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ELECTRON BEAM SCANNING IN THE DELTA-TYPE UNDULATORS FOR SIRIUS

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Abstract

We report on simulation studies to analyze the possibility of scanning the electron beam, and not scanning the sample, in CDI experiments using a Delta-Type undulator in the 3 GeV Sirius electron storage ring presently under construction at LNLS. This would allow much faster scans in diffraction limited storage rings such as Sirius. We study displaced beam trajectories through the undulators and analyze the effects on the emitted radiation. It is possible to show that displacements on the order of +/- 500 micrometers around the center will introduce variations in the radiation spectrum that are less that 1 per cent and thus acceptable for Coherent Diffraction Imaging experiments.

INTRODUCTION

The possibility of scanning the electron beam inside the undulators opens up space to accelerate data acquisition in image reconstruction techniques based on CDI (Coherent diffractive imaging), since it allows to scan the samples faster than using mechanical movement of the sample. However, changing the position in which electrons pass through the undulator implies changing the emitted radiation.

We will present the studies made in the displaced beam trajectories inside the Delta-type undulator modeled in the study described in [1], in order to show the main effects in the radiation and what magnitude they occur, so that it can be estimated the size of the region the beam could be moved in order that the data collected in the image reconstruction will not be significantly affected. The simulations in this study use Mathematica software (Ref. [2]) with the Radia computational package (Ref. [3]) for the calculation of the magnetic field of the undulators and the trajectories of the electron beam.

DEVELOPMENT

For this study, we have analyzed the characteristics of the electrons trajectory in an ideal undulator and the emitted radiation in this situation. Since the electrons are ultra-relativistic ($E = 3\,\text{GeV}$) and there are only undulators in straight sections of the ring, according to [4] and [5], we can describe the trajectory inside undulator with a magnetic induction field $\vec{B} = \vec{B}(\vec{r})$ by the equation

$$\vec{r}'' - \frac{ec}{E}\vec{B} \times \vec{r}' = 0$$

where $\vec{r}(s)$ is the trajectory of the electron parameterized by the length of the arc (s) of the ring and $\vec{r}'$ is the differentiation $d\vec{r}/ds$, e is the elementary charge and c is the speed of light.

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Thus, with Frenet coordinate system $\{\hat{x}, \hat{y}, \hat{s}\}$, we have

$$\begin{cases} x'' = \frac{ec}{E} B_y(x, y, s) \\ y'' = -\frac{ec}{E} B_x(x, y, s) \end{cases}$$

Near $x = y = 0$ we can approximate the field as a function of only s. In this situation, when we solve these equations,

$$x(s) = x(0) + x'(0)s + \frac{ec}{E}\int_0^s \int_0^{\bar{s}} B_y(s')ds'd\bar{s}$$

$$y(s) = y(0) + y'(0)s - \frac{ec}{E}\int_0^s \int_0^{\bar{s}} B_x(s')ds'd\bar{s}$$

inside the undulator the fields are designed so that when the beam passes through the center of the undulator with a zero angle it is not deflected neither displaced by the undulator. This means that by construction,

$$\int_0^L \int_0^{\bar{s}} \vec{B}(0, 0, s')ds'd\bar{s} = 0 \text{ and } \int_0^L \vec{B}(0, 0, s')ds' = 0$$

being L the length of the undulator. When the trajectory passes outside the center, the analytical approach used was to consider that the field is practically the same, but added to a uniform field $\vec{B}_0$ which depends only on the initial position of the beam. So,

$$x(s) = x(0) + x'(0)s + \frac{ec}{2E}B_{0y}s^2 + \frac{ec}{E}\int_0^s \int_0^{\bar{s}} B_y(s')ds'd\bar{s}$$

$$y(s) = y(0) + y'(0)s - \frac{ec}{2E}B_{0x}s^2 - \frac{ec}{E}\int_0^s \int_0^{\bar{s}} B_x(s')ds'd\bar{s}$$

In general, the term coming from the integral is oscillatory and responsible for the undulators radiation, while the constant term due to the transverse initial offset leads to a parabolic trajectory. Applying the conditions of null integrals, the result is that now the undulator has a total effect on the electrons,

$$\begin{cases} \Delta x = x'(0)L + \frac{ec}{2E}B_{0y}L^2 \\ \Delta y = y'(0)L - \frac{ec}{2E}B_{0x}L^2 \end{cases} \text{ and } \begin{cases} \Delta x' = \frac{ec}{E}B_{0y}L \\ \Delta y' = \frac{ec}{E}B_{0x}L \end{cases}$$

These angles and positions variations caused by the constant field term must be corrected in order to close the bump inside the undulator. To do that we will use two correctors placed near the undulator to compensate the field integrals. In the first corretor, we choose a value of $x'(0)$ and $y'(0)$ so that $\Delta x = \Delta y = 0$,

$$x'(0) = -\frac{ec}{2E}B_{0y}L \text{ and } y'(0) = \frac{ec}{2E}B_{0x}L$$

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where $\vec{B}_0$ can be obtained using an iterative algorithm to recalculate the trajectory with the new initial angle $\vec{r}'(0)$ from the data of $\vec{r}'(0)$ and $\vec{r}'(L)$ of the previous trajectory:

$$\vec{r}'_{i+1}(0) = \frac{1}{2}[\vec{r}'_i(0) - \vec{r}'_i(L)]$$

In this situation, the parabolic trajectory is symmetric and therefore the beam leaves the undulator with angle

$$\begin{cases} x'(L) = -x'(0) \\ y'(L) = -y'(0) \end{cases}$$

then the second corrector is used at the end of the undulator to cancel the residual angle satisfying the condition of not displacing the beam. It was necessary to study the magnitude of the curvature caused by the parabolic trajectory, since the shape of the trajectory can change the interference pattern of the emitted radiation.

As shown in [6], the radiation power is proportional to the square of the acceleration, thus the quadratic mean of the beam angle along the trajectory is a parameter that measures the characteristics of the radiation, which defines the parameters K_x and K_y, present in [7], as

$$K_x = \sqrt{\frac{2\gamma^2}{L}\int_0^L x'(s)^2 ds} \text{ and } K_y = \sqrt{\frac{2\gamma^2}{L}\int_0^L y'(s)^2 ds}$$

Based on these equations, to make the proposed analysis we use the Radia computational package that solves the equations of motion with the Runge-Kutta method and allows us to use an undulator model that had already been implemented by the group using the same computational package, published in [7].

Then, another important parameter is to verify the deviation that occurs in the parameters K_x and K_y of the beam, since they determine the wavelengths of undulator harmonics in spectrum radiation. The deviation map of K as a function of the electron beam displacement provides a method to determine the maximum scanning region for the electron beam.

RESULTS AND DISCUSSIONS

Figure 1 show the result of the trajectory when we move the beam from the center of the undulator. The direction y represents the azimuth direction, x is the radial direction and z the direction perpendicular to the ring plane. The orange curve is the trajectory when the beam passes in the center of the undulator and the blue curve when the beam passes outside the center of the undulator. The left axis shows the difference between the final and the initial position of the beam in the center trajectory and to the right this same parameter in the trajectory displaced. All distances in the graphs are given in millimeters.

From these results we can see that, without adjusting the initial angle, the trajectory doesn't have the symmetry that allows us to do a closed-bump inside the undulator.

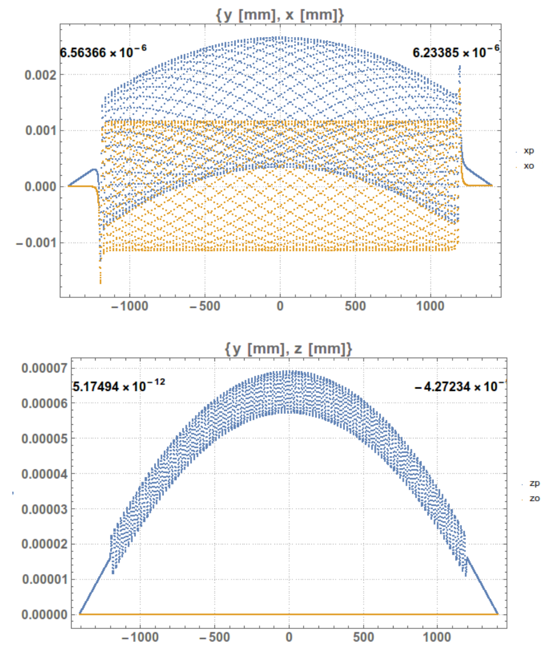

Figure 1: Trajectory with angles correction for $x = 0.3$ mm and $z = 0.2$ mm compared to the central orbit in the vertical polarizationoftheundulatorDelta21(See[8]).

Besides, it is possible to observe that the displaced trajectory introduces an oscillation in the direction perpendicular to the plane of oscillation, which can not be explained by the model developed in the previous section, where a constant field was added to account for the displacement. However, this deviation is very small, only two orders o magnitude.

Figure 2 shows the map of the kicks that are needed to correct the orbit, depending on the displacement that was given in the beam, respectively, for the vertical and circular polarization of the Delta21 undulator (See [8]). Notice that the kicks are on the order of micro-radians in the main direction of oscillation and on the order of 0.1 μrad in the direction perpendicular to the plane of oscillation. Additionally, the radius of curvature to the parabolic orbits are at least on the order of 100 km, with an equivalent field of at most 20 μT, that is the same order of the earth's magnetic field. Thus, it is safe to assume that this curvature will be imperceptible in experiments. In addition, the maps preserve the symmetry of the undulator polarization and show that the farther from the center, more the stronger is the kick needed to correct the parabolic trajectory.

Figure 3 shows the map for deviation in the undulator parameter K. The graph upper refer to the light emitted with horizontal polarization (K_x) and the bottom graph to the light emitted with the vertical polarization (K_z). In this case, where the magnetic field is only vertical, the values of K_x are an order greater than K_z. In terms of deviation, we see that K_x deviates less than 1% from its nominal value inside a region of radius 0.4 mm.

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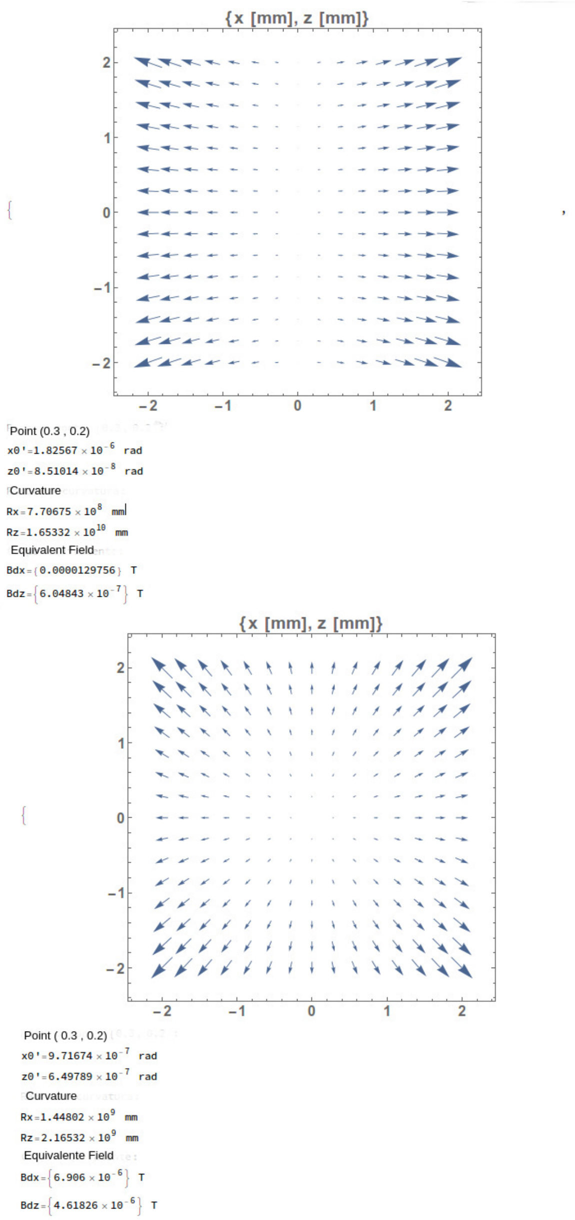

Figure 2: Map of correction kicks and numerical values for the point $x = 0.3$ mm and $z = 0.2$ mm in the vertical and circular polarization of the Delta21 (See [8] respectively.

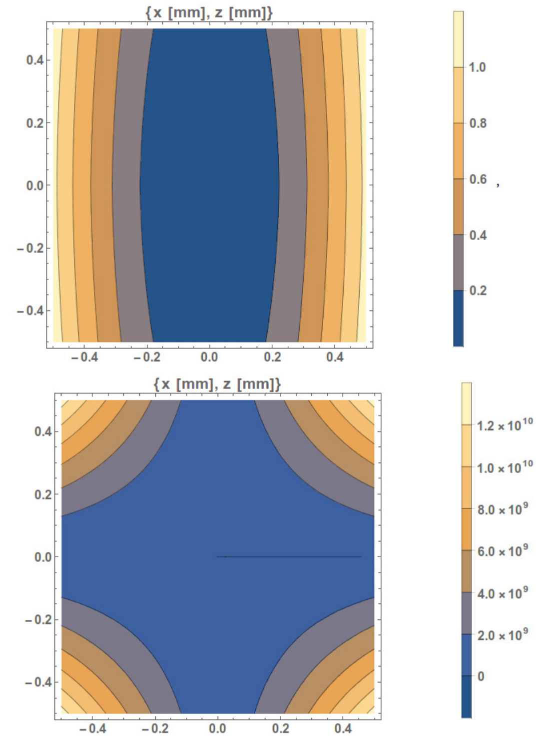

Figure 3: Map of values of K_x and K_z in values relative to the deviation of the value in the central trajectory, in percentage, in the vertical polarization of Delta21 (See [8]).

Figure 4 shows an analogous map for the circular polarization. In this case there is a directions and values of K_x are on the same order of magnitude as K_z.

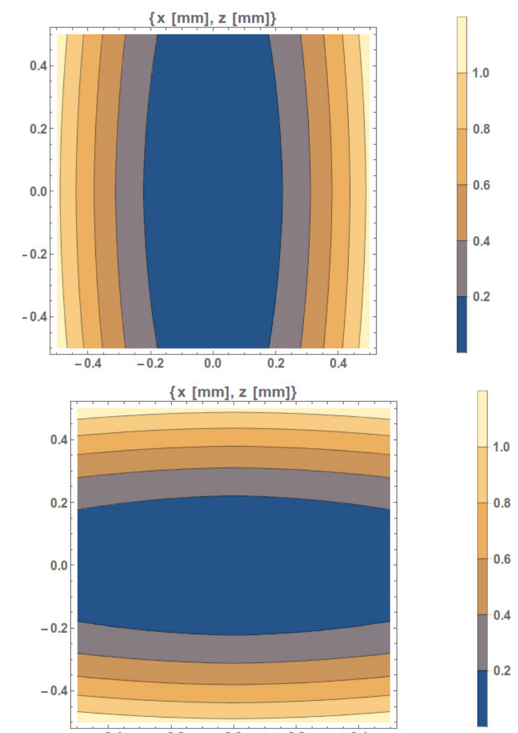

Figure 4: Map of values of K_x and K_z in values relative to the deviation of the value in the central trajectory, in percentage, for the circular polarization of Delta21 (See [8]).

CONCLUSIONS

The curvature that appears in the trajectory of the beam is on the order of curvature caused by the earth's magnetic field, which means that it should have little influence on the emitted radiation, but specifics simulations of radiation need to do yet to complete that the influence is really little. Still, using 1% tolerance in the values for K_x and K_z, the beam can be displaced from the center of up to 400 μm.

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