

# NUMERICAL METHOD FOR LONGITUDINAL DYNAMICS OF A TERAHERTZ CHERENKOV FREE ELECTRON LASER DRIVEN BY A MeV PICOSECOND ELECTRON BEAM

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## Abstract

Corrugated or dielectric structures have been widely used for producing electron bunch train or THz radiation source. Recently, a novel scheme of sub-terahertz free electron laser (FEL) from a metallic pipe with corrugated walls driven by a non-ultra-relativistic (<10 MeV) picosecond electron beam was proposed and analyzed using the Vlasov-Maxwell equations. In this paper, we use the dielectric loaded waveguide instead, and a numerical method for the longitudinal beam dynamics and electromagnetics of the FEL interaction is presented.

## INTRODUCTION

When relativistic electron beams pass through metallic pipes (or plates) with corrugations or dielectric structures, electromagnetic waves (wakefields) that propagate with the beams are excited. Such quasisingle frequency radiation is a promising candidate for intense THz source [1-7]. The modes can be coherently excited by a beam whose length is a fraction of the wavelength or an appropriate spaced electron bunch train.

Recently, a novel scheme of sub-terahertz free electron laser (FEL) from a metallic pipe with corrugated walls driven by a non-ultra-relativistic (<10 MeV) picosecond electron beam was proposed and analyzed using the Vlasov-Maxwell equations [8]. Such a beam will be driving a resonant mode in the pipe, and, if the pipe is long enough, it will become modulated and microbunched through the interaction with the mode. A numerical method for the longitudinal beam dynamics and electromagnetics of the FEL interaction is developed. The basic idea of the approach is to use the analytic expressions for the longitudinal mode fields to compute the wakefields at each time step using the macroparticle currents as sources. This allows fast time-dependent calculation of the wake-field while the Lorentz equations for the particles can be integrated numerically.

## NUMERICAL METHOD

When a relativistic point charge entering the pipe at the longitudinal coordinate  $s=0$  and moving along the pipe axis excites the resonant mode and wakefields is generated. Ignoring the attenuation and only taking into consider-

ing of the first monopole mode (TM<sub>01</sub>), the longitudinal wakefield can be mathematically expressed by the following equation [8]:

$$E_z(z, s) = \begin{cases} 2\kappa q \cos(k_z(s-z)), & s(1-v_g/\beta c) < z < s \\ \kappa q, & z = s \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

where  $z$  is observation point,  $\kappa$ ,  $k_z$  and  $v_g$  are respectively the loss factor, wave vector and group velocity associated to the mode supported by the DLW structure. The mode parameters can be obtained following the methodology as described in Ref. [9].

The total wakefield is divided into slices with thickness  $\Delta\zeta$  much smaller than the radiation wavelength. When the electron beam travels with time step  $\Delta t = \Delta\zeta/(\beta c - v_g)$  at speed  $\beta c$ , the wake happens to extend behind the electron beam over a finite length  $\Delta\zeta$ . Due to the slippage, the radiation field slices propagate in the backward direction with respect to the electron beam. For  $N$  time steps (for example  $N=4$ ), the order of radiation slices is schematically drawn in Fig. 1.

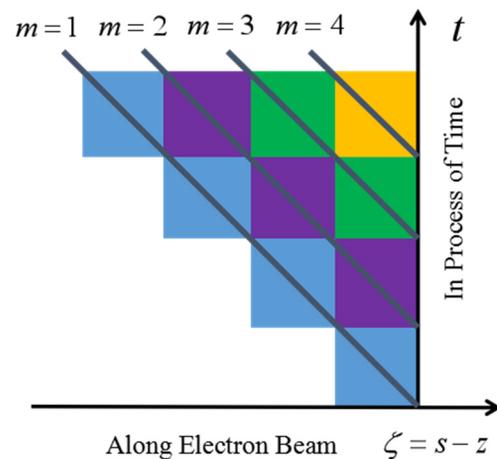


Figure 1: Schematic order of radiation slices.

A discretization of the radiation field in  $z$  and  $t$ , separated by  $\Delta\zeta$  and  $\Delta t$  and indicated by the indices  $m$  and  $n$ , respectively, yields the modified field equation

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$$E_z^{m,n+1} = E_z^{m,n} \exp(jk_z \Delta \zeta) + \sum_j q_j \exp(k_z (s_j - z^{m,n+1})) \text{IF}(z^{m,n+1} \leq s_j < z^{m,n}) \quad (2)$$

where  $q_j$  and  $s_j$  are the charge and position of the  $j$ -th marco particle,  $z^{m,n} = (m-n-1)\Delta\zeta + n\beta c\Delta t$  is the longitudinal position of the sampling radiation field  $E_z^{m,n}$ , and IF is the IF function. The first term of the right-hand side of Eq. (2) comes from the past electromagnetic wave oscillation in phase and the second term is the newly radiation generated by the electron beam source located on the slippage path of the  $m$ -th field slice during the  $n$ -th time step.

The energy exchange due to electron-radiation interaction is

$$m_e c \frac{d\gamma_j}{dt} = e \cdot \text{real}(E_z^j) \quad (3)$$

The electric field strength  $E_z^j$  can be obtained using the interpolation of the radiation field sampling points  $E_z^{m,n}$ .

The longitudinal advance of the  $j$ -th marco particle is

$$\frac{ds_j}{dt} = c \sqrt{1 - \frac{1}{\gamma_j^2}} \quad (4)$$

The integration of the radiation field (Eq. (2)) and the longitudinal variables of the macro particles (Eqs. (3) and (4)) are advanced using the 'leapfrog' method.

## NUMERICAL EXAMPLE

The parameters of the DLW structure are listed in Table 1. For simplicity, we assume that an electron beam has an uniform temporal distribution with time length 10 ps and beam current of 100 A; the beam energy is chosen at 3.8 MeV; the electron beam enters the pipe at the longitudinal coordinate  $s=0$  and the time  $t=0$ .

Table 1: Parameters of the DLW Structure

Structure Parameter	Value
Inner radius ( $r_1$ )	0.5 mm
Outer radius ( $r_2$ )	0.55 mm
Length (L)	15 cm
relative dielectric permittivity ( $\epsilon_r$ )	5.7
<b>Radiation Parameter</b>	
Frequency ( $f_0$ )	0.4615 THz
Relativistic group velocity ( $\beta_g$ )	0.6
Loss factor $\kappa$	50.84 MV/pC/m

The total FEL process, including the electron motion and radiation field propagation, can be obtained numerically following the methodology as described in the previous section. When the head of the electron beam reaches the DLW end, the longitudinal electric field strength distribution along the pipe axis and the electron beam

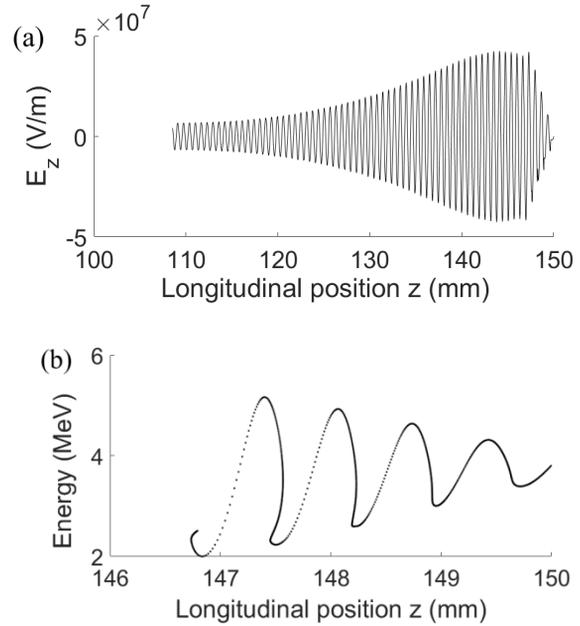


Figure 2: (a) Longitudinal electric field strength distribution along the pipe axis (b) Longitudinal phase space distribution.

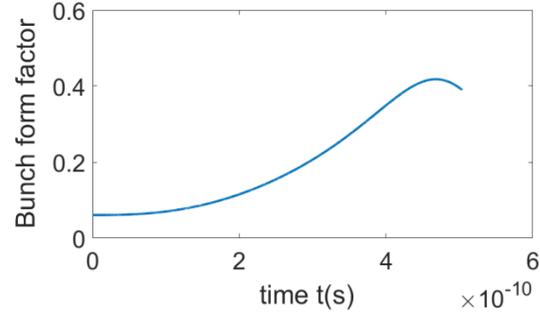


Figure 3: Bunch form factor evolution as with time.

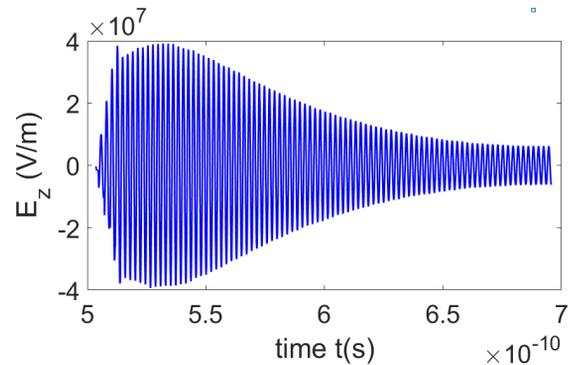


Figure 4: Electric field strength output at the DLW end.

phase space distribution are plotted in Fig. 2. To characterize the temporal structure of the bunch, we compute the bunch form factor  $\langle \exp(ik_z s) \rangle$  and the form factor evolution as with time is shown in Fig. 3. The output electric field strength at the end of the DLW is shown in Fig. 4. The pulse power  $P_w$ , is related to  $E_z$ , through [10]

$$P_z = \frac{v_g |E_z|^2}{4\kappa(1 - v_g/\beta c)} \quad (5)$$

The maximum gradient in Fig. 4 is 39 MV/m, corresponding to the pulse power of 5.8 MW.

## SUMMARY AND FUTURE WORK

We present a numerical method for longitudinal dynamics of a terahertz Cherenkov free electron laser driven by a MeV picosecond electron beam. The approach can fast track total FEL process, including the electron motion and radiation field propagation. We plan to expand the capabilities of this method with some other effects considered in future work, such as the resistive wall and the dielectric losses, the higher order mode of the DLW and the energy spread of the electron beam.

## ACKNOWLEDGEMENTS

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