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# SIMULATING NON-RELATIVISTIC BEAMS USING HELICAL PULSE LINES*

Chris Richard†, Michigan State University, East Lansing, USA
Steve Lidia, Facility for Rare Isotope Beams, East Lansing, USA

*Abstract*

Benchtop calibration of capacitive beam position monitors (BPMs) in low energy beamlines is challenging due to non-relativistic effects. Typical benchtop calibrations cannot account for these effects because they rely on speed of light fields transmitted along a straight wire. However, it is possible to replicate the electromagnetic fields generated by non-relativistic beams using a helical line pulse instead of a straight wire. In order to properly replicate the fields from a beam, a method must be developed for tailoring input pulses into the helical line to match bunch shape and a model of the impedance of the helix should be developed to assist with matching. This paper uses the sheath helix model to analyze signal propagation along a helical line in the time domain, with attention to dispersive effects and impedance matching. The results from this model are then compared to Microwave Studio simulations.

## INTRODUCTION

Button beam position monitors (BPMs) measure the position of the beam center of charge by comparing the signals generated on buttons on opposite of the beam pipe. The standard analysis of the signals assumes the electric field on each button has the same frequency spectrum and the beam position offset only affects the amplitude on each button. This is an appropriate assumption for relativistic beams where the electric field from the beam is flattened into the plane perpendicular to the beam velocity due to relativistic effects causing the fields to have the same profile on opposite sides of the pipe.

However, for non-relativistic beams, the electric field generated by the beam resembles the field from a charge at rest. If the beam is offset, the extents of the electric field on opposite sides of the pipe differ resulting in opposite buttons measuring different amplitudes and frequency spectra. The disparity in the frequency domain causes an error in the calculated beam position if the standard difference over sum calculation for the beam position is used [1]. For example, in the Facility for Rare isotope Beams (FRIB) medium energy beam transport beamline the beam is traveling with $\beta = 0.032$, non-relativistic effects cause a 50 percent error in the measured positions [2].

While the effects of non-relativistic beams can be estimated analytically [1] and can be measured using simulations [3], it is desirable to also have method to calibrate for these effects on a test stand. The standard method for calibrating BPMs is performed using straight wire which propagates a signal at the speed of light. This method cannot be used to account for non-relativistic effects. To account for non-relativistic effects, a similar method needs to be developed but is capable of propagating signals at the beam velocity.

### *Helical Pulse Lines*

A helical pulse line is one possible method for bench testing non-relativistic BPMs. Helical lines with the correct geometry are capable of propagating signals with velocities less than 10 percent the speed of light [4]. Therefore, replacing the straight wire used for BPM calibration with a wire wound in a helix could allow for a test stand that is capable of calibrating for non-relativistic effects. However, before a helical line can be used in a test stand to simulate non-relativistic beams, its properties must be fully understood; particularly the impedance and the effects of dispersion.

To guide the development of a helical pulse line, the sheath helix approximation was used as an analytic model to guide simulations. The sheath helix approximates a helix by a thin cylinder that is only conducting along a helical path on its surface. This simplifies the boundary conditions and allows the fields to be solved analytically. While this is a very simple model of a helix, it has been found to have good agreement with simulations of helices [5].

## DISPERSION

A dispersion relation can be found for a helix of radius $a$ and pitch angle $\psi$ in a pipe of radius $R$ using the sheath helix model and assuming perfect conductors:

$$\frac{\gamma^2}{k^2 \cot^2(\psi)} \frac{I_1(\gamma R) I_0(\gamma a)}{I_0(\gamma R) I_1(\gamma a)} = \frac{I_1(\gamma a) K_1(\gamma R) - I_1(\gamma R) K_1(\gamma a)}{I_0(\gamma a) K_0(\gamma R) - I_0(\gamma R) K_0(\gamma a)} \tag{1}$$

where $I_n$ and $K_n$ are the modified Bessel functions of the fist and second kind respectively, $k$ is the wave number in free space, $h$ is the propagation constant of the helix, and $\gamma^2 = h^2 - k^2$. Equation 1 only holds for the lowest order azimuthal mode, however the sheath helix model can also be used to calculate dispersion for higher modes. But, this model predicts the next highest mode does not get excited until $f \approx c/(2\pi a)$. For a 5-mm-radius helix, the next higher mode is excited around 10 GHz, which is higher than needed to replicate bunches.

From this dispersion relation, the low frequency limit of the phase velocity is less than the speed of light and decreases with the pitch angle (Fig. 1). In the limit as $\psi$ goes

* Work supported by the US Department of Energy, Office of Science, High Energy Physics under Cooperative Agreement award number DE-SC0018362
† richard@nscl.msu.edu

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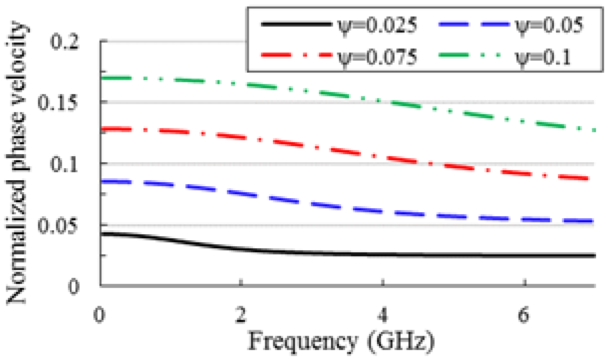


Figure 1: Phase velocities for 5 mm radius helices centered in a 20 mm radius pipe with different pitch angles, $\psi$.

to zero, the phase velocity also goes to zero. This limit can be approximated within a 5% error from the sheath helix predictions for helices with $\psi < 0.1$:

$$\beta_{\text{phase}} \approx \left| \left( \frac{a^2}{R^2} - 1 \right) \frac{\cot^2(\psi)}{2\ln(R/a)} + 1 \right|^{-\frac{1}{2}} . \quad (2)$$

This low frequency reduction of the phase velocity allows helical lines to be created such that a pulse can propagate at any chosen velocity using the correct pitch angle.

However, as the frequency increases the phase velocity decreases and approaches the sine of the pitch angle. This causes large changes in the phase velocity for different frequencies and results in a pulse quickly deforming while propagating down a helical line. For example, after propagating 200 mm down a 5-mm radius helix with pitch angle of 50 mrad, a 0.6-ns Gaussian pulse will deform into multiple peaks (Fig. 2).

The deformation of a pulse can be corrected at a chosen location, by propagating a desired pulse along the helix and measuring the deformed pulse at the desired location. The deformed pulse is then reversed in time and propagated along the helix. At the measurement location the deformation will be corrected and the original pulse will be reproduced. This method can be used for helical lines for BPM calibration to reproduce the desired pulse at the BPM.

Generating the necessary pulse shapes to recreate a Gaussian pulses can be accomplished using a 1D model of a helical line. At the buttons on a BPM, only the radial electric field is non-zero and the distortion to this field from propagating along the helical line can be calculated using the dispersion relation from the sheath helix model. The distorted pulse can then be input into CST studio for simulations of helical pulse lines. Overall, using this method to generate the required input pulse agrees with CST simulations. However, in general the original pulse is not perfectly reproduced, some minor distortions typically remain, such as small bumps near the ends of the pulse (Fig. 2). These are likely caused either from shortcomings of the sheath helix model or from distortions of the pulse as it encounters the helix.

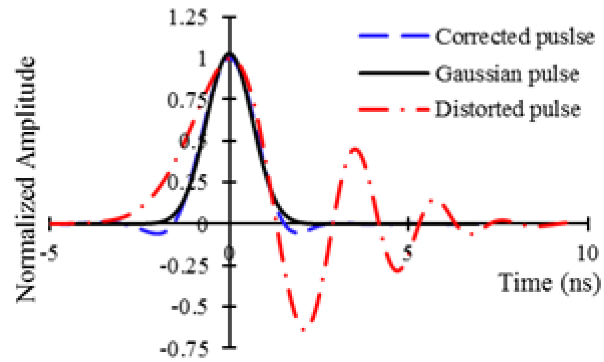


Figure 2: Original pulse, distorted pulse from 1D model, and corrected pulse for 1-ns Gaussian pulse propagating 200 mm along a helical line with $R = 20$ mm, $a = 5$ mm, and $\psi = 47.7$ mrad.

## IMPEDANCE MATCHING

In simulations of helical lines, the input pulse is excited on a coax line then the inner conductor transitions into a helix (Fig. 3). At the transition there are large reflections due to an impedance mismatch between the helix and the coax line. To improve matching, the impedance of the helical line is calculated based on the fields from the sheath helix model. Calculations of the impedance with this method agree with the results from reflection is CST simulations. For example, for a 5-mm radius helix with pitch angle of 47 mrad in a 40-mm diameter pipe, the measured impedance from CST simulations was 1020 Ω and the calculated impedance from the sheath helix model was 1016 Ω in the low frequency limit. Using the calculated impedance of the helix, an L-pad was used to match the coax input line to the helix. The impedance matching resulted in the amplitude of the reflected signal dropping from 60 percent of the input signal to less than 5 percent.

However, the sheath helix cannot be used to calculate the impedance for all helices. As the pitch angle of a helix is increased the helix approaches a straight wire, therefore the impedance should approach the impedance of a coax line with inner conductor radius equal to the wire radius. However, when the pitch angle is increased in the sheath helix model, which is based on a cylinder with radius equal to the helix radius, the impedance goes to the impedance of a coax line with inner conductor radius equal to the radius of the helix. The sheath helix approximation fails for loosely wound helices, however, to achieve low phase velocities the helix must be tightly wound thus the sheath helix calculations are valid for all cases of interest.

## FIELD COMPARISON

The benefits of using a helical line instead of a straight wire for non-relativistic BPM calibration can be demonstrated by comparing the electric fields generated by a helical line and a straight wire to the field from a pencil beam. This comparison was performed using CST studio to simulate Gaussian beams traveling at $\beta = 0.085$ in a 40-mm diameter

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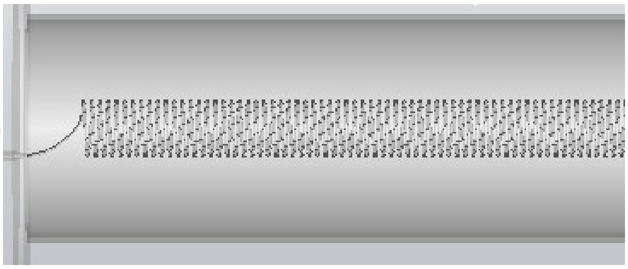

Figure 3: CST model of helical pulse line with pipe radius 20 mm, helix radius 5 mm, wire radius 0.2 mm, and helix pitch angle 47.7 mrad.

pipe and was offset 5 mm from the center of the pipe. To replicate this beam with a helical line, a 5-mm diameter helix was modeled with 0.2-mm radius wire with pitch angle of 0.0477 radians. The necessary input pulse to correct for disperion was generated using the 1D model. The straight wire was 0.2-mm radius and also displaced 5 mm. For each geometery, the radial electric field on either side of the beam pipe was measured to observe the effect of the off center beam. To simplify comparisons, the input pulses for each geometry was chooses such that a Gaussian pulse with a standard deviation of 0.51 ns was measured at the pipe wall closest to the beam or helix. The measured fields on both sides of the pipe were normalized to the peak field at the point closer source element.

The measured fields from the straight wire behaved differently from the pencil beam. Due to relativistic effects, the pulse shape on either side of the pipe were Gaussian with the same standard deviation of 0.51 ns. There is also a large difference in the normalized amplitude of the far side signal: 0.36 for the wire compared to 0.25 for the pencil beam. If a BPM is calibrated using the straight wire, these differences will cause an error in the measured position of the beam.

However, the helical line produces fields with similar behavior to the pencil beam (Fig. 4 top). The fields on opposite sides of the pipe from the pencil beam are Gaussian with an approximately 37% difference in their standard deviations, 0.51 ns on the near side and 0.7 on the far side. This effect is also seen in the field from the helix, however, the the standard deviation differs by about 25%: 0.51 ns on the near side and 0.64 ns on the far side. While the signals from the helix are still distorted, they show the capability to replicate the fields from a non-relativistic beam making them more ideal for BPM calibration.

## FUTURE WORK

Current work has shown the ability for helical pulse lines to reproduce the fields from a non-relativistic Gaussian beam. However, further work should be done to improve the pulse shape and determine sources of distortion other than dispersion. Another issue that must be address is the helices must be made from thin wire and therefore will deform unless they are supported. Any changes in the pitch angle and radius of the helix will cause local changes to the dispersion and impedance which will result in unwanted deformation

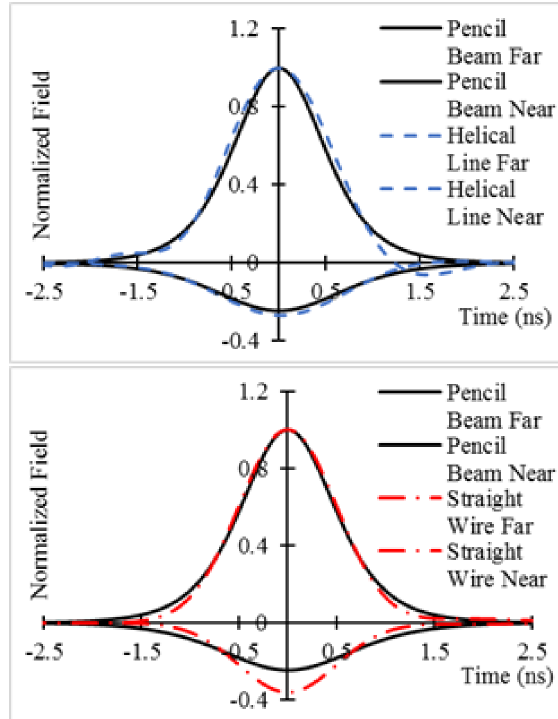


Figure 4: Radial electric field at pipe wall comparison from source 5 mm off center. Near side is shown positive, far side is negative. Top: Helical line compared to pencil beam. Bottom: Thin wire compared to pencil beam.

of the signal. To support the helix and fix the pitch angle, the helix can be wound around a dielectric rod that has a helical groove machined in it. An analytic model of helices wrapped around a dielectric rod in a pipe is being developed as well as simulations.

## ACKNOWLEDGMENTS

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## REFERENCES

[1] R. Shafer, "Beam position monitor sensitivity for low-$\beta$ beams," in *Proc. LINAC'94*, Tsukuba, Japan, Aug, 1994, pp. 905-907.

[2] O. Yair *et. al.*, "FRIB beam position monitor pick-up design," in *Proc. IBIC'17*, Monterey, CA, USA, Sep, 2014, paper TUP16, pp. 355-360.

[3] Kowina *et al.*,"FEM simulations - A powerful tool for BPM design," in *Proc. DIPAC'09*, Basel, Switzerland, May, 2009, paper MOOC03, pp .35-37

[4] S. Sensiper, "Electromagnetic wave propagation on helical conductors," Ph.D. thesis, MIT, 4954.

[5] C. Richard and S. Lidia, "Benchtop BPM calibration using helical pulse lines for non-relativistic beams," in *Proc. IBIC'17*, Grand Rapids, MI, USA, Aug. 2017, paper TUPCF14.