

# EFFICIENCY ANALYSIS OF HIGH AVERAGE POWER LINACS\*

M. Shumail†, V. A. Dolgashev

SLAC National Accelerator Laboratory, Menlo Park, CA 94025, USA

## Abstract

We present comprehensive efficiency equations and useful scaling laws to optimally determine design parameters for high efficiency rf linacs. For the first time we have incorporated the parasitic losses due to the higher order cavity modes into the efficiency analysis of the standing wave (SW) and travelling wave (TW) accelerators. We have also derived the efficiency equations for a new kind of attenuation-independent-impedance travelling wave (ATW) accelerators where the shunt impedance can be optimized independent of the group velocity. We have obtained scaling laws which relate the rf to beam efficiency to the linac length, beam aperture radius, phase advance per cell, and the type of accelerating structure: SW versus TW, disk-loaded (DL) versus nose-cone (NC). We give an example of using these scaling laws to determine a feasible set of parameters for a 10 MeV, 10 MW linac with 97.2% efficiency.

## INTRODUCTION

Typically, the main parameters of a high power linac are energy  $U_B = eV_A$  and the average power  $P_B$  of the beam. Here,  $e$  is the electron charge and  $V_A$  is the net accelerating voltage. The average beam current is  $I_B = P_B / (FV_A)$ , where  $F$  is the unit-less form factor characterizing the shape of the bunch. We define the beam impedance as:

$$R_B \equiv \frac{V_A^2}{P_B}. \quad (1)$$

In the subsequent derivations, we will be using an impedance ratio, which we call  $B$ -factor, defined as:

$$B = \frac{R_B}{r_s L}, \quad (2)$$

where  $r_s$  and  $L$  are the shunt impedance per unit length and the total linac length, respectively. The net accelerating voltage can be written in terms of this  $B$ -factor as:

$$V_A = BFI_B r_s L. \quad (3)$$

Denoting the power supplied by the rf generator as  $P_{GEN}$ , the rf to beam efficiency is defined as:

$$\eta \equiv \frac{P_B}{P_{GEN}}. \quad (4)$$

In continuous wave, all-rf-buckets-filled operation, we have  $I_B = qf$ , where the relativistic bunches of charge  $q$  pass at the rate equal to the fundamental rf frequency  $f$ . Besides the accelerating voltage  $V_{GEN}$  due to the rf generator or source, the beam also experiences a so-called beam loading voltage, which tends to decelerate the beam. We can further split this beam loading voltage into two parts: the one due to the coherent interaction with the fundamental mode  $V_{FND}$ , and the other due to the non-coherent interaction with all other higher order modes  $V_{HOM}$ . In the following sections, we will analyse these three voltage components one by one and this will naturally lead to the rigorous efficiency equations for both standing wave (SW) and travelling wave (TW) structures.

## ACCELERATING VOLTAGE DUE TO THE RF GENERATOR

### SW Structures

The accelerating voltage  $V_A$  is related to the power loss in the linac walls  $P_W$  as  $V_A = \sqrt{P_W r_s L}$ . In the absence of beam,  $P_W$  is equal to the total power coupled into structure, i.e.,  $P_W = (1 - |\Gamma|^2)P_{GEN}$  where  $|\Gamma| = |\beta - 1| / (\beta + 1)$  is the reflection coefficient and  $\beta$  is the rf coupling coefficient [1]. Thus, the voltage component just due to the rf generator is given as:

$$V_{GEN} = \frac{2}{\beta + 1} \sqrt{\frac{\beta}{\eta}} BFI_B r_s L \quad (\text{for SW}). \quad (5)$$

### TW Structures

We assume that bunches accelerate along the positive  $z$ -axis. In the case of double sign like  $\pm$  or  $\mp$ , the upper or lower sign would correspond to whether the rf energy is flowing in the positive or negative  $z$ -direction – the cases of positive or negative group velocity, respectively. The amplitude of the accelerating electric field  $E_A(z)$ , the power lost per unit length in the structure walls  $\tilde{P}_W(z)$ , the stored energy per unit length  $\tilde{U}(z)$ , and the total power flowing in the positive  $z$ -direction  $P(z)$  are related as follows:

$$E_A^2(z) = r_s \tilde{P}_W(z) = r_s \frac{2\pi f}{Q_0} \tilde{U}(z) = \pm 2r_s \frac{\left(\frac{\pi f}{Q_0 v_g}\right)}{\alpha} P(z). \quad (6)$$

Here,  $Q$  is the intrinsic quality factor,  $v_g$  is the group velocity, and  $\alpha$  is the attenuation constant per unit length. In the absence of the beam, the accelerating field is just due to the generator and we call it  $E_{GEN}(z)$ . Applying the energy conservation, i.e.,  $-\frac{d}{dz}P(z) = \tilde{P}_W(z)$ , we get,

$$\frac{d}{dz}E_{GEN}(z) = \mp \alpha E_{GEN}(z). \quad (7)$$

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†shumail@alumni.stanford.edu

It is straightforward to solve Eq. (7) with the boundary conditions  $E_{GEN}^2(0) = 2r_s\alpha P_{GEN}$  for the case of positive group velocity or  $E_{GEN}^2(L) = 2r_s\alpha P_{GEN}$  for that of negative group velocity. Then, by integrating  $E_{GEN}(z)$  from  $z=0$  to  $z=L$ , we obtain the expression for the accelerating voltage component due to the rf generator. This turns out to be the same for both positive and negative group velocity cases and is given in terms of the attenuation constant  $\tau = \alpha L$ , as:

$$V_{GEN} = \frac{1 - e^{-\tau}}{\sqrt{\tau}} \sqrt{\frac{2B}{\eta}} F I_{B r_s} L \quad (\text{for TW}). \quad (8)$$

## FUNDAMENTAL MODE BEAM LOADING VOLTAGE

### SW Structures

Using the fundamental theorem of beam loading [2, 3], the total beam loading voltage seen by the beam current in the fundamental mode is equal to:

$$V_{FND} = -Fk_0 L q - 2Fk_0 L q e^{-\frac{\tau}{T_f}} - 2Fk_0 L q e^{-\frac{2\tau}{T_f}} - \dots \quad (9)$$

Here,  $T = 1/f$  is the time between the arrival of the bunches,  $T_f = Q_L/(\pi f)$  is the filling time,  $Q_L = Q_0/(1 + \beta)$  is the loaded quality factor, and  $k_0$  is the wakefield fundamental mode loss factor per unit length defined as:

$$k_0 = \frac{\pi f (r_s)}{2(Q_0)} \quad (\text{for SW}). \quad (10)$$

In Eq. (9), the first term on the right corresponds to the voltage induced by the bunch itself while each of the subsequent term corresponds to the voltage induced by a preceding bunch. In the limit  $Q_L \gg \pi$ , Eq. (9) simplifies to:

$$V_{FND} = -\frac{1}{1 + \beta} F I_{B r_s} L \quad (\text{for SW}). \quad (11)$$

### TW Structures

In this case, the  $k_0$  factor is given as [3]:

$$k_0 = \frac{\pi f}{2(1 \mp v_g/c)} \left( \frac{r_s}{Q_0} \right) \quad (\text{for TW}). \quad (12)$$

Here,  $c$  is the speed of light. Also, we define a dimensionless number  $p$  as,

$$p = \frac{L/\lambda}{v_g/c} (1 \mp v_g/c), \quad (13)$$

where  $\lambda = c/f$  is the distance between the bunches. For positive group velocity structures [negative sign in Eq. (12, 13)], the net beam induced decelerating field in the fundamental mode witnessed by any bunch is [3]:

$$E_{FND} = -Fk_0 q$$

$$-2Fk_0 q \sum_{m=1}^{\infty} u(z - Lm/p) e^{-\tau m/p}. \quad (14)$$

Here,  $u(z)$  is the Heaviside step function. Similarly, in the case of negative group velocity, we have,

$$E_{FND} = -Fk_0 q - 2Fk_0 q \sum_{m=1}^{\infty} [u(z) - u(z - L + Lm/p)] e^{-\tau m/p}. \quad (15)$$

The first terms on the right in Eq. (14, 15) correspond to the field induced by the bunch itself while each term in the summation corresponds to the field induced by a preceding bunch. Either integrating Eq. (14) or Eq. (15) from  $z=0$  to  $z=L$ , and in the limit  $p \gg 1$ , we obtain the same result for the fundamental mode beam loading voltage for both positive and negative group velocity cases:

$$V_{FND} = -\frac{e^{-\tau} + \tau - 1}{\tau} F I_{B r_s} L \quad (\text{for TW}). \quad (16)$$

## HIGHER ORDER MODES BEAM LOADING VOLTAGE

Assuming that the wakefields corresponding to the higher order modes add up non-coherently for both SW and TW structures, the power lost in the higher order modes can be given in terms of the wakefield energy loss per unit length  $U$  as [3-4]:

$$P_{HOM} = U_{HOM} L f_0 = (U_{TOT} - U_0) L f_0, \quad (17a)$$

$$P_{HOM} = (k_{TOT} q^2 - F^2 k_0 q^2) L f_0. \quad (17b)$$

Here, the subscripts  $HOM$ ,  $TOT$ , and  $0$ , correspond to the higher order modes, total spectrum, and fundamental mode, respectively. Note that  $V_{HOM}$ , the higher order mode beam loading voltage is a construct that is related to the power lost in the higher order modes as:

$$P_{HOM} \equiv -F I_B V_{HOM}. \quad (18)$$

Defining higher order mode loss factor ratio as:

$$h \equiv \frac{k_{TOT} - F^2 k_0}{F^2 r_s f_0}, \quad (19)$$

we obtain following result which is applicable to both SW or TW.

$$V_{HOM} = -h F I_{B r_s} L \quad (\text{for both SW and TW}). \quad (20)$$

## RF TO BEAM EFFICIENCY

### SW Structures

Equating the net accelerating voltage given in Eq. (3) to the sum of voltage components given in Eq. (5, 11, 20) and solving for the rf to beam efficiency we obtain:

$$\eta = \frac{4\beta B}{[(1 + \beta)(1 + hB) + B]^2}. \quad (21)$$

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For maximum efficiency,  $\beta$  must optimally be chosen as:

$$\beta = \beta_{opt} \equiv 1 + \frac{B}{1+hB}. \quad (22)$$

With this choice of  $\beta$ , the corresponding expression of efficiency is:

$$\eta = \frac{B}{(1+hB)(1+(1+h)B)} = \frac{1-1/\beta_{opt}}{1+hB} \quad (\text{for SW}). \quad (23)$$

### TW Structures

Using  $V_A = V_{GEN} + V_{FND} + V_{HOM}$ , for TW case and solving for the rf to beam efficiency, we get:

$$\eta = \frac{2B\tau(1-e^{-\tau})^2}{(\tau + B(e^{-\tau} - 1 + (1+h)\tau))^2} \quad (\text{for TW}). \quad (24)$$

**Iris-coupled normal TW (NTW)** In these structures  $\alpha$  and  $r_s$  cannot be optimized independently as both are directly related to the beam aperture. In these structures, the efficiency versus  $L$  curves have a narrow peak, as one can see from Fig. 3. The maximum efficiency  $\eta_{max}$  and the corresponding optimal length  $L_{opt}$  are given as follows where  $A \equiv R_B\alpha/r_s$ .

$$\eta_{max} = \frac{2A}{(A + (h + (1+h)\alpha L_{opt}))^2}, \quad (25a)$$

$$L_{opt} = -\frac{1}{\alpha} \left( 1 + \frac{1}{A(1+h)} + W_{-1} \left( -e^{-1 - \frac{A}{(1+h)}} \right) \right). \quad (25b)$$

Here,  $y = W_{-1}(x) < -1$  is the solution of  $x = ye^y$ .

### Attenuation-independent-impedance TW (ATW)

These structures [5, 6] are mainly coupled through an external waveguide and hence, we can optimize  $\tau$  without affecting the shunt impedance. The optimal choice  $\tau = \tau_{opt}$  which maximizes the efficiency is given as:

$$\left. \begin{aligned} &\tau e^{\tau}(e^{\tau} - 1 - 2\tau) \\ &+ B(1 + e^{2\tau}(1 + \tau(1+h))) \\ &- B e^{\tau}(2 + \tau(1+h)(1+2\tau)) \end{aligned} \right|_{\tau = \tau_{opt}} = 0. \quad (26)$$

### SCALING LAWS

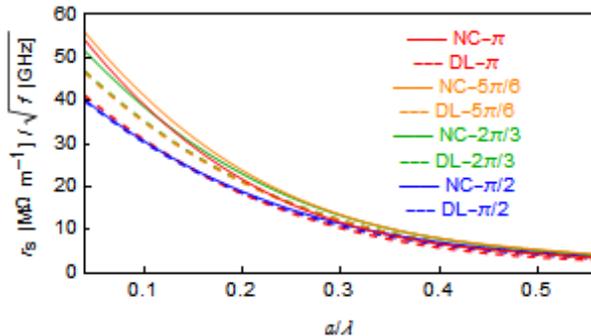


Figure 1: Shunt impedance per unit length vs. normalized aperture size for various values of phase advance.

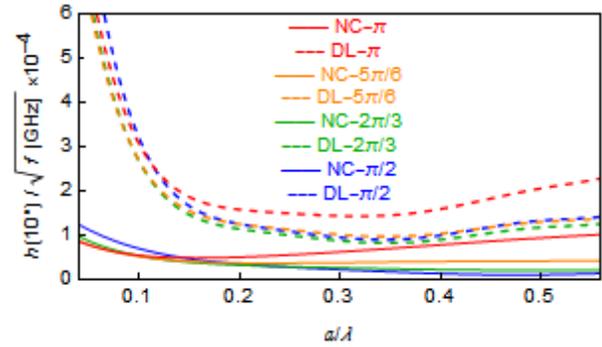


Figure 2: Higher order mode loss factor ratio for  $10^\circ$  bunch length vs. normalized aperture size.

We simulated nose-cone (NC) structures similar to those in Ref. [5–7] and disk-loaded (DL) structures similar to SLAC linac with various values of phase advance and aperture radius in HFSS from Ansys® [9] and ABCI [9,10]. From this simulation data, we obtained the scaling laws for  $Q$ ,  $v_g$ ,  $\alpha$ ,  $r_s$ , and  $k_o$ , as well as  $k_{TOT}$  and  $h$  for various bunch lengths [3]. Figures 1 and 2, respectively show the scaling laws of  $r_s$  and  $h$  (for  $10^\circ$  bunch length) by plotting them against the beam aperture radius  $a$  normalized to the free space wavelength  $\lambda$ . We observe that the nose-cone structures can yield higher efficiency due to their higher shunt impedance (Fig. 1) and lower higher order loss factor ratio (Fig. 2).

### EXAMPLE OF EFFICIENCY ANALYSIS

We consider a 10 MeV, 10 MW linac implying a beam impedance of 10 MΩ. Figure 3 shows efficiency verses length plots for various NC structures and  $10^\circ$  Gaussian bunch length. Here, frequency and aperture were kept constant as  $f = 1.3$  GHz and  $a/\lambda = 0.2$ . Note that SW and ATW structure exhibit higher efficiency for a broad range of linac length.

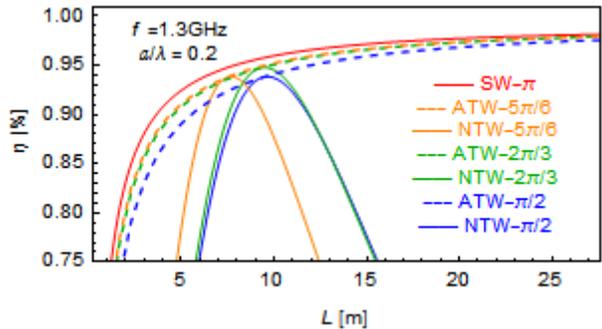


Figure 3: Efficiency vs. linac length at constant frequency and constant aperture for various NC structures.

Figure 4 shows the dependence of  $L$  on  $f$  for fixed efficiency and aperture. We have observed from various constant efficiency curves, like the ones in Fig. 4, that for any 10 MΩ beam impedance case L-band (1–2 GHz) rf accelerators will need smaller linac lengths.

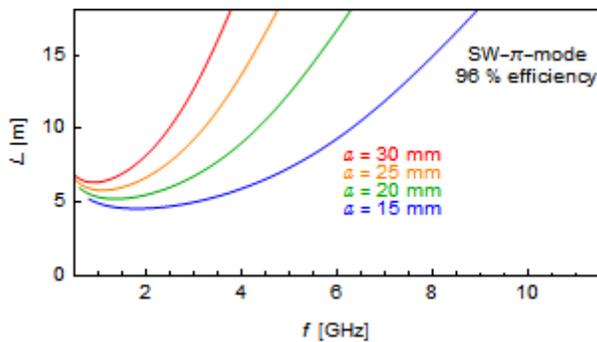


Figure 4: Linac length vs. frequency at constant efficiency and constant aperture for an NC-SW- $\pi$ -mode linac.

Based on these and similar analyses we chose  $f = 1.3$  GHz,  $a = 25$  mm, and  $L = 10$  m for an rf to beam efficiency of 97.4% for this linac. This implies that the rf power loss will be 26.2 kW/m. We have designed this linac using an innovative distributed coupling scheme [7]. The details are given in Ref. [11].

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