Beam Quality Limitations of Plasma-Based Accelerators

Sources of energy spread and bunch length

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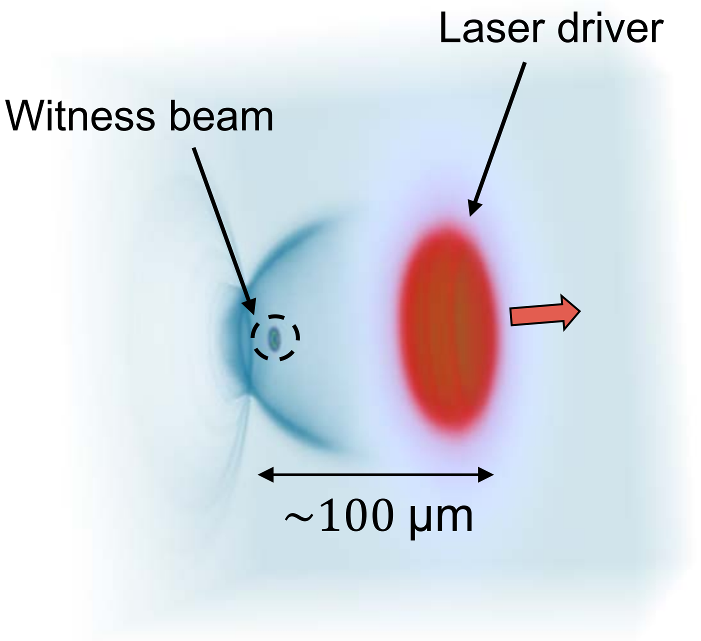
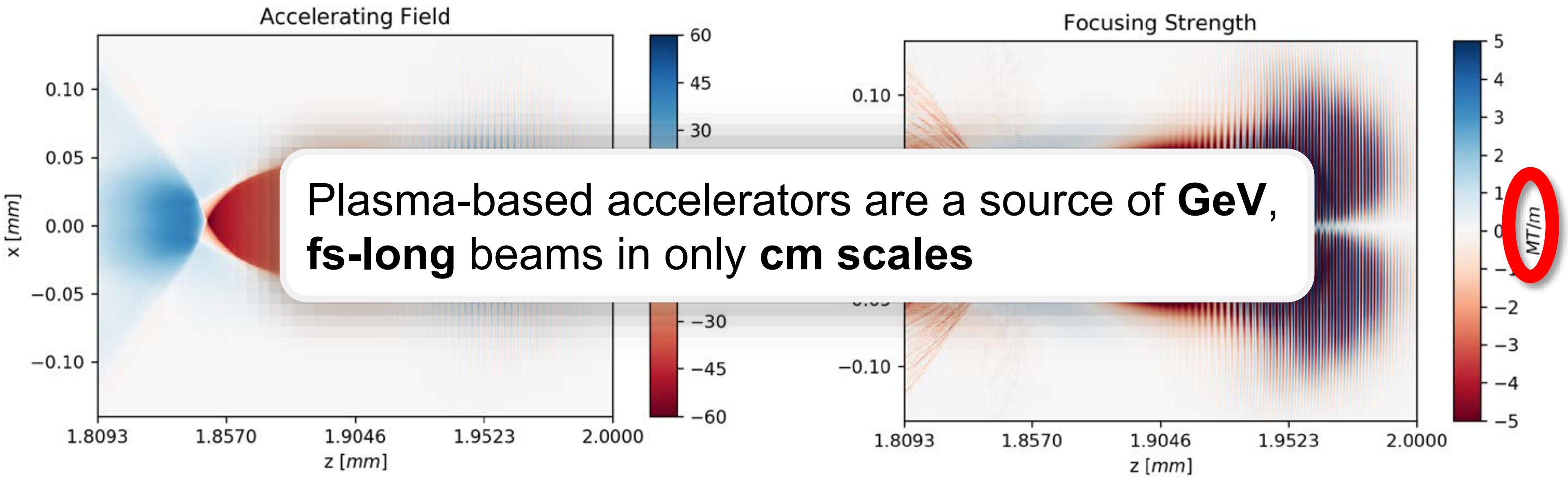
Introduction

Basics of Plasma Acceleration

- An intense **laser pulse** or **charged particle beam** is injected into a gaseous plasma.
- Extreme **accelerating** and **focusing** fields are generated behind the driver (laser or beam pulse), e.g. 100 **GV/m** acc. and 10 **MT/m** focusing.
- Witness can be externally injected or formed by trapping plasma electrons.





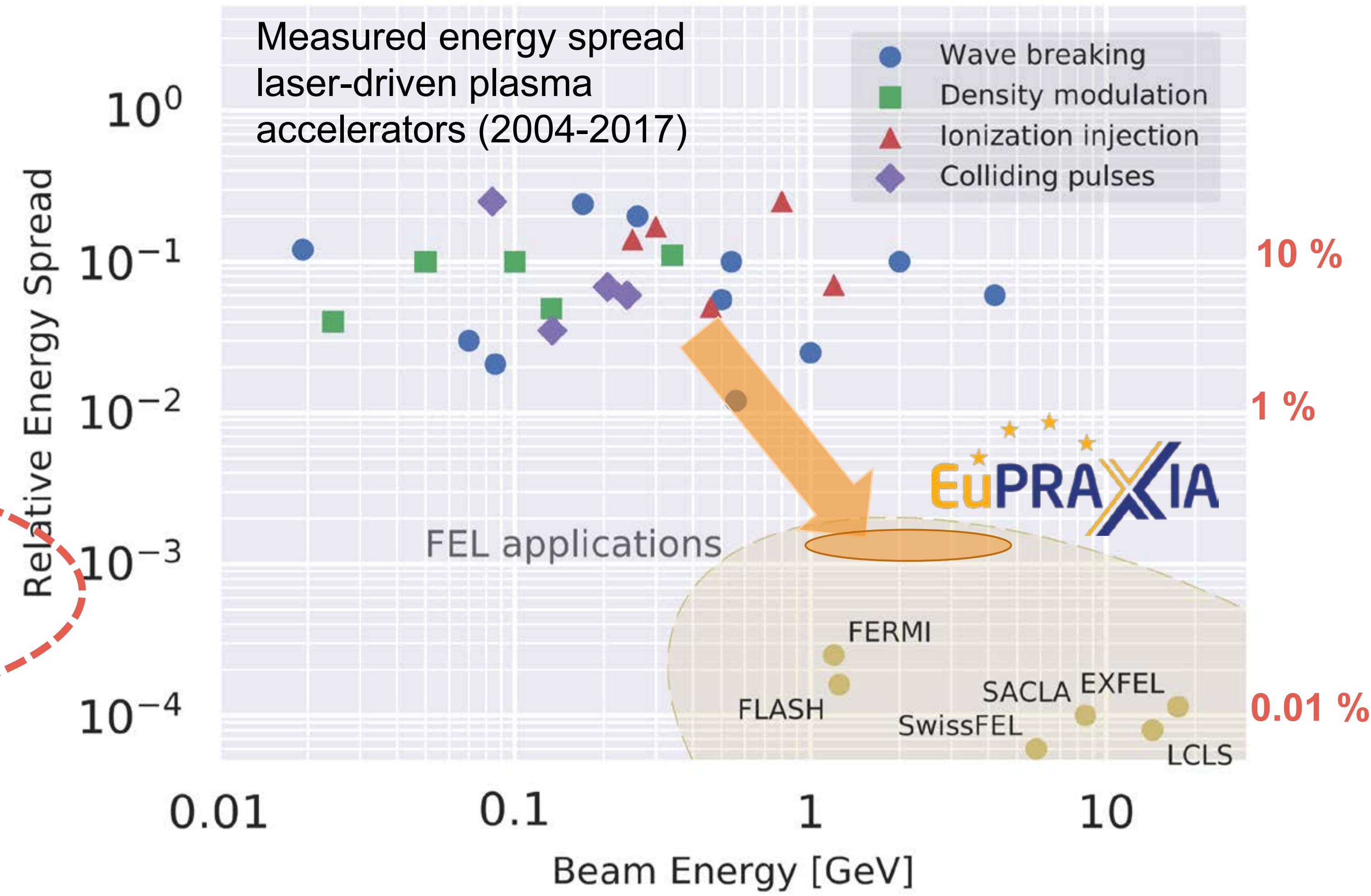
Plasma-based accelerators are a source of **GeV**, **fs-long** beams in only **cm scales**

Plasma Accelerators as compact and unique FELs?

Electron beam requirements

- Synchrotron light sources such as FELs would strongly benefit from **reduced size and cost**.
- FELs impose strict requirements on electron beam parameters:
  - **GeV** energies.
  - **Micron**-level emittance.
  - **Multi-kA peak** current.
  - **Femtosecond** long bunches.
  - Relative **energy spread** <10^{-3}.
- Could also offer unique ultra-fast X ray pulses:
  - Reaching stable **sub-femtosecond** bunches.

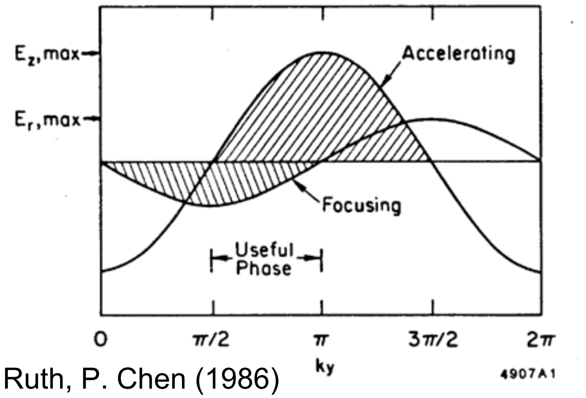
We will focus on these two parameters



Energy Spread in Plasma Accelerators: Typical Sources

Acceleration on slope

- In plasma accelerators: Accelerate off crest where fields are focusing.



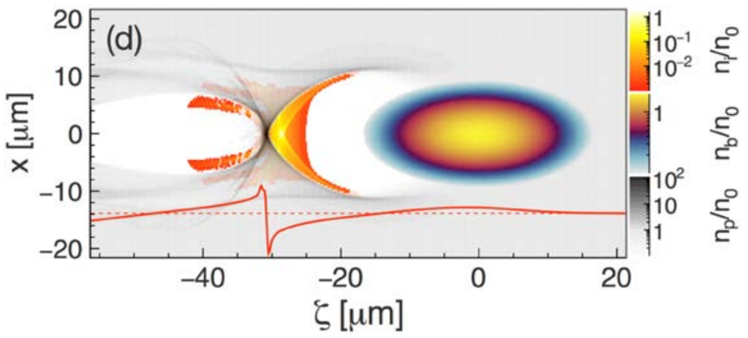
D. Ruth, P. Chen (1986)

How to mitigate:

- Beam-loading [v.d. Meer]
- Chirp compensation
  [G. Manahan, 2017, Nat. Comm.]
  [R. Brinkmann, 2017, PRL]

Energy Spread from Injection

- Potentially large for plasma injection schemes.



How to mitigate:

- Higher control over the injection process.

Synchrotron Radiation

Focusing fields

⇩

Transverse oscillations (betatron motion)

⇩

Photon emission

Radiated Power ∝ Oscillation Amplitude

How to mitigate:

- Reduce beam size or focusing strength (hollow plasma channels).
  [T. Chiou, 1995, Phys. Plasmas]

Community works for decades on reducing energy spread.

Are there additional significant sources?

Here we investigate additional sources of energy spread and bunch length

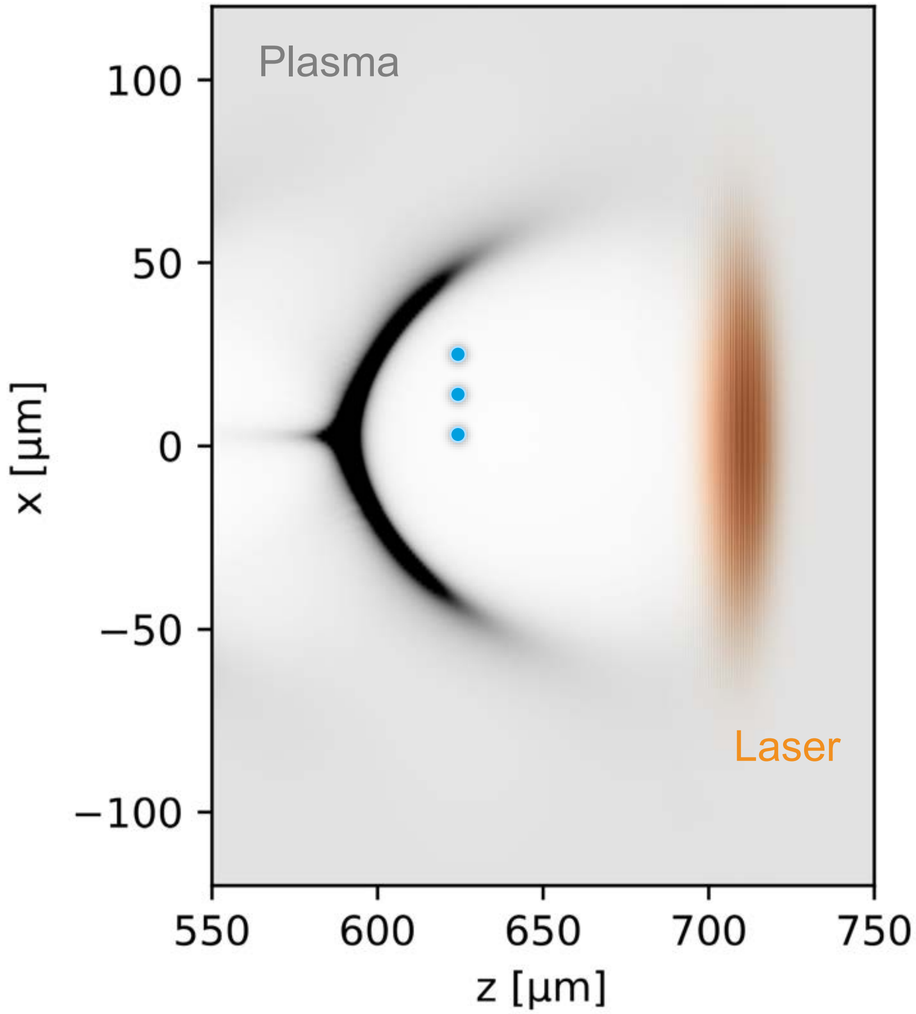
Differences in Path Length and Arrival Time

Another source for increased Energy Spread and Bunch Length

- Usually subtle effects become relevant for plasma accelerators with ultra-strong focusing fields and femtosecond-level bunch lengths.
- Beam electrons have different transverse oscillation amplitudes A_0 and therefore different path lengths.
- **Consequences**:

Increased bunch length → Correlated energy spread

Observed in 2014 by Assmann and Grebenyuk (*IPAC'14*)



Differences in Path Length and Arrival Time

Another source for increased Energy Spread and Bunch Length

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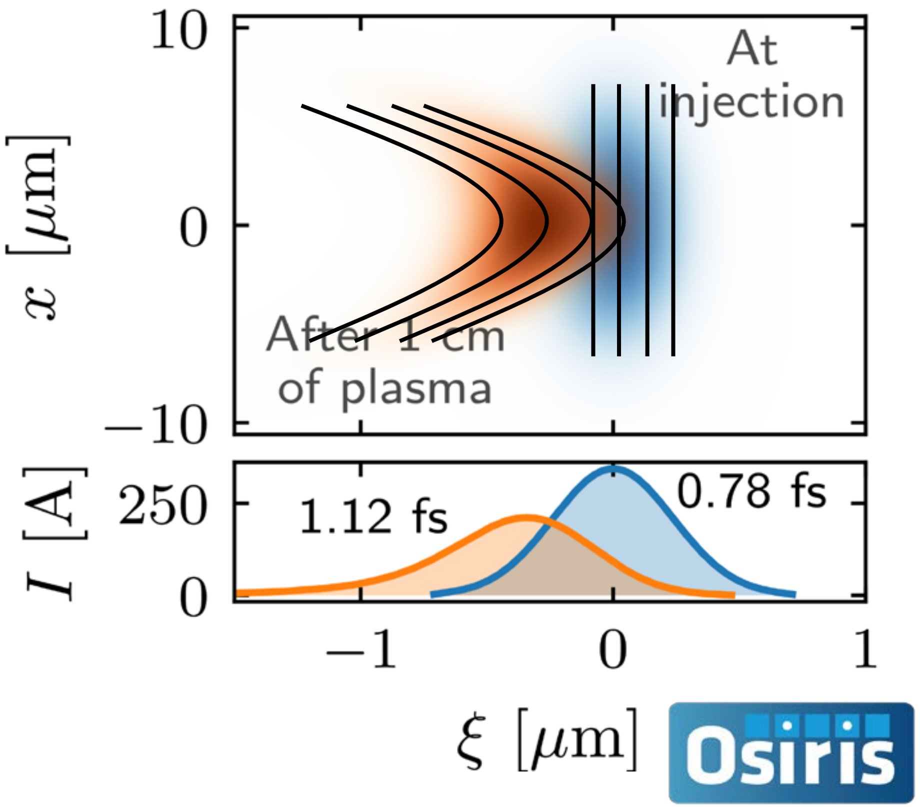
Slice mixing → Uncorrelated (slice) energy spread

Relevant for FEL applications

These dynamics were already pointed out by A. Reitsma and D. Jaroszynski, but no further studies (*Laser Part. Beams 2004*)

Here: Development of the first analytical model that describes these effects and limitations accurately for a particle bunch.



Realistic plasma accelerator simulation demonstrating bunch length generation and banana shape

Analytical Model for Single-Particle Motion

- **Complete 6D model** has been developed.
- Particle energy evolution including slippage and dephasing effects:

$$\gamma(t) \simeq \gamma_0 + \mathcal{E}_0 t - \frac{\mathcal{E}'}{2\mathcal{E}_0}\left(\frac{c}{\gamma_0} + \frac{A_0^2\mathcal{K}}{c}\right)t + \frac{\mathcal{E}'}{2}(c - v_w)\,t^2 + \frac{c\mathcal{E}'}{2\mathcal{E}_0^2}\ln\left(1 + \frac{\mathcal{E}_0}{\gamma_0}t\right) + \frac{\mathcal{E}' A_0^2\mathcal{K}\gamma_0}{c\mathcal{E}_0^2}\left[\left(1 + \frac{\mathcal{E}_0}{\gamma_0}t\right)^{1/2} - 1\right]$$

Full model and details can be seen in upcoming publication.

Analytical Model for Single-Particle Motion

- Energy difference between two particles with the same **initial energy** and **longitudinal position** but different **oscillation amplitude** after a time *t*:
  - P_1 does not oscillate.
  - P_2 has initial oscillation amplitude $A_{0,p2}$.

Higher oscillation amplitude → **Higher energy gain**

Slope of accelerating field

Focusing gradient

$$\gamma_{p_2} - \gamma_{p_1} \simeq \frac{1}{2c}\frac{\mathcal{E}'\mathcal{K}}{\mathcal{E}_0^2}\gamma_0 A_{0,p_2}^2\left[\left(1+\frac{\mathcal{E}_0}{\gamma_0}t\right)^{1/2}-1\right]^2$$

Accelerating field at initial position

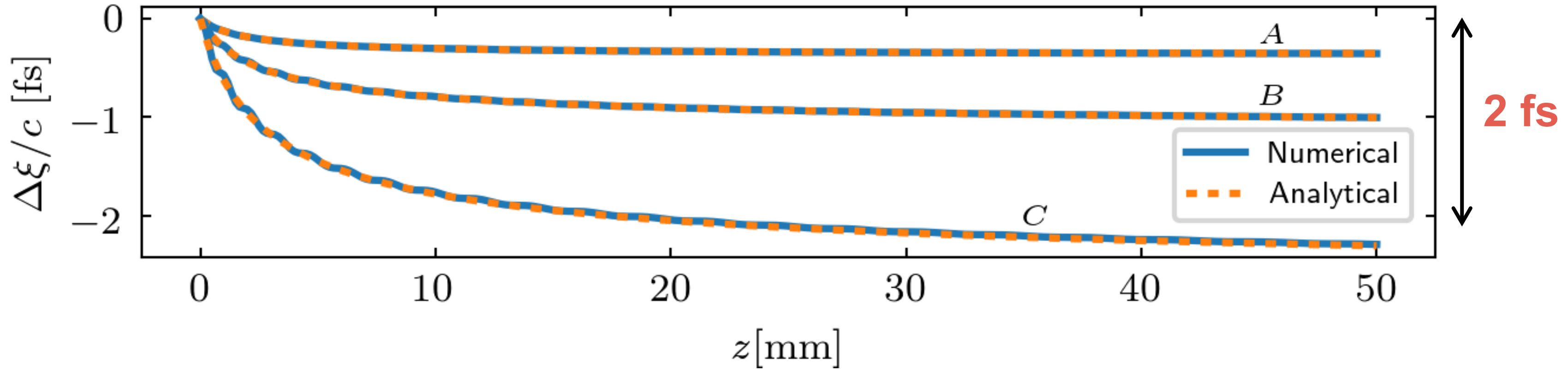
Initial particle energy

Initial oscillation amplitude

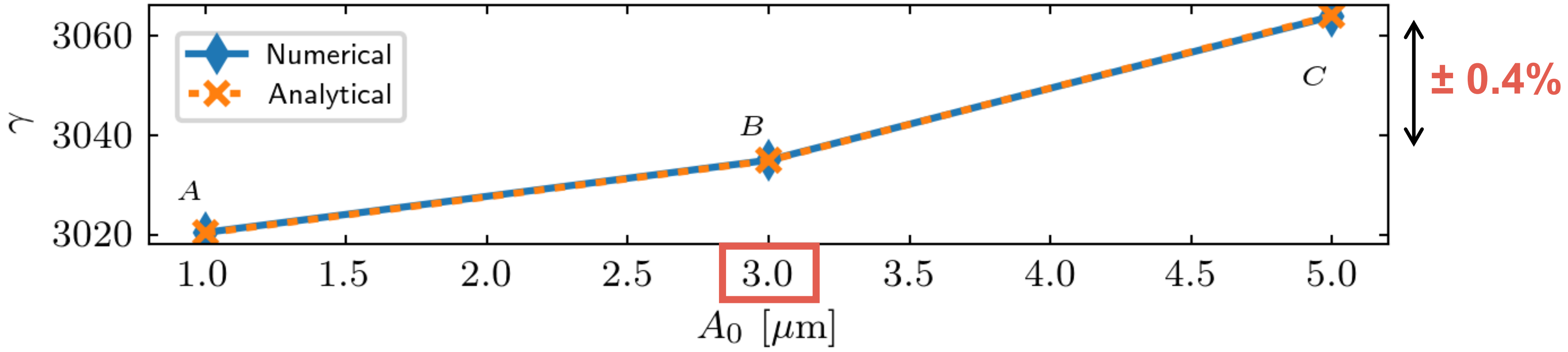
Cross check with a numerical model

- Equations of motion have been solved numerically.
- **Analytical model has assumptions** on relativistic properties, averaged oscillations, …
- Comparison for three different electrons with:
  - **Same initial energy**.
  - **Same initial longitudinal position**.
  - But **different transverse oscillation amplitudes**.
- **Excellent agreement in predictions.**

Slippage of three particles A, B, C with different oscillation amplitudes



Final energy of the three particles after 5 cm of plasma



Significant impact from micron-level oscillations

Uncorrelated Energy Spread of an Electron Bunch

- Induced energy spread in an **infinitesimally short beam slice** is

$$\frac{\sigma_{\gamma_s}^{\Delta\xi}}{\bar{\gamma}_s}(t) \simeq \frac{\mathcal{E}'\mathcal{K}\sigma_{A^2}}{2c\mathcal{E}_0^2}\frac{\left(\bar{\Gamma}_s(t)^{1/2}-1\right)^2}{\bar{\Gamma}_s(t)^{3/2}}$$

Slope of accelerating field → $\mathcal{E}'$

Focusing gradient → $\mathcal{K}$

Standard deviation of A_0^2 → σ_{A^2}

Accelerating field at the slice → $\mathcal{E}_0$

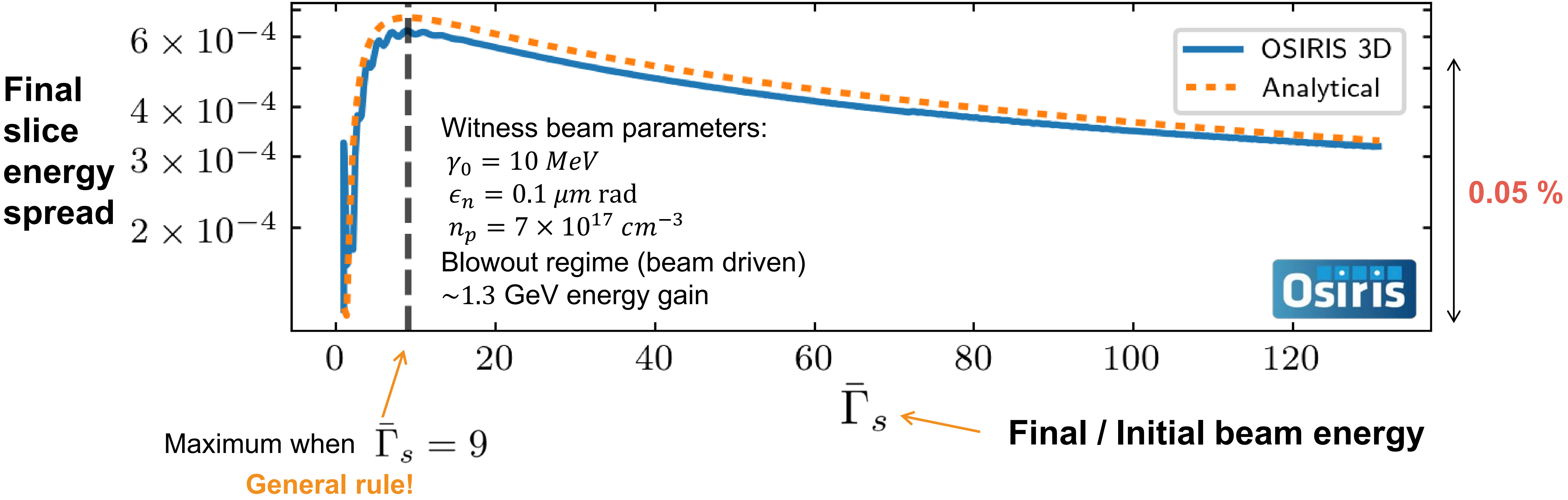
Mean slice energy increase = $\frac{\text{Final energy}}{\text{Initial energy}}$ → $\bar{\Gamma}_s(t)$

For a matched and cylindrically symmetric beam ➡ $\sigma_{A^2} \simeq \sqrt{\frac{8c^2}{\mathcal{K}\bar{\gamma}_{0,s}}}\epsilon_n$

Normalized emittance → ϵ_n

Uncorrelated Energy Spread of an Electron Bunch

- Comparison between analytical model and a full PIC simulation (OSIRIS) for an initial zero longitudinal momentum spread.



There is always this maximum when the energy is increased by **factor 9**

Excellent agreement with simulations

Uncorrelated Energy Spread of an Electron Bunch

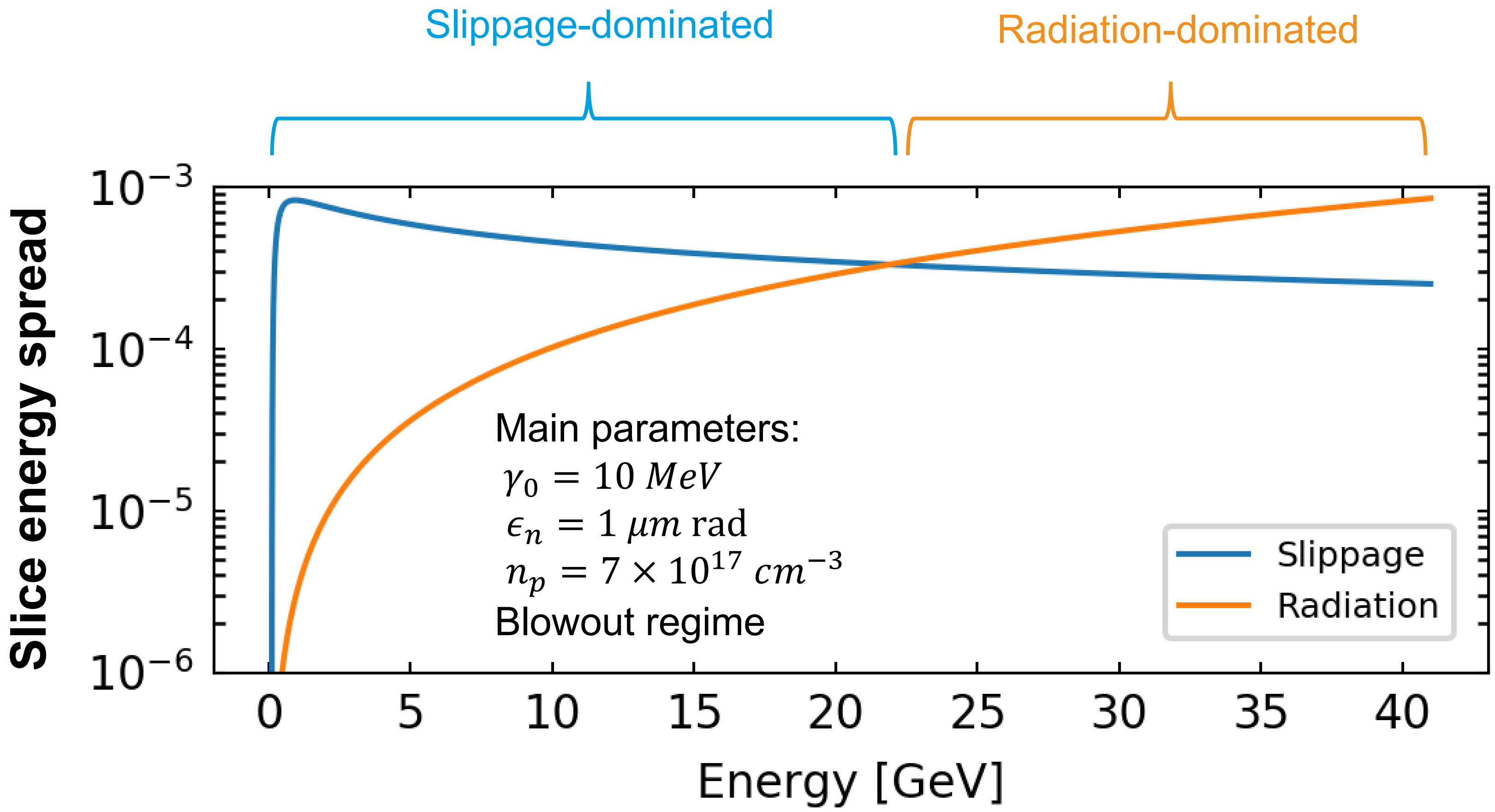
Comparison with betatron radiation

Betatron slippage

$$\frac{\sigma^{\Delta\xi}_{\gamma_s}}{\bar{\gamma}_s}(t) \simeq \frac{\mathcal{E}'\mathcal{K}\sigma_{A^2}}{2c\mathcal{E}_0^2}\frac{\left(\bar{\Gamma}_s(t)^{1/2}-1\right)^2}{\bar{\Gamma}_s(t)^{3/2}}$$

Betatron radiation (P. Michel, 2006, Phys. Rev. E)

$$\frac{\sigma^{R}_{\gamma_s}}{\bar{\gamma}_s} \simeq \frac{2r_e}{15c^3}\frac{\mathcal{K}^2\sigma_{A^2}\bar{\gamma}_{0,s}^2}{\mathcal{E}_0}\frac{\bar{\Gamma}_s^{5/2}-1}{\bar{\Gamma}_s}$$



Slippage will typically dominate for energies up to ~10 GeV. **Relevant for FEL applications.**

Induced Electron Bunch Length and Corr. Energy Spread

- Induced bunch length in an initially **infinitesimally short beam**:

Mean beam energy increase

$$\sigma_{\Delta\xi}(t) \simeq \frac{\mathcal{K}\sigma_{A^2}}{2c\mathcal{E}_0}\left(1 - \bar{\Gamma}(t)^{-1/2}\right)$$

Minimum achievable bunch length

- Energy spread generated from bunch lengthening:

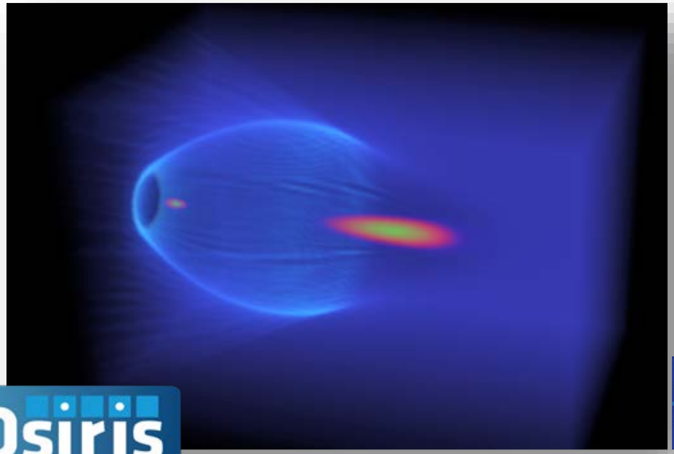
$$\left.\frac{\sigma_\gamma^{\Delta\xi}}{\bar{\gamma}}(t)\right|_{\sigma_{z,0}\to 0} \simeq \frac{\mathcal{E}'\mathcal{K}\sigma_{A^2}}{2c\mathcal{E}_0^2}\frac{\left(\bar{\Gamma}(t)^{1/2} - 1\right)^2}{\bar{\Gamma}(t)}$$

Explains observed limitations in 2014 by Assmann and Grebenyuk (*IPAC’14*)

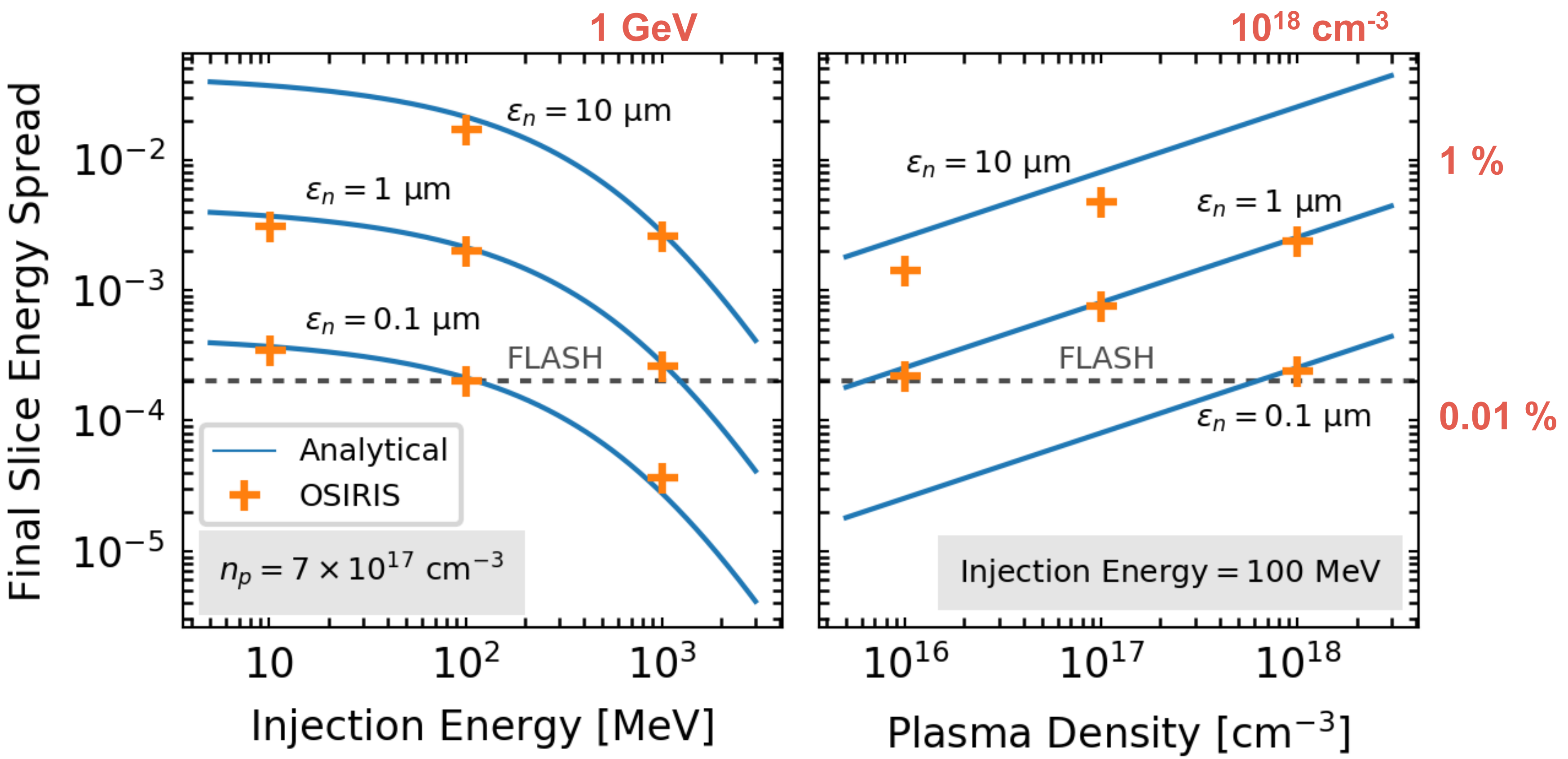
Validation against simulations: Slice Energy Spread

- Test for different **injection energies, emittance** and **plasma densities** on a **1GeV stage**.
- Beam driver.
- Blowout regime.
- Matched witness beam.

  $\sigma_z/c = 3$ fs and 1 pC



Made with VisualPIC

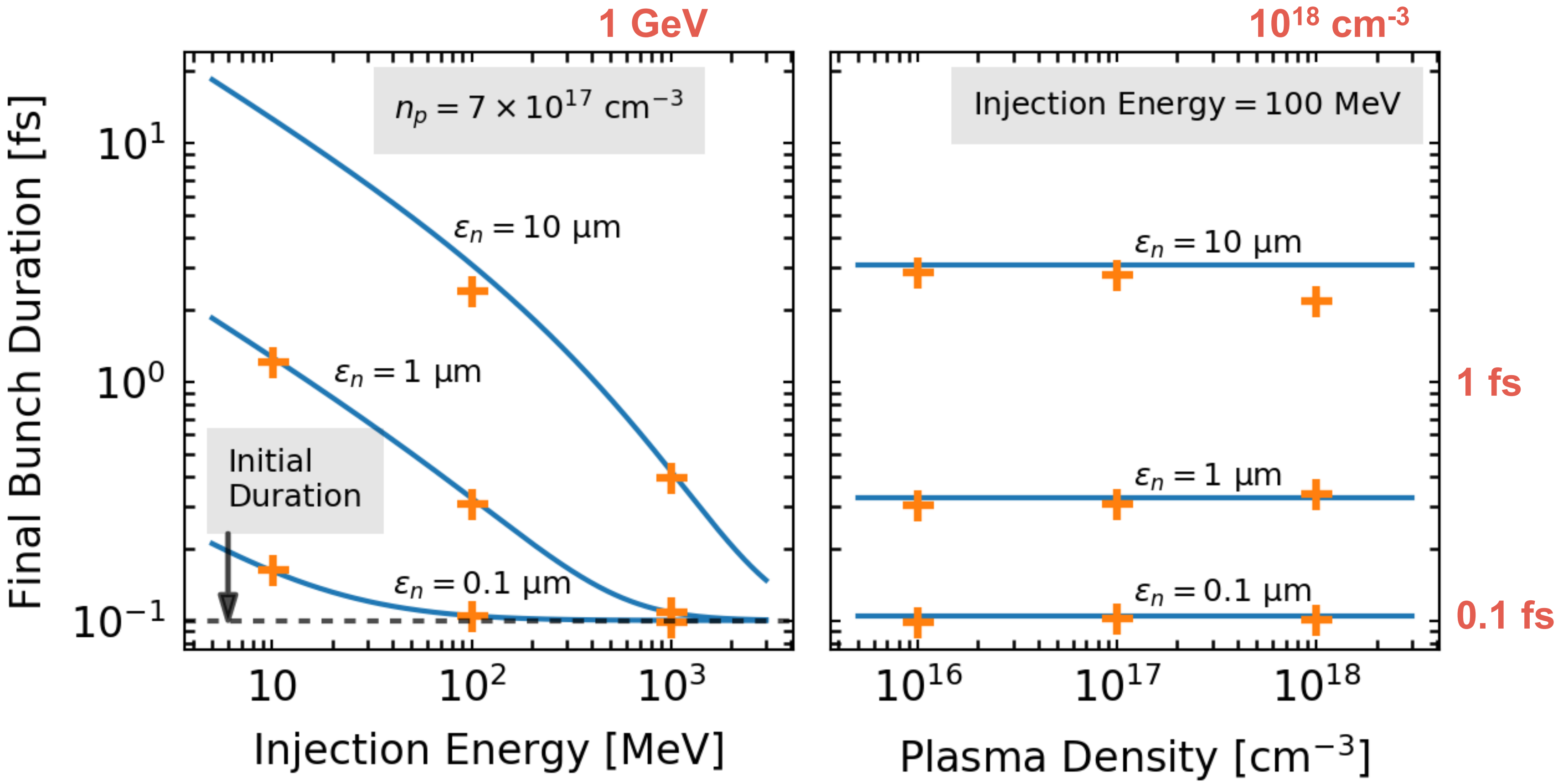


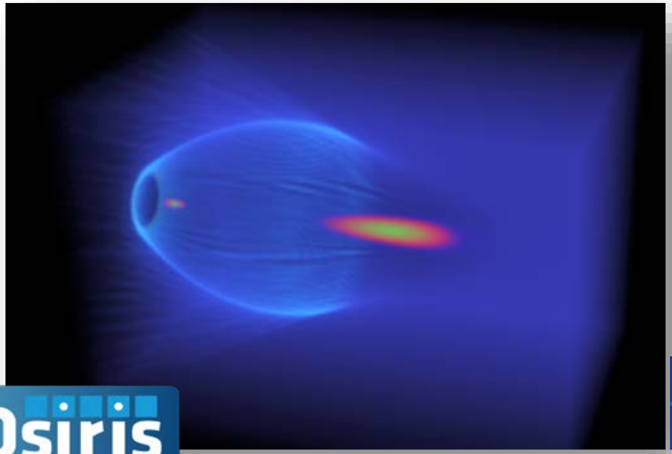
Inject with **low emittance**, **high energy**. Use **low plasma density**.

Validation against simulations: Bunch Length

- Test for different **injection energies, emittance** and **plasma densities** on a **1GeV stage**.
- Beam driver.
- Blowout regime.
- Matched witness beam.

$$\sigma_z/c = 0.1 \text{ fs and } 0.1 \text{ pC}$$





Made with VisualPIC

Inject with **low emittance**, **high energy**. No influence of plasma density.

How to Minimize Energy Spread and Bunch Lengthening

Directions for high-performance plasma-based FELs

Slice Energy Spread

$$\frac{\sigma_{\gamma_s}^{\Delta\xi}}{\bar{\gamma}_s} \propto \frac{\mathcal{E}'\mathcal{K}\sigma_{A^2}}{2c\mathcal{E}_0^2} \propto \sqrt{n_p}$$

Bunch Lengthening

$$\sigma_{\Delta\xi} \propto \frac{\mathcal{K}\sigma_{A^2}}{2c\mathcal{E}_0}$$

Independent of $\sqrt{n_p}$

Ideally can be suppressed by

- Hollow plasma channels ($\mathcal{K} \rightarrow 0$)
- Beam-loading ($\mathcal{E}' \rightarrow 0$)

Otherwise, minimized by

- Plasma density $\lesssim 10^{17}\mathrm{cm}^{-3}$
- Normalized emittance $\lesssim 1\mu\mathrm{m}$
- Injection energy $\gtrsim 100$ MeV

Plasma accelerator simulation with external injection

EuPRAXIA is aiming at finding a consistent set of parameters and solutions

Made with

Conclusions

- We have reviewed limitations on beam quality for electron beams accelerated in plasma accelerators. Focus is on the 1 – 10 GeV regime where ground-breaking FEL applications are possible.
- We have shown that individual **betatron oscillations** of electrons inside a beam of finite transverse size induce significant **path length and arrival time** differences, explaining previously puzzling observations.
- We have developed (to our knowledge for the first time) an **analytical theory** that describes the **slice energy spread,** the **correlated energy spread** and the **bunch length** induced by these phenomena.
- The model has been cross checked with numerical calculations and **full PIC simulations** (OSIRIS).
- Detailed analysis shows that highly **significant contributions** to slice energy spread (order **0.1%**) and bunch length (order **fs**) should be expected and must be **taken into account for a plasma-based FEL design**.
- The analytical formulas give us the tools to understand the dependencies, to quickly analyze many working points and thus to find optimized solutions for future designs.
- EuPRAXIA is using these insights and developing a future plasma research infrastructure.

Thank you