ISBN: 978-3-95450-184-7 doi:10.18429/JACoW-IPAC2018-TUPAF039

ELECTRON COOLING SIMULATION AND EXPERIMENTAL BENCHMARKS AT LEIR

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Abstract

A simulation of the electron cooling has recently been implemented in the tracking code RF-Track. The implementation was based on a "hybrid kinetic" model, where the full 6-D phase space of a particle beam is immersed in a fluid plasma of electrons. The computation of the cooling force was based on analytic models derived using the "dielectric theory" and the "binary collision approximation", numerically integrated to consider the thermal properties of the electrons. This gives the code the flexibility needed to simulate a large variety of realistic scenarios, including imperfections such as gradients in the electron density and hollow electron beam; relative misalignments of electron beam, ion beam, and solenoid field. Benchmarks of the simulations against results in the literature as well as against measurements performed at LEIR using Xenon ions are presented.

INTRODUCTION

Electron cooling is a powerful technique to improve the phase space structure of ion beams in storage rings. In the cooling section, the ion beam is superimposed to a co-moving electron beam, and the thermal exchange between the two beams effectively reduces the phase space area occupied by the ions.

Although electron cooling is routinely used at many facilities, the peculiarity of the interaction between the heavy particles (ions, or antiprotons) and the electron beam makes the modeling of the cooling process particularly difficult. Since the first successful experimental demonstrations of electron cooling [1], several models have been proposed to describe it [2–4], but even if these models capture correctly the phenomenology of electron cooling, the experience proved that it is hard to extrapolate predictions [5].

The underlying description of electron cooling is based on the theory of the interaction between charged particles with matter, developed during the whole last century by e.g. Bohr, Bethe, and Bloch [6–8], with the important difference that, in the case of electron cooling, the medium is a thermal plasma of electrons in a magnetizing solenoid field. Furthermore, due to the acceleration in the electron gun, the thermal velocity distribution of the electrons in their rest frame makes the electron plasma highly anisotropic. The temperature parallel to the electron beam and its magnetic-guiding field is lower than the transverse temperature by some order of magnitude. So, whilst the transverse motion of the electrons is driven by the magnetic field, the very small longitudinal thermal velocity of the electrons causes a strong velocity dependence of the stopping power.

We based our implementation on the expression of the cooling force given in [5], where a detailed theoretical study of this stopping force is carried out using two different and independent formalisms: the "dielectric theory" (a continuum theory in which the response of charge and current densities to external perturbations is calculated), and the "binary collision approximation" (where the motion of the ion is described as the aggregate of subsequent pairwise interactions with the target electrons). After detailed calculations, the book demonstrates that the two approaches yield the same mathematical expressions of the cooling force.

To validate their formulation the authors of reference [5] present a detailed comparison against experimental results and predictions from other models in literature (as in [2,4,9]). Since in many cases their results show better agreement with the experiment than the others, we chose to base our numerical model on their formulation.

THE STOPPING FORCE

The stopping force implemented in the tracking code RF-Track [10] has been based on Eq. (6.1) in [5]. According to this model the cooling force is divided in two contributions which are functions of the impact parameter: the non-magnetized interactions, $\vec{\mathcal{F}}_U$, and the magnetized ones, $\vec{\mathcal{F}}_M$. Both contributions need to be considered simultaneously, as in the presence of a solenoid magnetic field the non-magnetized force describes the interaction at impact parameters smaller than the pitch of the electron helices.

For convenience of calculation the cooling force is typically derived considering the interaction between a single ion and a sea of monochromatic electrons. To take into account the notion that the electrons are thermal, it is necessary to fold the equations of the stopping force with the probability distribution function of the electron velocities, a Maxwell-Boltzmann distribution:

$$f(\vec{v}_e) = \frac{1}{(2\pi)^{\frac{3}{2}}} \frac{1}{\Delta_\perp^2 \Delta_\parallel} \exp - \left(\frac{v_{e_\perp}^2}{2\Delta_\perp^2} + \frac{v_{e_\parallel}^2}{2\Delta_\parallel^2} \right),$$

where $\Delta_\perp$ and $\Delta_\parallel$ are the electrons' transverse and longitudinal thermal velocities; $v_{e_\perp}$ and $v_{e_\parallel}$ are the transverse and longitudinal components of the electrons' velocity. The anisotropic character of electron cooling emerges from the fact that $\Delta_\parallel \ll \Delta_\perp$. The thermal velocities $\Delta_{\perp/\parallel}$ are related to the electron temperatures through $k_B T_{\perp/\parallel} = m_e \Delta_{\perp/\parallel}^2$, where k_B is the Boltzmann constant, and m_e is the electron mass. $T_\perp$ and $T_\parallel$ are respectively the transverse and longitudinal

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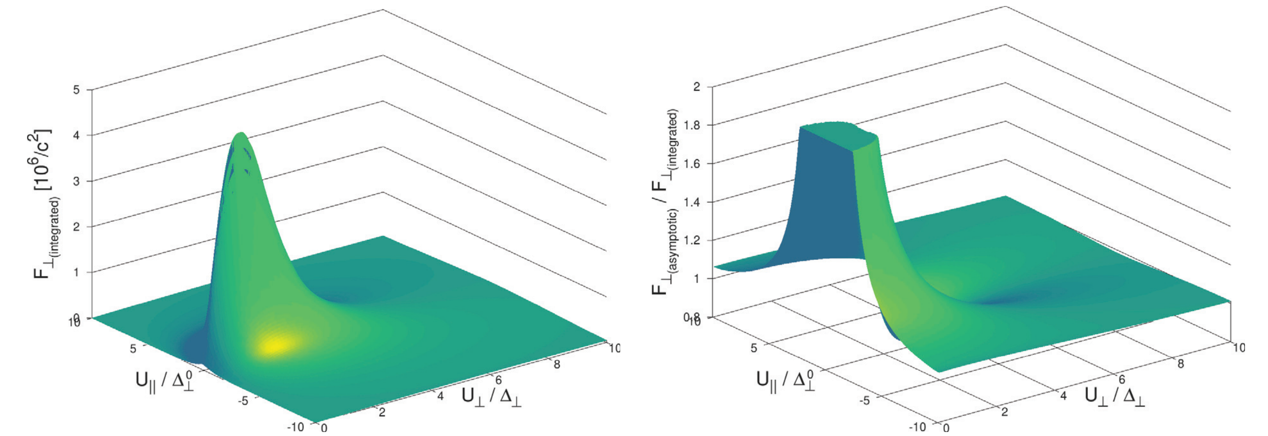

Figure 1: Numerical integration of the unmagnetized radial force, result of the interaction of one ion with a thermal plasma of electrons, over a mesh 160×200. The left-hand-side plot shows the result of the numerical convolution of the single-particle cooling force with the electron velocities distribution, see Eq. (1) in the text. The right-hand-side plot shows the ratio between the asymptotic expression of the force and the integrated one (limited to 2 for clarity of the plot). It is visible that the ratio between the asymptotic and the integrated forces converges to 1 only at very large relative velocities, of the order of 10 times the transverse thermal velocity of the electrons.

temperatures. At LEIR, for example [11], the transverse and longitudinal temperatures are approximatively: $k_B T_\perp = 100$ meV, and $k_B T_\parallel = 1$ meV. The folding between the stopping force and the velocity distribution can be expressed with the two convolution integrals:

$$\left\langle \vec{F}_U \right\rangle = \iiint \left[\vec{\mathcal{F}}_U \right] f\left(\vec{v}_e\right) d\vec{v}_e, \quad (1)$$

$$\left\langle \vec{F}_M \right\rangle = \int \left[\vec{\mathcal{F}}_M \right] f\left(v_{e\parallel}\right) dv_{e\parallel}. \quad (2)$$

The second (single) integral accounts for the fact that the contribution to the magnetized force comes only from the longitudinal component of the electrons' velocity, parallel to the solenoid magnetic field. Analytic solutions to these integrals, especially the first, are very hard to find. Even powerful symbolic mathematical toolboxes available on the market [12, 13] fail. Other codes specialized in the simulation of electron cooling, like BETACOOL [9], use approximated solutions for different regions of the velocity $\vec{v}_e$, typically $v_e < \Delta$ and $v_e > \Delta$, where Δ is either $\Delta_\perp$ or $\Delta_\parallel$ depending on the case. At large relative velocities they use asymptotic expressions. We believe that such approximations are too rough to accurately represent the interaction, as shown in the following paragraphs. In our implementation the integrals (1) and (2) are solved exactly, numerically, in the range:

$$-10\Delta_\perp \le U_\parallel \le 10\Delta_\perp, \quad (3)$$

$$0 \le U_\perp \le 10\Delta_\perp, \quad (4)$$

where $U_\perp$ and $U_\parallel$ are, respectively, the radial and the longitudinal components of the relative ion-electron velocity, which is defined as

$$\vec{U} = \vec{v}_i - \vec{v}_e.$$

These integrals are computed and tabulated in 2-D mesh grids to be interpolated at run time. At relative ion velocities, $\vec{U}$, larger than specified in Eq. (3) and (4) the analytic asymptotic expression of the cooling force is used, since at large velocity differences the thermal properties of the electrons can be neglected. Figure 1 shows the result of the integration for the radial component of the unmagnetized force.

The "Hybrid Kinetic" Model

The interaction between the ions and the electrons is modeled using a "hybrid kinetic" model, where the full 6-D phase space of an ensemble of several-thousand macro particles, representing the ion beam, is immersed in a fluid plasma of electrons, represented with a 3-D Cartesian mesh of an arbitrary number of cells. The local density and velocity of the electrons surrounding an ion is computed with a tri-cubic interpolation of the electron mesh at the ion's location. Both the ions and the electron mesh are immersed in a solenoidal magnetic field.

The ions' time evolution takes into account the friction force experienced by each ion while it traverses the plasma. The electron plasma is currently static, as the effect of the ion passage on the plasma itself is negligible. Yet, we are working on the implementation of the plasma's time evolution with use of the equations of magnetohydrodynamics. This will enable detailed studies of the electrons' response to the ions passage.

BENCHMARK

Literature

The RF-Track implementation of the cooling has been benchmarked against the experimental results proposed in [5]. Figure 2 shows the longitudinal cooling force for various fully stripped Xe ions as function of the relative ion velocity with respect to the rest frame of the electron beam, obtained

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from measurements at the electron cooler of the ESR storage ring. The dashed curve corresponds to the model presented in [4]. The solid curve corresponds to the model presented in [5]. The theoretical descriptions of the cooling force and the simulations are calculated for an electron beam with $n_e = 10^{12}$ m^{-3}, $k_B T_\perp = 0.11$ eV and $k_B T_\parallel = 0.1$ meV in a magnetic field of $B = 0.1$ T, and are fitted to the experimental results at low relative velocities by treating the transverse ion velocity $v_{i\perp}$ as a free parameter. The number which best fits the data is $\langle x' \rangle \equiv \left\langle v_{i\perp}/v_{i\parallel} \right\rangle = 0.3$ mrad, compatible with the estimated beam divergence [5].

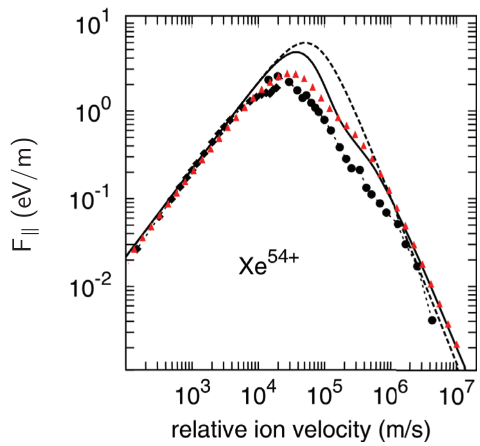

Figure 2: Longitudinal cooling force for various fully stripped Xe^{54+} ions as function of the relative ion velocity with respect to the rest frame of the electron beam. Black marks: experimental data. Solid curve: binary collision approximation. Dashed curve: empiric formula as proposed in BETACOOL. Red triangles: simulated data with RF-Track.

LEIR

A similar benchmark has been performed also at LEIR, the Low Energy Ion Ring of CERN, with Xe^{39+} ions. The cooler was set with electron beam current 200 mA, temperatures $k_B T_\perp = 0.1$ eV and $k_B T_\parallel = 1$ meV, in a solenoid magnetic field of $B = 0.07$ T. In this experiment the cooling force was estimated measuring the variation of the ions average momentum with time, as measured by the Schottky signal.

Figure 3 shows that, during the initial part of the cooling, this signal was quite noisy, which lead us to apply a 15% threshold to filter the data. Yet the estimate of the cooling force was subjected to significant uncertainty: the peak of the distribution seems to converge faster (100 ms) than the mean, suggesting a dilution effect of the electron density due to the already-cooled ions. Figure 4 shows a direct comparison between measurements and simulated data that seems to confirm this hypothesis: for the given electron density the measured cooling force was smaller than expected.

We realize that to improve the estimate of the cooling force one should change the electron energy *after* cooling has been completed. In this way most particles in the ion beam will have the same velocity and behave like a single-particle. This condition will ease the momentum dragging measurement and the overall evaluation of the cooling force. This measurement will be performed in the near future.

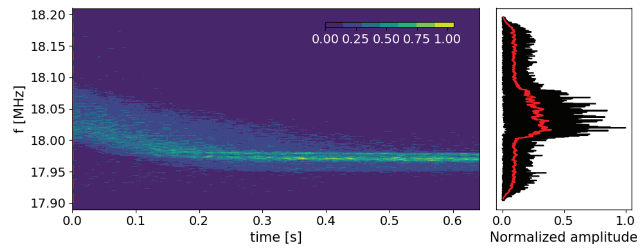

Figure 3: Schottky signal as measured at LEIR during the cooling process. The black histogram shows the power spectrum at the 50th harmonic, at 0 ms. The red curve shows a rolling average over 500 samples (<1% of the total number of samples).

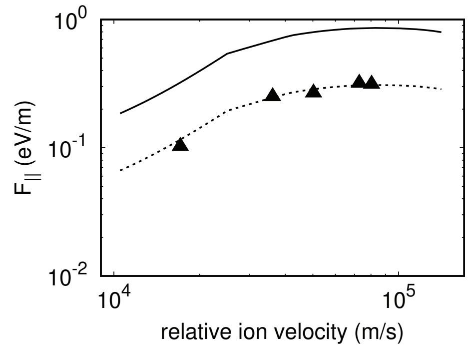

Figure 4: Longitudinal cooling force for Xe^{39+} ions at LEIR, as a function of the relative ion velocity with respect to the rest frame of the electron beam. The solid curve is the result of a simulation with nominal electron density $n_e = 8 \cdot 10^{13}$ m^{-3}. The dashed curve is a simulation with $n_e = 5 \cdot 10^{11}$ m^{-3}. The markers are the measured points.

CONCLUSIONS

A flexible module for the simulation of electron cooling has been added to the tracking code RF-Track. The aim of this module is to understand the performance limitations of the cooling process, at LEIR as well as in other machines, and help overcome them. The implementation is based on a hybrid kinetic model, where the cooling force is obtained with a semi-analytical / semi-numerical calculation. A benchmark of the code has been performed both against results presented in literature and with experimental measurements obtained at LEIR, focusing the attention on the cooling force. The comparison with the results in literature gave very good agreement. The comparison with the measurements at LEIR matched if one assumes a lower electron density than expected. This could be due to an overall dilution of the electron density once most of the ions are cooled. Additional measurements at LEIR will be required. Future development of the code include the implementation of the time evolution of the electron plasma.

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