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ACCURATE MODELING OF THE HOSE INSTABILITY IN PLASMA BASED ACCELERATORS*

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Abstract

The hose instability is a long standing challenge for plasma-based accelerators. It is seeded by initial transverse asymmetries of the beam or plasma phase space distributions. The beam centroid displacement is thereby amplified during the propagation in the plasma, which can lead to an unstable acceleration process. A witness beam can itself cause hosing and/or may be affected by the hosing of the drive beam. The accurate study of hosing including a witness beam is of utmost importance to facilitate stable plasma-based accelerators. In this contribution, we discuss novel methods for the mitigation of hosing and present a new model for the evolution of the plasma centroid, which enables the accurate investigation of the hose instability of drive and witness beam pair in the nonlinear blowout regime. This work enables more precise and comprehensive studies of hosing and hence, for the potential stabilization of future compact plasma-based accelerators.

INTRODUCTION

Plasma wakefield accelerators (PWFAs) [1,2] can sustain accelerating fields beyond 10 GV/m and are therefore considered a promising technology candidate for future compact and affordable particle accelerators. While the extreme longitudinal fields allow for an energy gain of several GeVs over distances of only tens of centimeters [3,4], the comparable magnitude of the transverse fields in the nonlinear blowout regime [5] implies a rapid growth rate for the hose instability [6]. Applications driven by particle accelerators require a high degree of stability and hence, the hose instability poses a critical challenge for the applicability of PWFAs.

The current mathematical description of the hose instability in PWFAs is given by the coupled differential equations [7,8]

$$\frac{\partial^2 X_b}{\partial t^2} + \lambda(\xi,t)\frac{\partial X_b}{\partial t} + \Omega^2(\xi,t)X_b = \Omega^2(\xi,t)X_p\,, \quad (1)$$

$$\frac{\partial^2 X_p}{\partial \xi^2} + \frac{c_\psi(\xi)c_r(\xi)}{2}X_p = \frac{c_\psi(\xi)c_r(\xi)}{2}X_b\,, \quad (2)$$

where $\xi = t - z$ is the co-moving variable, z the propagation distance, and t the time. Length-scales within this work are normalized by the plasma wavenumber $k_p = \omega_p/c$ and timescales by the plasma frequency $\omega_p = \sqrt{4\pi n_0 e^2/m}$, where c is the speed of light. Furthermore, densities are normalized to the ambient plasma electron density n_0, charges to the elementary charge e, masses to the electron mass m and potentials to mc^2/e.

The coefficients $\lambda(\xi,t)$ and $\Omega(\xi,t)$ in Eq. (1) were introduced in Ref. [8]. A finite $\lambda(\xi,t)$ accounts for the damping/amplification of the beam centroid oscillations owing to a relativistic mass gain/loss of beam electrons and for the damping of the centroid oscillations from a finite uncorrelated beam energy spread. The coefficient $\Omega(\xi,t)$ incorporates the effect of the change of the betatron frequency for a changing energy. These effects intrinsically lead to a saturation or damping of the hosing for the drive beam in PWFAs [8], similarly to the mitigation of hosing in self-modulated PWFAs owing to a varying betatron wavenumber along the beam [9] and similarly to Balakin-Novokhatsky-Smirnov damping in conventional accelerators [10].

The coefficients $c_r(\xi)$ and $c_\psi(\xi)$ in Eq. (2), introduced in Ref. [7], account for the ξ-dependent blowout radius and current and for relativistic velocities of electrons in the sheath. These coefficients were derived through investigation of the dynamics of a plasma electron at the blowout-sheath boundary, so as to infer the perturbation of the channel centroid.

While the model in Ref. [7] posed a dramatic improvement for the modeling of hosing in PWFAs in the nonlinear blowout regime, compared to the original model ($c_r c_\psi = 1$) [6], it still is insufficiently accurate and features unphysical properties. Here, we outline a generalization of Eq. (2), introduced recently by investigation of the collective dynamics of all sheath electrons subject to beam and plasma centroid deviations [11]. As shown, this model is more accurate and physical, and therefore can provide an important basis for crucial studies on the hosing of drive and witness beam pairs in PWFAs in the blowout regime.

DERIVATION OF THE PLASMA CENTROID EQUATION

Quasi-static Moment Equation

In Ref. [11], a general equation for the evolution of the moments of the plasma-electron phase space density f_p along ξ is derived, when the electrons are subject to the fields of a beam with a finite centroid displacement and to the fields of a blowout wake with a finite transverse displacement. This derivation is outlined in the following.

The use of the quasi-static approximation [12–14] $\partial_t f_p \ll \partial_\xi f_p$ for the plasma-electron phase space density implies that in the azimuthally symmetric case the density function can be only a function of the radial position r, radial momentum p_r, and ψ [resulting from the constant of motion

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$\psi + p_z - \gamma = -1$ (see Ref. [15])], for a given time t and a co-moving position ξ, such that $f_{p,0} = f_{p,0}(r, p_r, \psi; \xi, t)$.

From the conservation of the phase space density $df_{p,0}/dt = 0$, we obtain the Vlasov equation for $f_{p,0}$ in the quasi-static approximation,

$$\partial_\xi f_{p,0} = -\frac{\gamma}{1+\psi}\left(\frac{p_r}{\gamma}\partial_r + F_r\partial_{p_r} + F_\psi\partial_\psi\right) f_{p,0}\,, \tag{3}$$

with the forces $F_r = dp_r/dt$ and $F_\psi = d\psi/dt$, and where the Lorentz factor is expressed as $\gamma = [1 + p_r^2 + (1+\psi)^2]/(2+2\psi)$. The asymmetric plasma electron phase space distribution f_p can be expressed as an expansion of $f_{p,0}(r^*, p_r^*, \psi; \xi, t)$ for small perturbations $\langle x\rangle$ and $\langle p_x\rangle$, where $r^* = \sqrt{(x-\langle x\rangle)^2 + y^2}$ and $p_r^* = (p_x - \langle p_x\rangle)\cos\theta + p_y\sin\theta$, such that

$$f_p \simeq -\cos\theta(\langle x\rangle\,\partial_r + \langle p_x\rangle\,\partial_{p_r})f_{p,0}\,, \tag{4}$$

and $f_p = f_p(r, \theta, p_r, \psi; \xi, t)$. The Vlasov equation for the asymmetric plasma electron distribution is given by

$$\partial_\xi f_p = -\frac{\gamma}{1+\psi}\left(\frac{p_r}{\gamma}\partial_r + \dot\theta\partial_\theta + F_r\partial_{p_r} + F_\psi\partial_\psi\right) f_p\,. \tag{5}$$

Multiplication of Eq. (5) by quantities Φ and integration by parts with the assumption that the phase space density decays to zero at the integration limits, yields the quasi-static moment equation

$$\begin{aligned}\partial_\xi\langle\Phi\rangle = &\left\langle\frac{p_r}{1+\psi}\partial_r\Phi\right\rangle + \left\langle\frac{\gamma\dot\theta}{1+\psi}\partial_\theta\Phi\right\rangle \\ &+ \left\langle\frac{\gamma F_r}{1+\psi}\partial_{p_r}\Phi\right\rangle + \left\langle\frac{\gamma F_\psi}{1+\psi}\partial_\psi\Phi\right\rangle\,.\end{aligned} \tag{6}$$

This equation expresses how an averaged property $\langle\Phi\rangle$ of the plasma electron phase space distribution at a given time and comoving position changes along the comoving coordinate.

Plasma centroid equation

Application of Eq. (6) to the transverse spatial average ($\Phi = x$) and transverse momentum average ($\Phi = p_x = p_r\cos\theta$), after some simplifications and algebra, yields

$$\begin{aligned}\partial_\xi^2\langle x\rangle &= \frac{\left\langle\frac{\gamma}{1+\psi}F_r\cos\theta\right\rangle}{\langle 1+\psi\rangle} - \frac{\langle p_r\cos\theta\rangle}{\langle 1+\psi\rangle^2}\left\langle\frac{\gamma F_\psi}{1+\psi}\right\rangle \\ &= \frac{\left\langle\frac{\gamma}{1+\psi}F_r\cos\theta\right\rangle}{\langle 1+\psi\rangle} - \frac{\partial_\xi\langle x\rangle}{\langle 1+\psi\rangle}\left\langle\frac{\gamma F_\psi}{1+\psi}\right\rangle\,.\end{aligned} \tag{7}$$

The forces are now cylindrically expanded w.r.t. the plasma electron centroid $X_p = \langle x\rangle$ and w.r.t. the beam centroid X_b and expressed in terms of electromagnetic potentials. After application of a cold fluid approximation to $f_{p,0}$ one obtains the second-order differential *plasma centroid equation*

$$\frac{\partial^2 X_p}{\partial\xi^2} + C_d(\xi)\frac{\partial X_p}{\partial\xi} + \frac{C_p(\xi)}{2}X_p = \frac{C_b(\xi)}{2}X_b\,. \tag{8}$$

This equation is equivalent to a driven, damped harmonic oscillator for the plasma centroid $X_p(\xi, t)$ along ξ. The second term thereby acts as a damping/amplifying term for $C_d \neq 0$ and the coefficient $C_p/2$ is the square of the undamped oscillation wavenumber of the system. The oscillator is driven by the beam centroid X_b for a finite C_b.

Ceofficients for the Blowout Regime

The coefficients C_d, C_p, and C_b can be explicitly derived by the use of a mathematical description for the potentials and currents within the blowout and within the sheath, e.g., by use of the blowout model in Ref. [16]. In addition, we consider a beam which is sufficiently narrow, such that the beam current density does not significantly overlap with the plasma electron sheath. These assumptions allow for the formulation of C_d, C_p, and C_b in terms of the instantaneous beam current $I_b(\xi)$, the local blowout radius $R(\xi)$ and the blowout curvature $R'(\xi) = \partial_\xi R(\xi)$ to first order accuracy of an expansion w.r.t. $\Delta_\rho/R \ll 1$, where Δ_ρ is the characteristic sheath thickness.

The term responsible for the restoring effect is

$$C_p = \frac{1-R'^2}{4} - \frac{\Delta_\rho}{4}\left[\frac{1+\Lambda}{R} + \frac{R\left(R'^2 - R^2 - 1\right)}{4} - \frac{R^2R'}{2}\right]\,, \tag{9}$$

where $\Lambda(\xi) = 4I_b/I_A$ is the normalized beam current, where the Alfvén current by $I_A \simeq 17$ kA. Equation (9) implies a restoring effect of the plasma wakefields onto the plasma centroid if $R'^2 \lesssim 1$ and an amplifying effect if $R'^2 > 1$.

The coefficient, defining the magnitude of the force exerted onto the plasma centroid by the beam centroid, is

$$C_b = \frac{\Lambda}{R^2}\left[1 - \Delta_\rho\left(\frac{2}{R} + \frac{R}{4}\right)\right]\,. \tag{10}$$

The centroid deviation of an electron beam acts as a force onto the plasma centroid, where the magnitude of the effect scales with the beam current and inversely with the square of the blowout radius. A finite sheath thickness Δ_ρ reduces this effect.

The damping-coefficient is

$$C_d = \frac{R'}{4}\left[R - \frac{\Delta_\rho}{2}\left(\frac{R^2}{2} - 1\right)\right]\,, \tag{11}$$

and hence implies a damping of the plasma centroid due to a relativistic mass gain of electrons in the sheath if $R' > 0$ and an amplification owing to a relativistic mass loss for $R' < 0$.

Discussion and Comparison to Previous Works

We compare these results to the plasma centroid equations obtained in previous works. Whittum *et al.* [6] assumed an adiabatically generated ion channel with non-relativistic sheath electrons at the charge neutralization radius. The respective channel centroid equation therefore is equivalent to Eq. (8) with coefficients $C_p = C_b = 1$ and $C_d = 0$.

In a more recent work [7], the plasma centroid equation was generalized by introduction of the coefficients

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$c_r = \Lambda/R^2$ and $c_\psi = [1 + \Psi(R)]^{-1}$ where $\Psi = \phi - A_z$ is the wakefield potential, ϕ the scalar potential, and A_z the longitudinal component of the vector potential. This generalization incorporates the effects of a varying beam current, changing blowout radius along the beam, and relativistic velocities of electrons in the sheath, and is equivalent to Eq. (8) if the coefficients are set to $C_p = C_b = c_r c_\psi \simeq \Lambda/R^2 (1 - \Delta_\rho R/2) + O(\Delta_\rho^2/R^2)$ and $C_d = 0$.

In the limit $\Delta_\rho \to 0$, the model in Ref. [7] and Eq. (10), recover the same result $C_b \to \Lambda/R^2$, for which the impact of the beam centroid deviation onto the plasma centroid deviation scales linearly with the beam current and inversely with the square of the blowout radius. However, for a finite sheath thickness, $\Delta_\rho > 0$, a reduced coefficient C_b for blowout radii $R \sim 1$ is found here.

In addition, and in contrast to previous models (cf. [6, 7]) we find a coefficient corresponding to the restoring force of the plasma centroid which differs from the coefficient for the driving force ($C_p \neq C_b$). It should be noted that $C_p \neq 0$ for $\Lambda = 0$ in Eq. (9). This is in contrast to Eq. (2), for which $C_p = 0$ for $\Lambda = 0$ implies the unphysical property that $\partial_\xi X_p = \text{const}$ if the beam current drops to zero. Furthermore, our model includes the effect of damping or amplification of the channel centroid deviation ($C_d \neq 0$) owing to a relativistic mass gain or loss of the sheath electrons along the blowout.

COMPARISON TO SIMULATION RESULTS

In this section we compare the model from Ref. [11], outlined above, and the model in Ref. [7] to results from 3D particle-in-cell (PIC) simulations with the quasi-static code HiPACE [17]. We consider a Gaussian drive beam with an energy of 10 GeV, with a peak current $\Lambda_0 = 2.35$ (corresponding to 10 kA), and with dimensions $\sigma_{x,y} = 0.1$ and $\sigma_z = 1.0$. These parameters are close to the parameters anticipated for FACET-II [18]. The beam features a tilt of $dx/d\xi = 0.001$, which is introduced from a comoving position located at $\xi_0 - 1.0 \times \sigma_z$, where ξ_0 specifies the beam-center.

Figure 1 depicts the temporal evolution of the beam centroid at the tail, at $\xi_0 + 4.0 \times \sigma_z$. Shown in green is the curve obtained from numerical evaluation of Eqs. (1) and (2) and in red the result obtained from numerical evaluation of Eqs. (1) and (8). The black dashed curve depicts the result from a 3D PIC simulation. The sheath thickness observed in the simulation is compatible with $\Delta_\rho = 0.2$, which is the value used for the numerical solutions of the above differential equations. Furthermore, the longitudinal field $E_z(\xi)$, extracted from the PIC simulation, is used for the computation of $R(\xi)$ and $R'(\xi)$.

As seen in Figure 1, all curves predict the beam centroid to initially grow significantly owing to hosing and to saturate for greater times because of the betatron decoherence from a differing energy change along the beam (cf. [8]). This effect is incorporated in the beam centroid equation (1).

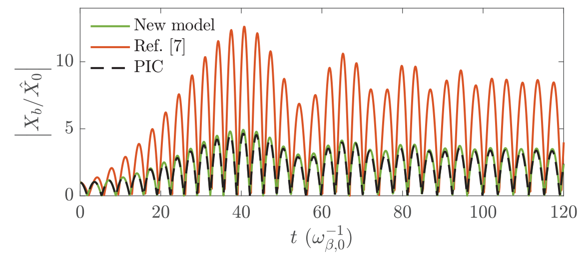

Figure 1: Beam centroid evolution at the tail of a 10 GeV, 10 kA drive beam.

However, while the channel centroid, Eq. (2) overestimates the beam centroid amplitude, the new equation (8) is in good agreement with the PIC simulation result. The accurate reproduction of hosing at the tail of the beam is possible by means the new model since Eq. (9) implies a finite restoring force for the channel centroid for $\Lambda \to 0$. This is in contrast to the unphysical behavior of $X_p' \to \text{const}$ for $\Lambda \to 0$ implied by Eq. (2). In addition, Eq. (10) implies a reduced growth rate of the hose instability compared to Eq. (2).

CONCLUSION

In this work, a generalized plasma centroid equation for the description of hosing in PWFAs, derived in Ref. [11], is outlined and used to model the beam centroid evolution at the tail of a 10 GeV, 10 kA Gaussian drive beam in a PWFA. We find an excellent agreement between results from PIC simulations and the new model. The accurate investigation of the channel and beam centroid evolution at comoving positions behind the drive beam is relevant for the study of the impact of the hosing of a drive beam onto the dynamics of a trailing beam in PWFAs. Hence, the presented model provides an important basis for the accurate modeling of hosing in PWFAs and for the possibility to understand and improve the stability of compact plasma-based accelerators.

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