ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-TUPMK016

USING TIME EVOLUTION OF THE BUNCH STRUCTURE TO EXTRACT THE MUON MOMENTUM DISTRIBUTION IN THE FERMILAB MUON g-2 EXPERIMENT

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Abstract

Beam dynamics plays an important role in achieving the unprecedented precision on measurement of the muon anomalous magnetic moment in the Fermilab Muon $g-2$ Experiment. It needs to find the muon momentum distribution in the storage ring in order to evaluate the electric field correction to muon anomalous precession frequency. We will show how to use time evolution of the beam bunch structure to extract the muon momentum distribution by applying a fast rotation analysis on the decay electron signals.

INTRODUCTION

A charged elementary particle has a magnetic dipole moment aligned with its spin:

$$\vec{\mu} = g\frac{q}{2m}\vec{s} \tag{1}$$

where q is the electric charge, m is the mass and g is the dimensionless gyromagnetic factor. Dirac theory predicts that $g = 2$ for a spin 1/2 particle [1, 2] , however hyperfine structure experiments conducted in the 1940's showed that $g \neq 2$ [3, 4]. The deviation is called the magnetic dipole moment anomaly, defined by $a \equiv (g-2)/2$.

Standard Model (SM) contributions to the muon anomaly a_μ come from the quantum electrodynamics, electroweak and quantum chromodynamics sectors, which contains all the virtual loops involving with the SM particles. So far, a_μ^{SM} has been calculated to a precision of around 0.3-0.4 parts per million (ppm) [5, 6].

The most recent measurement of a_μ was done at Brookhaven National Laboratory (BNL) by the E821 Experiment. It achieved a precision of 0.54 ppm [7], which differs from the SM prediction by about 3.5 standard deviation. The E989 experiment at Fermi National Accelerator Laboratory (Fermilab) reuses the E821 muon storage ring to measure a_μ with a goal of 0.14 ppm [8]. In order to study the beam dynamics and estimate the electric field correction to $g-2$, we discuss the beam debunching process and show how to use the time evolution of the beam bunch structure to extract the muon momentum distribution by applying a so-called fast rotation analysis on the decay positron signals.

PRINCIPLES OF THE EXPERIMENT

Both E821 and E989 measure a_μ by using the spin precession resulting from the torque experienced by the muon magnetic moment when placed in an external magnetic field. The rate of change of the component of spin $\vec{s}$ parallel to the velocity ($\vec{\beta} = \vec{v}/c$) is given by

$$\frac{d}{dt}(\hat{\beta}\cdot\vec{s}) = -\frac{e}{mc}\vec{s}_\perp\cdot\left[\left(\frac{g}{2}-1\right)\hat{\beta}\times\vec{B} + \left(\frac{g\beta}{2}-\frac{1}{\beta}\right)\vec{E}\right] \tag{2}$$

where $\hat{\beta}$ is the unit vector in the direction of $\vec{\beta}$ and $\vec{s}_\perp$ is the component of $\vec{s}$ perpendicular to the velocity [9]. The electric field $\vec{E}$ is provided by the Electrostatic Quadrupole System inside the storage ring [10]. For a uniform magnetic field $\vec{B}$ and a quadrupole electric field $\vec{E}$, the anomalous muon spin precession frequency is given by

$$\begin{aligned}\vec{\omega}_a \simeq & -\frac{q}{m}\left[a_\mu\vec{B} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)(\vec{\beta}\cdot\vec{B})\vec{\beta}\right. \\ & \left. - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]\end{aligned} \tag{3}$$

The third term in Eq. (3) vanishes by choosing the "magic" momentum $p_{magic} = m/\sqrt{a_\mu} \simeq 3.09$ GeV/c. However, muons have a momentum spread. Therefore, an electric field correction is needed.

In the experiment, the anomalous spin precession frequency $\vec{\omega}_a$ is measured by using the decay electron signal; while the magnetic field is measured by observing the Larmor frequency of stationary proton ($\vec{\omega}_p = (g_p q\vec{B})/(2m_p)$) with the nuclear magnetic resonance (NMR). The muon anomaly is then extracted and can be rewritten as

$$a_\mu = \frac{\omega_a}{\omega_p}\frac{\mu_p}{\mu_e}\frac{m_\mu}{m_e}\frac{g_e}{2} \tag{4}$$

where $\frac{\mu_p}{\mu_e}$ is the proton-electron magnetic moment ratio and g_e is the electron g-factor [8].

BEAM DEBUNCHING AND MUON DECAY

Muons are injected into the storage ring as a bunch and not all the muons sit on the "magic" radius ($r = 711.2$ cm), of which the corresponding $\gamma \simeq 29.3$ and the third term in Eq. (3) will vanish. Because muons have a radial distribution over the storage aperture, the knowledge of the momentum/radius distribution is critical to estimate the electric field correction to the anomalous spin precession frequency.

Muons at the "magic" radius travel around the ring in about 149 ns. Muons having higher or lower momentum will have larger or smaller radii and thus slightly different

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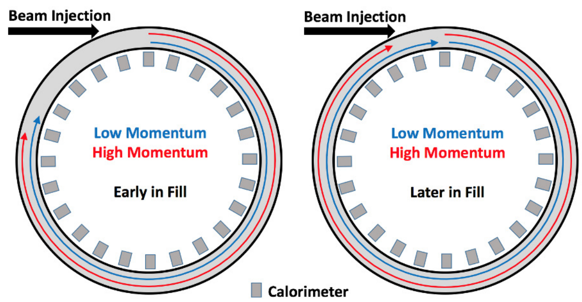

Figure 1: Beam debunching in the storage ring

periods. Those muons at inner equilibrium radii will steadily move ahead of those at outer equilibrium radii. This causes beam debunching, as shown in Fig. 1. The debunching process will give rise to the beam overlap in the late time.

As muons (μ^+) go around the storage ring, they will decay into positrons and neutrinos ($\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e$). Because of the parity violation, there exists a correlation between the muon spin and decay electron direction. This correlation allows the spin direction to be measured as a function of time [7]. By applying an energy-cut, positrons in a range of angles in the muon rest frame can be selected and used for the study of the anomalous muon spin precession. An example of simulated decay positron signal from the ideal beam injection is shown in Fig. 2.

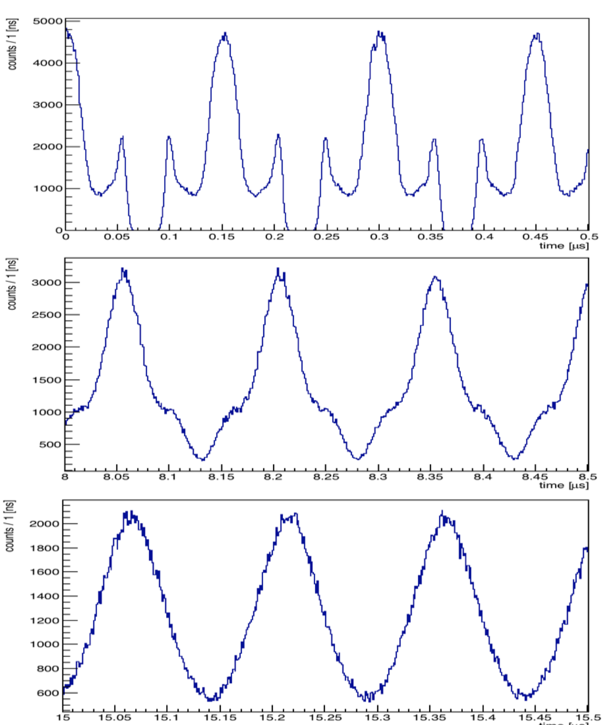

Figure 2: Example of simulated decay positron signal from the ideal beam injection at different times: beam bunches overlap at late times.

FAST ROTATION ANALYSIS

The so called *Fast Rotation Analysis* (FRA) is a technique for measuring the muon radial momentum distribution that uses the time evolution of the bunch structure. Muons in a bunch with momentum $p > p_{magic}$ will naturally assume higher orbits ($r > r_{magic}$), which take longer to complete one cyclotron revolution. The bunch will stretch and eventually the lower momentum muons will lap with the higher momentum muons in the same bunch. The stored muon momentum/radial distribution can be determined by analyzing the debunching beam as discussed in Ref. [11].

The technique of using beam debunching to extract the muon momentum/radial distribution was introduced in the CERN III $g-2$ Experiment [12] and also used in the BNL $g-2$ Experiment [7]. The momentum distribution can be also studied by applying the Fourier Transform analysis on the decay positron time histogram, as discussed in Ref. [13].

For the analysis here, we consider the contributions from the muon radial contents to the decay positron time bin contents. The expected decay positron count C_j for the jth time bin is given by

$$C_j = \sum_{i=1}^{I} f_i \beta_{ij} \tag{5}$$

where i is the radial momentum bin index, I is the total number of radial bins, f_i is the unknown content (fraction) of muons in the ith bin, and β_{ij} are the muon temporal-radial evolution coefficients. Then we can define a χ^2-function

$$\chi^2 = \sum_j \frac{(N_j - C_j)^2}{Z_j} = \sum_j \frac{(N_j - \sum_i f_i \beta_{ij})^2}{Z_j} \tag{6}$$

where Z_j is the weighting factor that should be equal to the expected C_j and N_j is the decay positron count in the jth bin. By minimizing the χ^2, we can resolve for the radial content f_i [8].

To calculate the geometry factors β_{ij}, we need to know the injection pulse time distribution, the time that the bunch injected into the ring (t_0) and the beam bunch revolution time (T_C). The injection pulse time distribution can be rebuilt from the realized decay positron bunch shape at the earliest times. The injection zero time (t_0) and bunch revolution time can be found by fitting the arrival time of bunches by number of turns: $t_{bunch} = nT_C + t_0$, as shown in Fig. 3. Here, the bunch arrival time t_{bunch} can be represented by the bunch peak time, which can be found by fitting a Gaussian function to the middle part of each bunch. The injection pulse temporal distribution and injection zero time are critical for extracting the muon momentum distribution because these determine the beam debunching in the storage ring.

Once we know the geometry factors β_{ij}, we can apply the χ^2 minimization analysis and solve for the momentum radial bin contents. As an example, Fig. 4 shows the preliminary equilibrium radius distribution of the muons found by the fast rotation analysis on the simulated signal shown in Fig.

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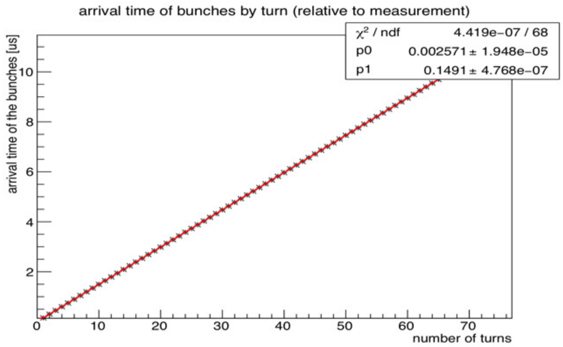

Figure 3: Fit the arrival time of bunches by number of turns.

2, where the beam injection is ideal and the magnetic field is uniform.

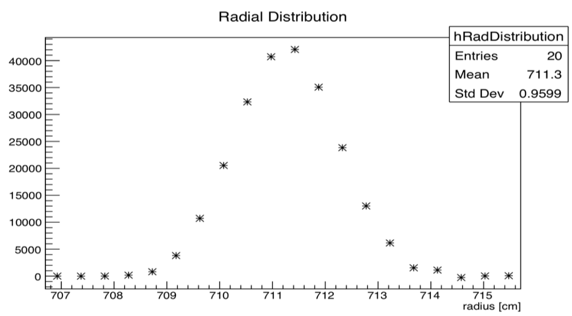

Figure 4: Muon equilibrium radius distribution (preliminary) obtained by the fast rotation analysis on simulated signal in Fig. 2 with ideal beam injection and perfect magnetic field.

For the fast rotation analysis in the experiment, the decay positron time histogram after applying an energy-cut usually contains many additional features such as those due to muon lifetime, the g-2 frequency (ω_a), and muon losses. Thus, we normally consider a weighted decay positron signal after dividing the original signal by a fitted function. For example, we can fit the decay positron signal with a "wiggle"-function: $N_e(t) = N_0 e^{-t/\gamma\tau}[1+A\cos(\omega_a t+\phi_a)]$ [8]. We then remove effects of the muon lifetime and the g-2 frequency to the fast rotation analysis by considering a weighted signal of $S(t)_{weighted} = S(t)/\{e^{-t/\gamma\tau}[1 + A\cos(\omega_a t + \phi_a)]\}$, where $S(t)$ is the original signal.

The equilibrium radius distribution can be used for the electric field correction to the anomalous muon spin frequency as shown in Eq. (3). After several mathematical steps, one can get

$$\langle\frac{\delta\omega_a}{\omega_a}\rangle = -2\beta^2 n(1-n)\langle(\frac{x_e}{R_0})^2\rangle \quad (7)$$

where $\beta = v/c$, x_e is the deviation of the mean equilibrium radius from the magic radius R_0 and $n = \frac{R_0}{v_z B}\frac{\partial E}{\partial R}$ is the field index [7].

CONCLUSION

The Fermilab Muon g-2 Experiment is running for its first physics data taking. The time evolution of the beam bunch structure is really useful to study the beam dynamics. The beam debunching can be explored in the fast rotation analysis and can be used to extract the muon momentum distribution as shown in this work. With the improvements on many techniques, the Fermilab Muon g-2 Experiment will test the SM by measuring the muon anomaly to an unprecedented accuracy.

ACKNOWLEDGMENT

This document was prepared by the Muon g-2 Collaboration using the resources of the Fermi National Accelerator Laboratory (Fermilab), a U. S. Department of Energy, Office of Science, HEP User Facility. Fermilab is managed by Fermi Research Alliance, LLC (FRA), acting under Contract No. DE-AC02-07CH11359. The authors thank the organizers of this conference and our E989 Collaborators. This manuscript has been authored by employees of The University of Mississippi supported by the U.S. Department of Energy Office of Science, Office of High Energy Physics, award DE-SC0012391.

REFERENCES

[1] P. A. M. Dirac, "The quantum theory of the electron," Proc. R. Soc. A (London) **117**, 610, 1928.

[2] P. A. M. Dirac, "The quantum theory of the electron. part II," Proc. R. Soc. A (London) **118**, 351, 1928.

[3] J. E. Nafe, E. B. Nelson, and I. I. Rabi, "The hyperfine structure of atomic hydrogen and deuterium," Phys. Rev. **71**, 914, 1947.

[4] D. E. Nagle, R. S. Julian, and J. R. Zacharias, "The hyperfine structure of atomic hydrogen and deuterium," Phys. Rev. **72**, 971, 1947.

[5] C. Patrigani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001, 2016.

[6] A. Keshavarzi, D. Nomura and T. Teubner, "The muon $g-2$ and $\alpha(M_Z^2)$: a new data-based analysis," arXiv preprint, arXiv:1802.02995, 2018.

[7] G. Bennett, *et al.*, "Final report of the E821 muon anomalous magnetic moment measurement at BNL," Phys. Rev. D **73**, 072003, 2006.

[8] J. Grange, *et al.*, "Muon (g-2) technical design report," arXiv preprint, arXiv:1501.06858, 2015.

[9] J. D. Jackson, *Classical Electrodynamics*, Third Edition, New York, NY, USA: Wiley, 1999, pp. 565.

[10] J. D. Crnkovic, *et al.*, "Commissioning the muon $g-2$ experiment electrostatic quadrupole system," IPAC-2018-WEPAF015.

[11] W. Wu and B. Quinn, "Beam dynamics in g-2 storage ring," IPAC-2017-MOPIK119.

[12] J. Bailey, *et al.*, "Final report on the CERN muon storage ring including the anomalous magnetic moment and the electric dipole moment of the muon, and a direct test of relativistic time dilation," Nucl. Phys. B **150**, 1, 1979.

[13] Y. Orlov, C. S. Ozben, and Y. S. Semertzidis, "Muon revolution frequency distribution from a partial-time Fourier transform of the g-2 signal in the muon g-2 experiment," Nucl. Instrum. Meth. **A482**, 767, 2002.