APPLICATION OF SVD ANALYSIS TO DEFLECTING CAVITY SPACE HARMONICS*

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Abstract

Singular value decomposition (SVD) analysis is a powerful tool for identifying different spatial and timing variation patterns in many fields of researches. Recently we applied complex SVD method to space harmonic analysis of a 13-cell defecting cavity that is built and installed in the APS linac injector for beam phase space characterization and emittance exchange experiments. Real and imaginary space harmonics components are extracted from CST simulated data. Fields inside the iris were expressed in analytic forms and produced good agreement. Work is underway to implement the results into elegant simulation model.

INTRODUCTION

SVD analysis [1] is a powerful tool for identifying various system spatial and timing variation patterns. Recently we applied complex SVD analysis to extract space harmonics from CST simulated deflecting cavity data. We were able to extract the real and imaginary parts of the space harmonics, from which the field inside the iris can be expressed analytically, which can be used to develop simplified model for beam simulations of deflecting cavities.

A BRIEF DESCRIPTION OF SVD ANALYSIS

A system matrix consists of m spatial columns, each contains n equal samples of time or another dimension.

$$A=\begin{bmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1m} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2m} \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ A_{n1} & A_{n2} & \cdots & \cdots & A_{nm} \end{bmatrix}$$

SVD analysis converts A into U, V and Σ matrices:

$$A=U^T\Sigma V$$
$$U=\begin{bmatrix} U_1 & U_2 & \cdots & U_m \end{bmatrix}$$
$$V=\begin{bmatrix} V_1 & V_2 & \cdots & V_n \end{bmatrix}$$

$$\Sigma=\begin{bmatrix} \lambda_1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda_2 & 0 & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \lambda_m \\ 0 & \cdots & \cdots & \cdots & 0 \end{bmatrix}$$

Here λ_1... λ_m are eigenvalues and each of the vector of U and V matrices are spatial and timing vectors corresponding to eigenvalues.

For analysis of cavities the field components are phasors and are generally expressed in complex values. For a complex matrix A, we construct a matrix AA that consists of 2×2 block matrices from a construction matrix Q [2]:

$$Q=\begin{bmatrix} 0 & -I \\ I & 0 \end{bmatrix}$$
$$Q*Q=-1$$

$$A=A_{\Re}+i\,A_{\Im}$$

$$AA=\begin{bmatrix} A_{\Re} & -A_{\Im} \\ A_{\Im} & A_{\Re} \end{bmatrix},$$

It can be shown [4] that the U- and V-matrices of AA and A are related by:

$$AA=\begin{bmatrix} U^{\Re} & -U^{\Im} \\ U^{\Im} & U^{\Re} \end{bmatrix}\begin{bmatrix} \Sigma & 0 \\ 0 & \Sigma \end{bmatrix}\begin{bmatrix} V^{\Re} & -V^{\Im} \\ V^{\Im} & V^{\Re} \end{bmatrix}^T$$

All singular values, U and V matrices of AA appear in pairs. Each of the two vectors in a pair is a sign and phase change of the other. Only one of the pair is necessary for SVD analysis. Both spatial and time-domain vectors can be split into real and image half and recombined back to complex vectors.

APPLICATION TO SPACE HARMONIC ANALYSIS OF A DEFLECTING CAVITY

Figure 1 shows deflecting cavity model with $2\pi/3$ phase advance mode [3]. The field can be separated in two regions: region I is inside of the iris (r < a); region II is outside of the iris (b > r > a). Because of the periodic structure the field in region I can be generally expressed as [4] with the $e^{-j\omega t}$ term removed for simplification:

$$E_z(r,\phi,z)=\sum_{-\infty}^{\infty} A_n \frac{J_m(\gamma_n r)}{J_m(\gamma_n a)}e^{-jk_n z}e^{-jm\phi} \quad ,$$

where:

$$K_n=\frac{2\pi}{\lambda}+\frac{2\pi n}{w}$$
$$\gamma_n^2+k_n^2=k_0^2 \quad .$$

Here m = 1 for deflecting mode, γ_n are the r direction wave number, and if it is imaginary the corresponding Bessel function terms become modified Bessel function.
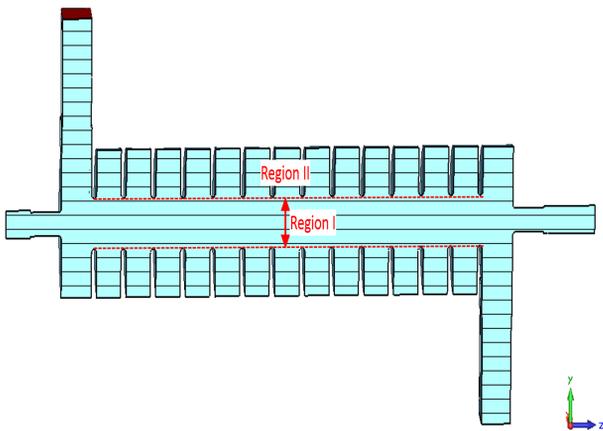
Figure 1: A simulation model of deflecting cavity.

For a periodic disk-loaded structure fields in the region I and region II can be uniquely determined by the field at the iris. In order to express the CST simulation data into finite-term analytic expressions we export the field on the surface of r = a_0 (a_0 <= a), and apply SVD analysis to extract complex space harmonic coefficients A_n.

The input matrix is expressed as following:

$$A=\begin{bmatrix} A_{11} & A_{12} & A_{13} & \cdots & A_{1m} \\ A_{21} & A_{22} & A_{23} & \cdots & A_{2m} \\ \vdots & \cdots & \cdots & \cdots & \vdots \\ A_{n1} & A_{n2} & \cdots & \cdots & A_{nm} \end{bmatrix} ,$$

with each row represents one ϕ angle value while each column represents one z value, both are equal spaced.

Because the E_z is complex, A is a complex matrix. In order to apply SVD analysis we first construct a 2×2 block matrix:

$$AA=\begin{bmatrix} A_{\Re} & -A_{\Im} \\ A_{\Im} & A_{\Re} \end{bmatrix},$$

We apply SVD pseudo-inverse to obtain Sigma, U- and V-matrix. Each of the S-U-V triplet represents a mode. Figure 2 shows a plot of the singular values. The singular values are in pairs. They are actually image of each other and we only need to count one of the pairs. The first few V-vectors are plotted in Figure 3. These vectors represent angular spatial modes. In particular SV0000 to SV0003 show a clear cosine/sine dependency on ϕ, a characteristic of deflecting mode. Figure 4 shows the first four U-vectors, which represent z dependency.
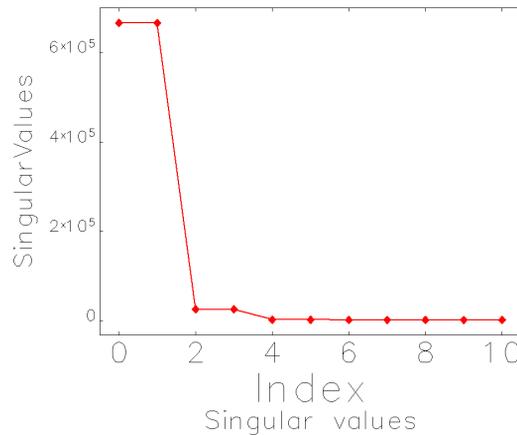
Figure 2: First ten singular values of the SVD analysis.
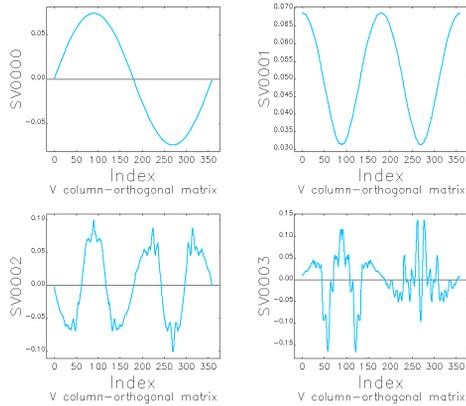
Figure 3: V-vector 00 to 03, which represents the fundamental and high harmonics in azimuthal space.
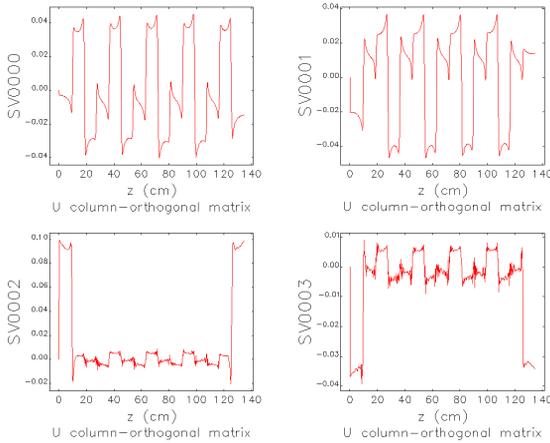
Figure 4: U-vectors 00 to 03 that represent longitudinal space harmonics in z.

For SV0000 mode we processed its amplitude, phase and FFT. Figure 5, 6 show the results.
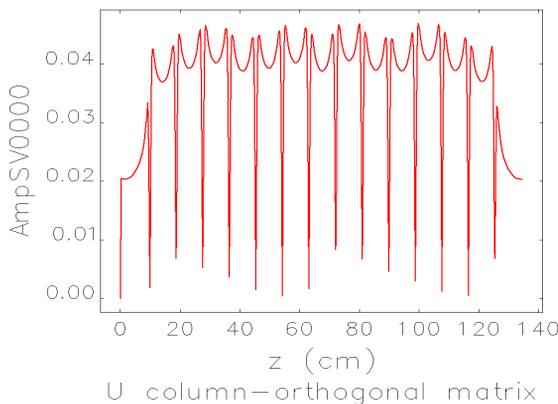
Figure 5: Amplitude waveform of mode 00 u-vector.

Figure 7 shows the amplitude of the space harmonics in region I of the deflecting cavity. There are only a dozen or so non-zero space harmonics. Once the A_n coefficients are known any field component can be expressed analytically for such purposes as particle tracking and deflecting angle evaluations, etc. For tracking of deflecting cavity we need only to use the m=1 mode:

$$E_z(r,\phi,z) = \sum_{-N}^{N} A_n \frac{J_1(\gamma_n r)}{J_1(\gamma_n a)} e^{-jk_n z} e^{-j\phi} \quad .$$

CONCLUSIONS

We applied SVD analysis to the deflecting cavity simulation data analysis, and obtained the space harmonic coefficients, which can be used to express the electric field analytically in beam simulation and calculation of deflecting angles.
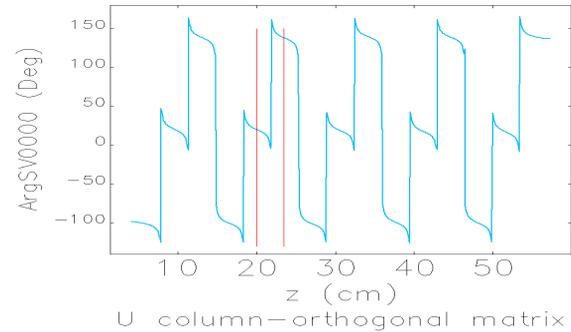
Figure 6: Phase waveform of mode 00 u-vector. The phase advance per cell is $2\pi/3$ as expected of the fundamental mode. The red line marks a cell boundary on which the phase shift is close to expected 120°.
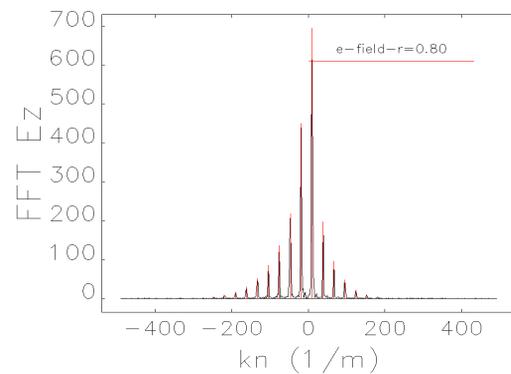
Figure 7: Space harmonics of the $2\pi/3$ mode of a deflecting cavity. Red line is the spectrum while the black lines mark the peaks. The lower harmonics are folded to negative K_n for presentation purpose. Positive and negative K_n represents backward and forward harmonics, respectively.

ACKNOWLEDGMENTS

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