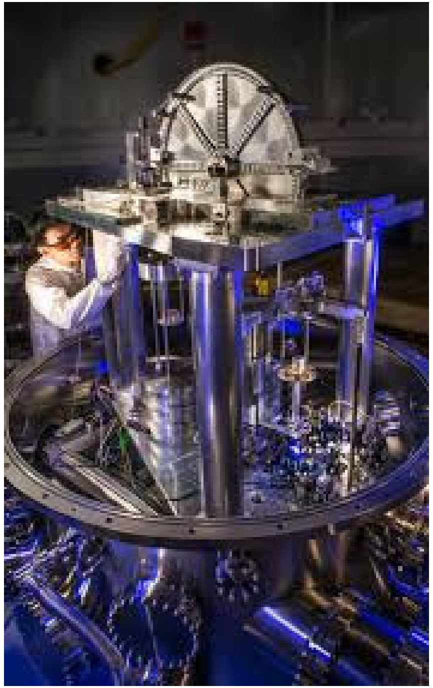






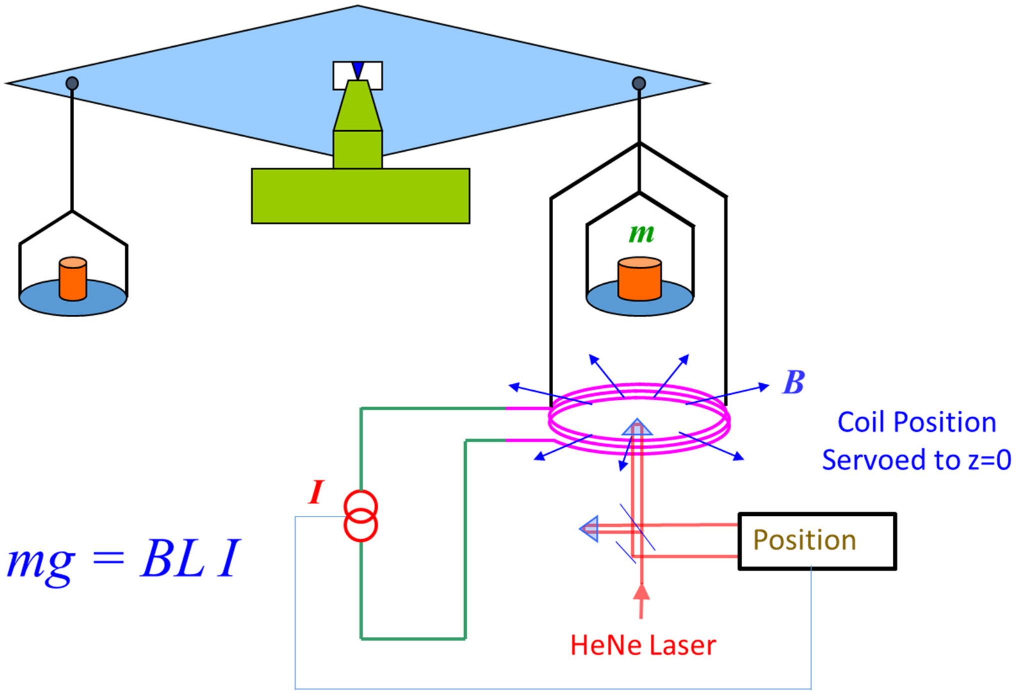
Or, Build a Kibble Balance…



1975
Kibble
his balance

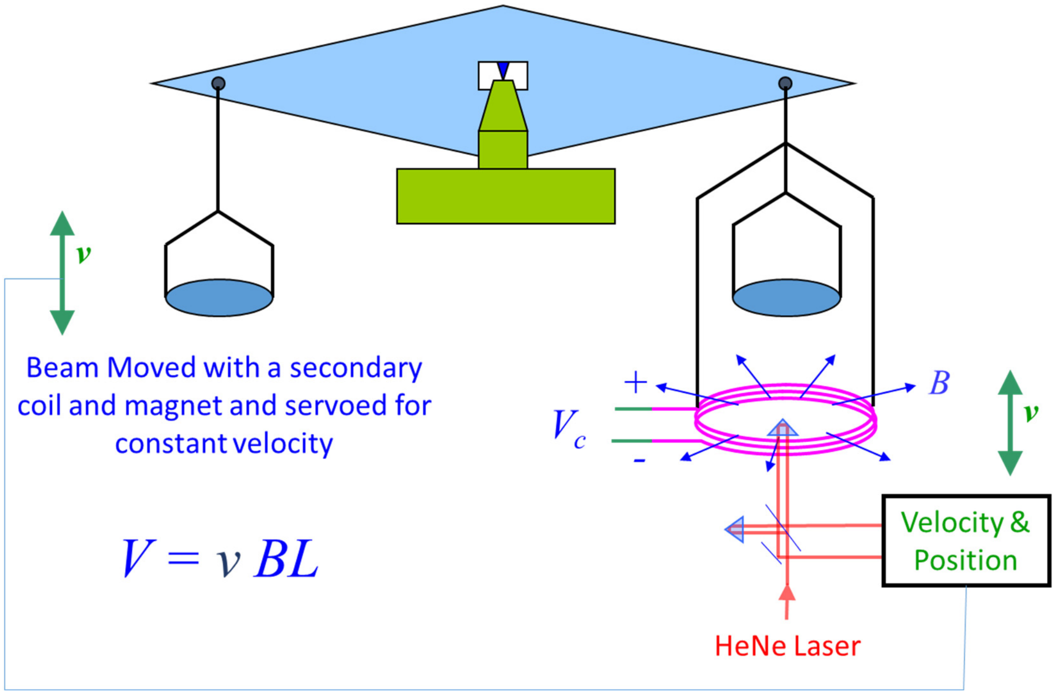
$$m\, g\, v = I\, V$$

Kibble Balance – weighing phase



$$F = mg = BL\,I$$

Kibble Balance – moving phase



Kibble Balance – combining both phases

$$mg = BLI$$ weighing

$$V_c = vBL$$ moving

$$m = \frac{1}{g}\frac{V_c}{v}\frac{V_R}{R}$$

Kibble Balance – combining both phases

$$mg = BLI$$ weighing

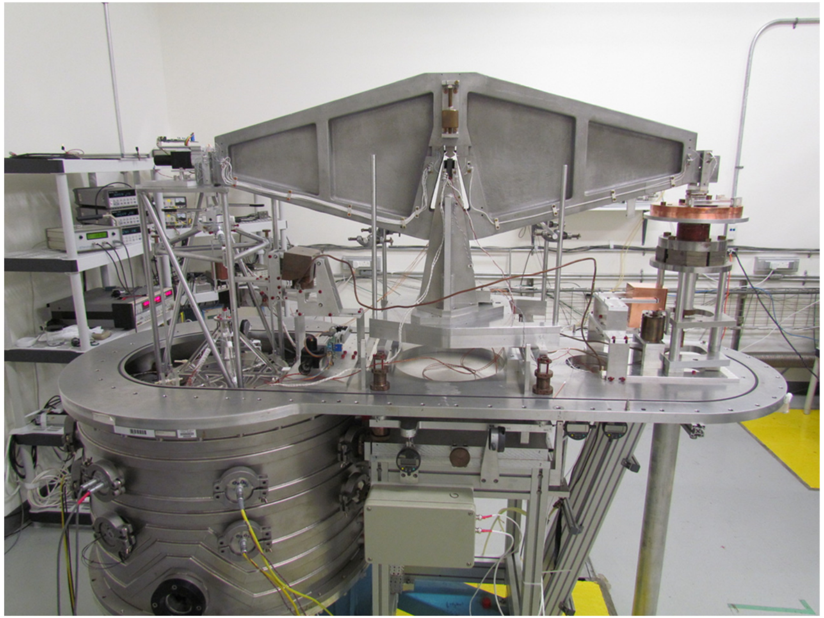
$$V_c = vBL$$ moving

$$m = \frac{1}{g}\frac{V_c}{v}\frac{V_R}{R}$$

$$\frac{h}{2e} \times \frac{h}{2e} \times \frac{e^2}{h} \longrightarrow \frac{h}{4}$$

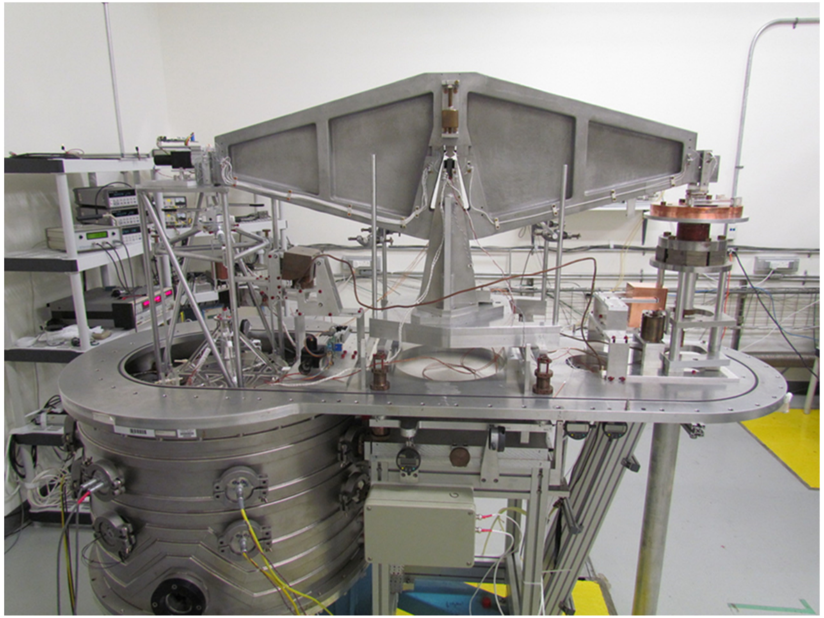
Measurement of the Planck Constant

- To achieve an overall uncertainty of 15 ppb, we need measurement capabilities on six primary quantities, all better than 10 ppb
  - Gravity
  - Voltage
  - Resistance
  - Length
  - Time
  - Mass
- Plus mechanical alignment, electrical timing, and other issues



NRC: Incremental Improvements for Best *u*(*h*)

- Digital Servo
- One Interferometer but Two Isolated Detection Systems
- Constant Velocity Drive with minimal noise
- Nanovolt Amplifier
- Tracking PJVS
- In Situ Resistor & Laser Calibrations
- Thermal Control
- Weighing : Exercising Pivots before & Demagnetization after
- Isolated Pads for balance and gravity measurements
- Tuned Acceleration Profile



All The Data That's Fit To Print...

Table 1. Key data for the determination of h, e, k, and N_A in the CODATA 2017 Special Adjustment. See Mohr *et al* (2017) for a complete list of input data.

Source	Identification[a]	Quantity[b]	Value	Rel. stand. uncert u_r
Schlamminger *et al* (2015)	NIST-15	h	$6.626\,069\,36(38) \times 10^{-34}$ J s	5.7×10^{-8}
Wood *et al* (2017)	NRC-17	h	$6.626\,070\,133(60) \times 10^{-34}$ J s	9.1×10^{-9}
Haddad *et al* (2017)	NIST-17	h	$6.626\,069\,934(88) \times 10^{-34}$ J s	1.3×10^{-8}
Thomas *et al* (2017)	LNE-17	h	$6.626\,070\,40(38) \times 10^{-34}$ J s	5.7×10^{-8}
Azuma *et al* (2015)	IAC-11	N_A	$6.022\,140\,95(18) \times 10^{23}$ mol^{-1}	3.0×10^{-8}
Azuma *et al* (2015)	IAC-15	N_A	$6.022\,140\,70(12) \times 10^{23}$ mol^{-1}	2.0×10^{-8}
Bartl *et al* (2017)	IAC-17	N_A	$6.022\,140\,526(70) \times 10^{23}$ mol^{-1}	1.2×10^{-8}
Kuramoto *et al* (2017)	NMIJ-17	N_A	$6.022\,140\,78(15) \times 10^{23}$ mol^{-1}	2.4×10^{-8}

… And the Resulting Value for *h*

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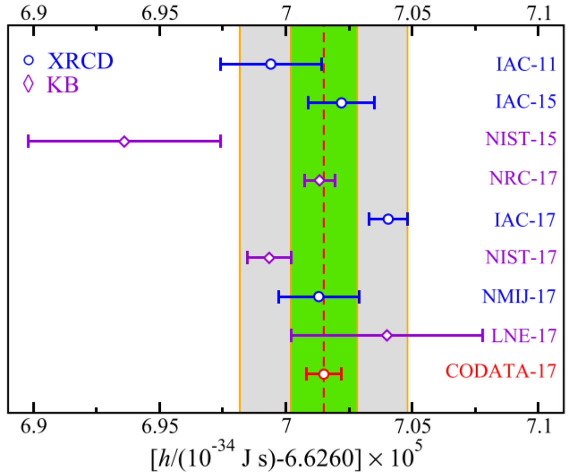


Figure 1. Values of the Planck constant h inferred from the input data in table 1 and the CODATA 2017 value in chronological order from top to bottom. The inner green band is ± 20 parts in 10^9 and the outer grey band is ± 50 parts in 10^9. KB: Kibble balance; XRCD: x-ray-crystal-density.

Table 2. The CODATA 2017 adjusted values of h, e, k, and N_A.

Quantity	Value	Rel. stand. uncert u_r
h	$6.626\,070\,150(69) \times 10^{-34}$ J s	1.0×10^{-8}
e	$1.602\,176\,6341(83) \times 10^{-19}$ C	5.2×10^{-9}
k	$1.380\,649\,03(51) \times 10^{-23}$ J K^{-1}	3.7×10^{-7}
N_A	$6.022\,140\,758(62) \times 10^{23}$ mol^{-1}	1.0×10^{-8}

Table 3. The CODATA 2017 values of h, e, k, and N_A for the revision of the SI.

Quantity	Value
h	$6.626\,070\,15 \times 10^{-34}$ J s
e	$1.602\,176\,634 \times 10^{-19}$ C
k	$1.380\,649 \times 10^{-23}$ J K^{-1}
N_A	$6.022\,140\,76 \times 10^{23}$ mol^{-1}

26th CGPM : November 13 – 16, 2018

- Equipped with the necessary science to improve the International System of Units
- We will adopt the best SI value for *h*, *e*, and N_A to minimize disruption
- The last artefact, the IPK, will no longer define the unit of mass
- The unit of electric current will no longer be impossible to realize

Draft Resolution A

On the revision of the International System of Units (SI)

The General Conference on Weights and Measures (CGPM), at its 26th meeting,

considering

- the essential requirement for an International System of Units (SI) that is uniform and accessible world-wide for international trade, high-technology manufacturing, human health and safety, protection of the environment, global climate studies and the basic science that underpins all these,
- that the SI units must be stable in the long term, internally self-consistent and practically realizable being based on the present theoretical description of nature at the highest level,
- that a revision of the SI to meet these requirements was proposed in Resolution 1 adopted unanimously by the CGPM at its 24th meeting (2011) that laid out in detail a new way of defining the SI based on a set of seven defining constants, drawn from the fundamental constants of physics and other constants of nature, from which the definitions of the seven base units are deduced,
- that the conditions set by the CGPM at its 24th meeting (2011), confirmed at its 25th meeting (2014), before such a revised SI could be adopted have now been met,

decides that, effective from 20 May 2019, the International System of Units, the SI, is the system of units in which:

kg Redefined : Mass Scales Won't Change, BUT…

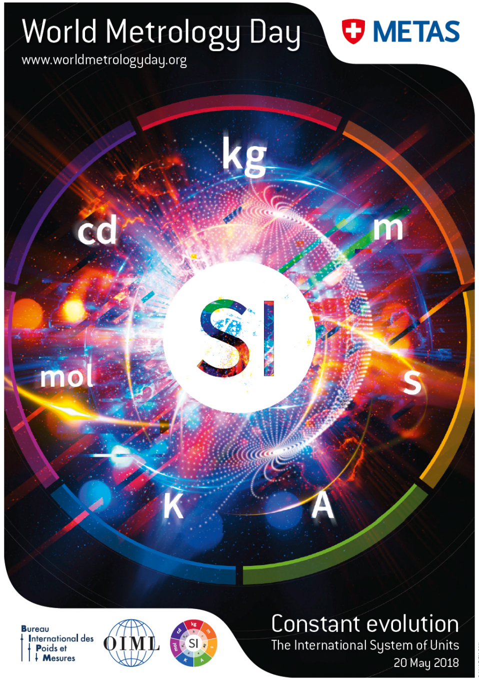
- Once *h* the value of the Planck constant is fixed, the International Prototype of the kilogram will no longer define the unit of mass

- Other techniques – specifically the *XRCD Si spheres* and the *Kibble (Watt) balances* – will become the way to realize the unit of mass from its definition (at any required mass value, not just at 1 kg)

The Consequences

- Redefinition: Josephson voltage standard will increase by 98 ppb
- Best voltage calibrations have uncertainty (k=2) of 1.5 ppb
  - This represents a 13 σ change at the highest level of precision
- Redefinition: QHR resistance standard will increase by 18 ppb
- Best resistance calibrations have uncertainty (k=2) of 12 ppb
  - This represents a 1.5 σ change at the highest level of precision
- Redefinition: IPK will not change its mass but gains uncertainty
  - The new "realization" uncertainty (transferred from h) is 10 ppb

World Metrology Day: May 20th



Thanks for listening!