

SIMULATION OF INTEGRABLE SYNCHROTRON WITH SPACE-CHARGE AND CHROMATIC TUNE-SHIFTS

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Abstract

We present a nonlinear rapid-cycling synchrotron designed as a high-intensity replacement of the Fermilab Booster. The design incorporates integrable optics, an innovation in particle accelerator design that enables strong nonlinear focusing without generating parametric resonances. We use the Synergia space-charge tracking code to demonstrate the stability of a beam in this lattice with a space-charge tune-shift up to 0.4 and a rms momentum spread up to 0.4%. We demonstrate the benefit of increased lattice periodicity.

BACKGROUND

Integrable optics is a development in particle accelerator technology that enables strong nonlinear focusing without generating new parametric resonances [1]. A promising application of integrable optics is in high-intensity hadron rings, where nonlinearity is known to suppress halo formation [2,3] and enhance Landau damping of charge-dominated collective instabilities [4]

In this work we present a new integrable lattice design for a rapid-cycling synchrotron designed to replace the Fermilab Booster as part of a multi-MW upgrade of the Fermilab proton complex [5, 6]. We perform two simulation studies indicating that this integrable lattice is robust against realistic space-charge and chromatic tune-shifts.

INTEGRABLE LATTICE & DESIGN UPDATES

In [1] a procedure for integrable accelerator design is derived based on an alternating sequence of linear and nonlinear sections. The linear-sections, referred to as T-inserts, are arc sections with π -integer betatron phase-advance in the horizontal and vertical plane. The lattice should be dispersion-free in the nonlinear section, and the horizontal and vertical beta functions should be matched. A special nonlinear elliptic potential magnet is matched to the beta functions to provide the nonlinear focusing. The manipulation of the beta functions and phase-advances removes the time-dependence of the nonlinear kick so as to avoid introducing parametric resonances.

In [6] we introduced a specific design of an integrable rapid-cycling synchrotron (iRCS) that meets the essential single-particle requirements of an iRCS - periodicity, bounded beta function, low momentum compaction factor, long dispersion-free drifts, and Danilov-Nagaitsev integral accelerator design.

In this work, we produce a new iRCS lattice design 'iRCSv3' which improves on the previous lattice design in several ways - it doubles the periodicity from 6 to 12, reduces

the beta function from 35 m to 30 m, and reduces the momentum compaction factor. The circumference is increased 16%, the sextupoles were removed, and the chromaticity per periodic cell is roughly the same (i.e double overall). The ring circumference is compatible with a five-batch injection into the Fermilab Main Injector.

Figure 1 shows latest version of the iRCS lattice and Table 1 shows the key parameters of the lattice.

The lattice can be optimized to anticipate linear space-charge tune depression - i.e. the phase advances between the nonlinear inserts are correct for particles in the beam core at a specific beam intensity.

The phase advance through the nonlinear insert of $Q_0 = 0.3$, the nonlinear strength parameter is $t = 0.3$, and elliptic potential parameter is $c = 0.14 \text{ m}^{1/2}$. (see [1, 7]).

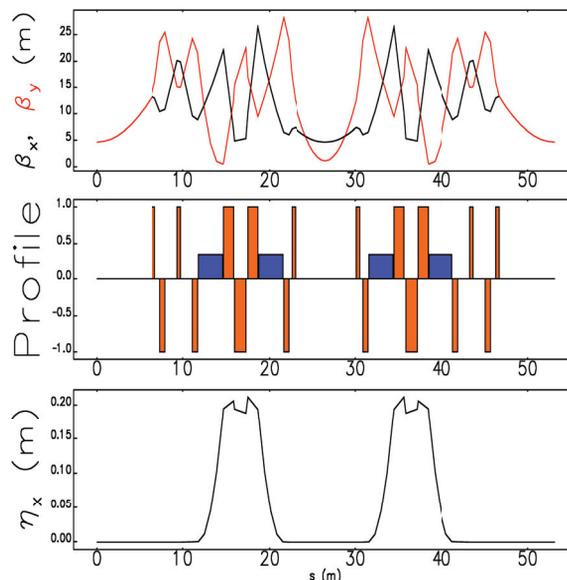


Figure 1: Twiss parameters for one of the twelve periodic cells. (top) Horizontal and vertical beta functions shown in black and red, respectively. (middle) Location and length of magnetic lattice, elements where dipoles are shown as short blue rectangles and quadrupoles as tall orange rectangles. (bottom) Linear dispersion function.

SIMULATION PARAMETERS

We use Synergia to calculate 6D particle tracking for nonlinear optics. Synergia is a Python-based parallel code for multiparticle tracking with imported CHEF functionality [8]. There is an ongoing effort to simulate the IOTA lattice using Synergia [3] and IMPACT-Z [9].

Space-charge forces were calculated using the Synergia 2D-Hockney solver, a 2D particle-in-cell method described

Table 1: Parameters of iRCSv3 Lattice

Parameter	Value
Circumference	636 m
Periodicity	12
Bend Radius	15.4 m
Max Beta Function	30 m
Max Dispersion	0.22 m
Betatron Tune	21.6
Linear Chromaticity	-79
Second-Order Chromaticity	1600
Momentum Compaction	5.9×10^{-4}
Insertion lengths per cell	7.2 m, 4×1.3 m
RF Voltage	1.680 MV
Synchrotron Tune	0.08
NL Insertion Length	12.7 m
Phase-advance over insert	0.3
Nonlinear Strength t-value	0.3
Elliptic Distance c-value	$0.14 \text{ m}^{1/2}$
95% Transverse Emittance	20 mm mrad
95% Longitudinal Emittance	0.09 eV·s
Vertical Lattice Tune Spread	0.52
Horizontal Lattice Tune Spread	0.34
Space-charge Tune Spread	0.20
Chromatic Tune spread	0.52

in [10] and [11]. For this work we used 8 steps per element, 32×32 space-charge grid, map order 6, and 40,960 macroparticles. The simulation did not include an acceleration of the beam energy.

HALO SUPPRESSION IN SPACE-CHARGE DOMINATED BEAM

For the integrable RCS to serve as a high-intensity replacement for the Fermilab Booster, it should be stable under a Laslett tune-shift of 0.4 corresponding to a ring intensity of 32×10^{12} protons [6]. The number of integrable cells is a critical design feature, as it reduces the tune-shift per integrable periodic cell to only 0.033.

The strong nonlinear focusing provided by the integrable lattice should actively suppress the formation of space-charge driven beam halo. An important source of transverse beam halo is described by the particle-core model [12] in which breathing mode oscillations of a uniform-density beam cores can drive particles into the halo. In [3], a KV beam with 99% of the particles mismatched rapidly drives the remaining 1% particles into the halo.

To investigate the susceptibility of the beam to halo formation, we populate a beam with a generalized transverse Waterbag distribution following the equipotential contours of the nonlinear potential [13]. Then we introduce a 20% beam mismatch into 98% of a beam and observe the equilibration. We find the beam is stable and that halo is suppressed by the presence of the nonlinear optics. For this study focused on the transverse space-charge dynamics, the beam is mono-energetic and unbunched.

Figure 2 shows the horizontal particle distribution over time, for the space-charge simulation. The beam rapidly equilibrates, halo formation is suppressed and the beam distribution remains stable. The vertical plane is similarly stable, and the simulations have been conducted out to 5000 revolutions to verify the continued beam stability.

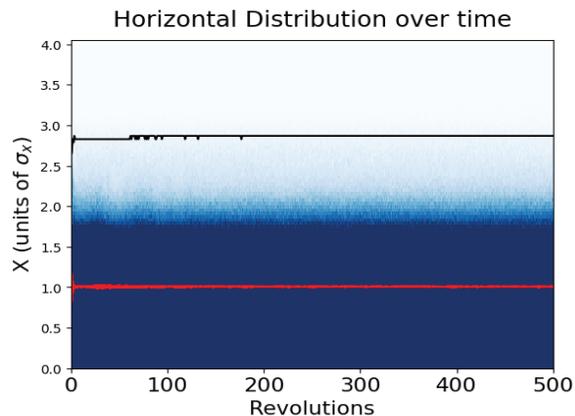


Figure 2: Horizontal particle distribution over time for a space-charge driven beam in the integrable RCS lattice. The color axis is scaled to express the variation in the halo density instead of the full density range. The black line indicates the 99.9th percentile of the beam (40 macroparticles) and the red line indicates the rms beam size relative to its initial value.

Figure 3 shows the tune distribution across one periodic cell (1/12 of the iRCS ring) for the space-charge simulation. The main-diagonal tune-spread of the beam is driven by space-charge and the off-diagonal tune-spread of the beam is driven by the nonlinearity of the lattice. The nonlinear insert also has a quadrupole component which has shifted the operating point off the main diagonal.

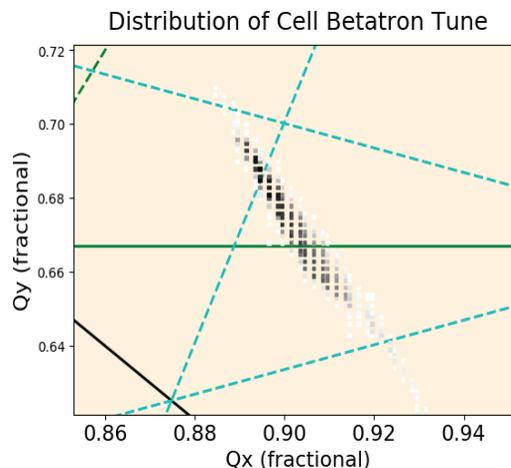


Figure 3: Betatron tune diagram across one periodic cell for a space-charge driven beam in the integrable RCS lattice. Darker points indicate greater density of particles.

STABILITY OF SYNCHROTRON OSCILLATIONS

It is an unresolved question whether chromaticity-correcting sextupoles can be accommodated in integrable lattice designs. Consequently it is important to demonstrate the stability of beams with a realistic momentum spread in integrable lattices with significant uncorrected chromaticity.

Recent work shows that integrability is preserved to lowest-order if the horizontal and vertical chromaticity are matched and the momentum varies only adiabatically (synchrotron tune per periodic cell much less than one) [14].

Our iRCS lattice is uniquely situated to validate the stability of synchrotron oscillations under chromaticity matching conditions. The horizontal and vertical tunes were matched to a precision of 0.01 across the momentum span of $\pm 0.5\%$. For an off-momentum particle of 0.4%, the chromatic tune-shift is 0.32 with a deviation between the horizontal and vertical tune-shift of 3×10^{-5} . The iRCS lattice has the flexibility necessary to finely adjust the betatron tune-matching and chromaticity matching independently.

For the chromaticity simulation study, there are no space-charge forces. The beam is divided into 9×12 bunches, each with a parabolic longitudinal distribution with a realistic emittance.

Figure 4 shows the horizontal particle distribution over time, for the simulation of the chromatic bunched beam. The vertical plane is similarly stable, and the simulations have been conducted out to 5000 revolutions to verify the continued beam stability.

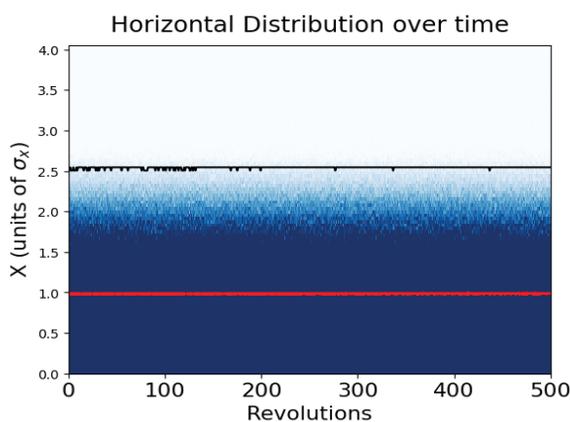


Figure 4: Horizontal particle distribution over time for a chromatic beam in the integrable RCS lattice. The color axis is scaled to express the variation in the halo density instead of the full density range. The black line indicates the 99.9th percentile of the beam (40 macroparticles) and the red line indicates the rms beam size relative to its initial value.

Figure 5 shows the tune distribution across one periodic cell (1/12 of the iRCS ring) for the space-charge simulation. The beam is remarkably stable under the synchrotron oscillations, so long as the tune-spread of the beam does not cross the structural sum resonance at $Q_x + Q_y = 36$ (thick black line in Figure 5).

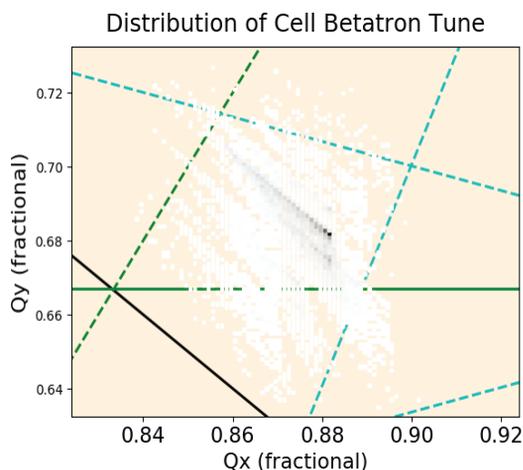


Figure 5: Betatron tune diagram across one periodic cell for a chromatic beam in the integrable RCS lattice. Darker points indicate greater density of particles.

CONCLUSION AND FUTURE WORK

We have provided an updated integrable RCS lattice that demonstrates robust performance under significant space-charge and chromatic effects. The improved periodicity of the lattice design enables the integrable lattice to maintain beam stability under a space-charge tune-shift of 0.4, which corresponds to an RCS beam intensity benchmark for a multi-MW proton facility at Fermilab. A subsequent study may investigate the interaction between chromatic and space-charge effects with a full 3D space-charge simulation of bunched beam.

The natural chromaticity-matching of this iRCS lattice makes it a useful platform to study the implications of chromatic effects in integrable optics. Subsequent tests may investigate stability under stronger chromatic effects and the tolerance required for effective chromaticity matching.

We have also prepared a version of this iRCS lattice with random quadrupole errors that explicitly break the overall symmetry of the lattice. This lattice with errors was then retuned for integrability in order to understand the implications of an integrable lattice composed of aperiodic integrable cells. The performance of this lattice will be used to evaluate the performance of integrable lattices under betatron resonances.

The efficacy of an accelerator design incorporating integrable optics will undergo comprehensive experimental tests at the Fermilab Integrable Optics Test Accelerator (IOTA) [15] and the University of Maryland Electron Ring (UMER) [16] over the next several years.

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