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MICROBUNCH ROTATION AND COHERENT UNDULATOR RADIATION FROM A KICKED BEAM*

J. P. MacArthur,† A. A. Lutman, J. Krzywinski, Z. Huang
SLAC National Accelerator Laboratory, Menlo Park, USA

Abstract

Recent observations of x-rays from a microbunched beam that has been kicked off-axis have shown coherent radiation at surprisingly large angles, in some cases reaching 30-50 µrad. Previous work on the topic has suggested that radiation at such large angles is inconsistent with classical radiation theory because microbunches cannot tilt. Here we show that, when kicked in a quadrupole lattice, microbunches can automatically tilt toward a new direction of propagation. This allows for coherent radiation farther off axis.

INTRODUCTION

During the commissioning of the arbitrary polarization Delta undulator at LCLS, a pre-microbunched electron beam was given a transverse kick immediately before the undulator. This produced an angular separation between the background linearly polarized x-ray pulse and the circularly polarized x-ray pulse from the Delta undulator [1]. This technique leads to nearly 100% circularly polarized soft x-rays at LCLS [2].

With a carefully chosen undulator K parameter in the Delta undulator, a powerful diverted beam was observed at several times the intrinsic beam divergence. It is generally understood that a transverse kick does not change the microbunch orientation, and hence suppresses the radiation emitted in the direction of the electron motion. This understanding has been formalized into a theory of far-field radiation from a kicked beam [3,4], but is inconsistent with Delta undulator experiments.

In the following section we begin with the conventional viewpoint that the microbunch angle is not immediately affected by a transverse kick. We depart from the standard theory and find that microbunch rotation develops as a consequence of quadrupole focusing, as illustrated in Fig. 1.

MICROBUNCH ROTATION IN A FOCUSING CHANNEL

Corrector dipole magnets are integrated into the undulator lattice quadrupoles at LCLS. The corrector immediately upstream of the Delta undulator can be used to divert a microbunched beam vertically. This situation is depicted in Fig. 1. In this section we calculate the microbunch tilt, $\alpha_b(z)$. We do so with an eye toward calculating the far-field radiation distribution in the following section.

The dynamics of a beam of N_e electrons moving in a helical undulator are described by the Klimontovich distribution

* Work supported by DOE contract DE-AC02-76SF00515.
† jmacart@slac.stanford.edu

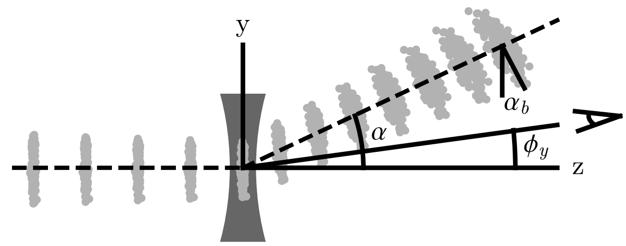

Figure 1: Microbunched electrons (light gray) traveling left to right along the z-axis are kicked by an angle α in the $+y$ direction. The microbunches acquire an automatic tilt angle, α_b, as a result of the defocusing quadrupole (dark gray) at $z = 0$. Microbunch smearing is also present. An observer in the far field sits at an angle ϕ_y above the z-axis.

function

$$\mathcal{F} = \frac{k_1}{I/ec}\sum_{j=1}^{N_e}\delta(\theta-\theta_j)\delta(\eta-\eta_j)\delta(\mathbf{x}-\mathbf{x}_j)\delta(\mathbf{x}'-\mathbf{x}'_j),$$

where j indexes each electron, $\theta_j = (k_1 + k_u)z_j - \omega_1 t$ is the ponderomotive phase, $k_1 = \omega_1/c$ is the wavenumber resonant to the helical undulator, $k_u = 2\pi/\lambda_u$ with λ_u representing the undulator period, $\eta_j = (\gamma_j - \gamma_r)/\gamma_r$ is the energy relative to the resonant electron energy γ_r, $\mathbf{x}_j$ is the transverse position, and $\mathbf{x}'_j = d\mathbf{x}_j/dz$ is the transverse momentum. $\mathcal{F}$ is normalized by the current, I.

$\mathcal{F}$ can be decomposed into a background, stationary distribution $\bar{F}$, and microbunched perturbation F that contains FEL induced modulation, $\mathcal{F} = \bar{F} + F$. At a particular frequency $\nu = k/k_1$, the interaction between the field $E_\nu(\mathbf{x}; z)$ and the perturbation distribution $F_\nu(\mathbf{x}, \mathbf{x}', \eta; z) = \int e^{-i\nu\theta} F d\theta$ is described by the linearized Maxwell-Klimontovich equations,

$$\left[\frac{\partial}{\partial z} + i\Delta\nu k_u + \frac{ik}{2}\boldsymbol{\phi}^2\right]\tilde{E}_\nu = -\kappa_1 n_e \int d\mathbf{x}' d\eta \tilde{F}_\nu \quad (1)$$

$$\left[\frac{d}{dz} + i\left(2\nu\eta k_u - \frac{k}{2}\mathbf{x}'^2\right)\right]F_\nu = -\chi_1 E_\nu \frac{\partial \bar{F}}{\partial \eta}, \quad (2)$$

where the tilde indicates an angular transform,

$$\tilde{E}_\nu(\boldsymbol{\phi}; z) = \frac{1}{\lambda^2}\int d\mathbf{x} E_\nu(\mathbf{x}; z) e^{-ik\mathbf{x}\cdot\boldsymbol{\phi}},$$

$$\tilde{F}_\nu(\boldsymbol{\phi}, \mathbf{x}', \eta; z) = \frac{1}{\lambda^2}\int d\mathbf{x} F_\nu(\mathbf{x}, \mathbf{x}', \eta; z) e^{-ik\mathbf{x}\cdot\boldsymbol{\phi}},$$

$\chi_1 = eK/\sqrt{2}\gamma_r^2 mc^2$, $\kappa_1 = eK/2\sqrt{2}\epsilon_0\gamma_r$, n_e is the electron volume density, K is the helical undulator strength parameter, ϵ_0 is the vacuum permittivity, and $\Delta\nu = \nu - 1$. The d/dz on the left side of Eq. (2) is a total derivative along the trajectory, and the natural focusing of the helical undulator is ignored

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because it is much weaker than quadrupole focusing in an x-ray FEL.

We will solve Eqs. (1)-(2) for the situation depicted in Fig. 1. The transverse position of a given electron for $z \geq 0$ is

$$x(z) = x_0 + \left(x_0' - x_0/(2f)\right) z \quad (3)$$

$$y(z) = y_0 + \left(y_0' + y_0/(2f) + \alpha\right) z, \quad (4)$$

where f is the signed focal length of the quadrupole. Previous treatments implicitly set $f = \infty$. A positive focal length represents a quadrupole that is defocusing in the y-dimension in this notation.

The electron beam in an FEL is confined by a focusing-drift-defocusing-drift (FODO) lattice, wherein $\langle x_0 \rangle = \langle y_0 \rangle = \langle x_0' \rangle = \langle y_0' \rangle = \langle x_0 x_0' \rangle = \langle y_0 y_0' \rangle = 0$. The matched rms beam size at $z = 0$ in the middle of the quadrupole is

$$\sigma_x^2 = 2\epsilon\,|f|\sqrt{\frac{2f + L_u}{2f - L_u}}, \quad \sigma_y^2 = 2\epsilon\,|f|\sqrt{\frac{2f - L_u}{2f + L_u}}, \quad (5)$$

where L_u is the FODO lattice half period, approximately equal to the single undulator module length, and ϵ is the transverse geometric emittance, typically the same in both dimensions. The lattice must satisfy $L_u < 2|f|$. The rms divergence in x and y may be calculated from the geometric emittance, $\sigma_{x'} = \epsilon/\sigma_x$, $\sigma_{y'} = \epsilon/\sigma_y$.

In experiments at LCLS, a reverse tapered undulator upstream of the Delta undulator produces a highly microbunched beam with minimal background radiation. We therefore expect $E_\nu(\boldsymbol{\phi}; 0) \approx 0$ in Eq. (2). The Delta undulator is only one or two gain lengths long, so we ignore the FEL interaction and hence the right side of Eq. (2). While this assumption removes self-consistency from Eqs. (1)-(2), it leads to an expression that may be compared with experiment and simulation. With this assumption, Eq. (2) can be solved to give

$$F_\nu = F_\nu(\mathbf{x}_0, \mathbf{x}_0', \eta; 0) \exp\left(-2ik_u z\eta\nu + ikz\frac{\mathbf{x}'^2}{2}\right), \quad (6)$$

where $\mathbf{x}'$ is the z derivative of Eqs. (3)-(4). If $\mathbf{x}_0$, $\mathbf{x}_0'$, and η are independent and normally distributed,

$$F_\nu(\mathbf{x}_0, \mathbf{x}_0', \eta; 0) \propto b_\nu(\mathbf{x}_0; 0) e^{-\frac{x_0^2}{2\sigma_x^2} - \frac{y_0^2}{2\sigma_y^2} - \frac{x_0'^2}{2\sigma_{x'}^2} - \frac{y_0'^2}{2\sigma_{y'}^2} - \frac{\eta^2}{2\sigma_\eta^2}}, \quad (7)$$

where σ_η is the rms energy spread. After a change of variables equivalent to an application of Liouville's theorem, the right side of Eq. (1) becomes

$$\int d\mathbf{x}' d\eta \tilde{F}_\nu = \int d\mathbf{x}_0' d\eta d\mathbf{x}_0 e^{-ik\mathbf{x}\cdot\boldsymbol{\phi}} F_\nu = b_\nu(\boldsymbol{\phi}; z). \quad (8)$$

Eq. (8) can be integrated exactly with the integrand given by Eqs. (6) and (7). The result generalizes expressions seen elsewhere to include the effects of focusing, emittance, and energy spread for a matched beam,

$$\frac{b_\nu(\phi_x = 0, \phi_y; z)}{b_\nu(\phi_x = 0, \phi_y; 0)} = \frac{e^{-2\left(k_u\sigma_\eta\nu z\right)^2 + \frac{|f|k(i\psi + \zeta)}{\sqrt{1-\hat{L}^2} - 2i\hat{\epsilon}|\hat{z}|}}}{i + \frac{2\hat{\epsilon}|\hat{z}|}{\sqrt{1-\hat{L}^2}}}, \quad (9)$$

where $\hat{L} = L_u/(2f)$, $\hat{z} = z/(2f)$, $\hat{\epsilon} = \epsilon k$,

$$\psi = |\hat{z}|\sqrt{1-\hat{L}^2}\left(\alpha^2 - 2\alpha\phi_y + \phi_y^2\hat{\epsilon}^2\right)$$

$$\zeta = -\hat{\epsilon}\phi_y\left(\left(1-\hat{L}\right)\left(\phi_y - 2\alpha\hat{z} + 2\phi_y\hat{z}\right) + 2\phi_y\hat{z}^2\right),$$

and $\phi_x = 0$ for brevity. The $\phi_x \neq 0$ behavior may be recovered by setting α to zero, negating f, and changing ϕ_y to ϕ_x.

The angle at which $\left|b_\nu(\phi_y; z)\right|^2$ reaches a maximum indicates how much the microbunches have rotated toward the new direction. This angle of maximum bunching is

$$\alpha_b(\hat{z}) = \alpha\hat{z}\frac{1}{1 - 2\hat{z}\frac{\hat{L}-\hat{z}-\hat{z}\hat{\epsilon}^2}{1-\hat{L}+2\hat{z}}}. \quad (10)$$

Eq. (10) can be simplified when the emittance is small and the quad separation is larger than $2f$,

$$\hat{\epsilon}^2\hat{L}^2 \ll 1 - \hat{L}^2. \quad (11)$$

Condition 11 is hardly a constraint, for most FELs satisfy $\hat{\epsilon} < 1/2$ by design, and $\hat{L}$ is kept well below unity to optimize the gain length. With Eq. (11) satisfied, the microbunch angle after a single undulator drift $\hat{z} = \hat{L}$ is

$$\alpha_b(\hat{z} = \hat{L}) \approx \alpha\hat{L} = \frac{\alpha L_u}{2f}. \quad (12)$$

Evidently the microbunches rotate toward (away from) the kick direction after passing through a defocusing (focusing) quadrupole. This microbunch rotation can help or hinder off-axis lasing. The magnitude of the rotation can approach the kick angle. This effect is entirely geometric – no gain is needed to explain the microbunch rotation.

This rotation comes at a cost, however. During the microbunch rotation, smearing occurs. The bunching magnitude at the angle specified by Eq. (10) is

$$\left|b_\nu(\phi_x = 0, \phi_y = \alpha_b; z)\right|^2 \propto e^{-\alpha^2/\phi_c^2},$$

where the critical angle ϕ_c is given by

$$\phi_c^2\,|f|\,k = \frac{\hat{\epsilon}}{\sqrt{1-\hat{L}^2}} + \frac{1 - 2\hat{L}\hat{z} - \hat{L} + 2\hat{z}^2 + 2\hat{z}}{2\hat{\epsilon}\hat{z}^2\sqrt{1-\hat{L}^2}}. \quad (13)$$

This is a generalized version of the critical angle discussed in [3]. Again applying Eq. (11), ϕ_c reduces to

$$\phi_c(\hat{z} = \hat{L})^2 \approx \frac{1}{2|f|k\hat{\epsilon}\hat{L}^2}\sqrt{\frac{1+\hat{L}}{1-\hat{L}}} = \frac{1}{k^2\hat{L}^2\sigma_y^2}, \quad (14)$$

where σ_y is the beam size in the middle of the quadrupole, Eq. (5). The importance of microbunch tilt can be estimated by calculating the tilt when the beam is kicked at the critical angle. If the condition in Eq. (11) is met, the result is

$$\alpha_b(\hat{z} = \hat{L}, \alpha = \phi_c) \approx \frac{1}{k\sigma_y}\frac{f}{|f|}. \quad (15)$$

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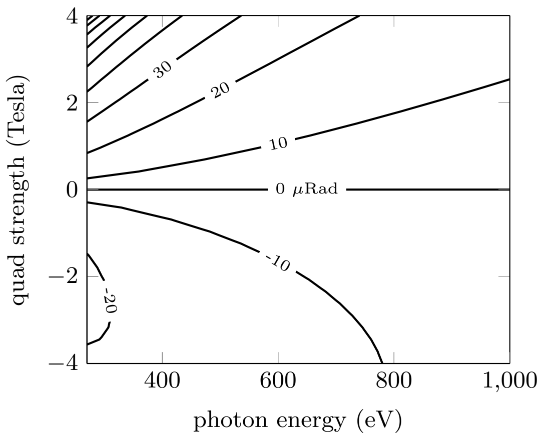

Figure 2: The microbunch tilt angle $\alpha_b(\hat{z} = \hat{L}, \alpha = \phi_c)$ (in μRad) is plotted as a function of photon energy and integrated quadrupole strength using the LCLS beam and lattice parameters. The kick angle α is chosen to be the critical angle, Eq. (14), and the drift length is $\hat{z} = \hat{L}$. At LCLS the integrated quadrupole strength can vary from −4 T to 4 T, and the photon energy can reach down to 270 eV.

Eq. (15) is plotted in Fig. 2 as a function of photon energy and quadrupole strength for LCLS-like parameters. In this figure quadrupole strength refers to the quadrupole gradient integrated along the central axis.

The microbunch angle in Eq. (15) can exceed the coherent undulator radiation divergence, $\sigma_{r'} \approx \sqrt{\pi/kL_u}$. When it does, the smooth focusing approximation is inadequate, and microbunch rotation is expected to have an effect on the radiation produced by a kicked beam.

COHERENT RADIATION FROM A KICKED BEAM

The electric field may be calculated numerically from Eq. (1), but more physical insight can be gained through approximation. At LCLS the energy spread before saturation satisfies

$$2\left(k_u \sigma_\eta z\right)^2 \ll 1. \qquad (16)$$

Henceforth we assume Eqs. (11) and (16) are satisfied.

Equation 1 can be rewritten with the relabeling $\tilde{E}_\nu(\boldsymbol{\phi}; z) \rightarrow \exp\left(i\Delta\nu k_u z + \frac{1}{2} i k \boldsymbol{\phi}^2 z\right) \tilde{E}_\nu(\boldsymbol{\phi}; z)$ as

$$\frac{\partial \tilde{E}_\nu}{\partial z} \propto b_\nu(\boldsymbol{\phi}; z) e^{i\Delta\nu k_u z + \frac{1}{2} i k \boldsymbol{\phi}^2 z}. \qquad (17)$$

This is valid because $(\Delta\nu k_u + k\boldsymbol{\phi}^2/2)$ is independent of z in the afterburner, and an overall phase will not affect $|E|^2$.

Inserting Eq. (9) into Eq. (17) and applying Eq. (11), the resultant expression for the field growth is

$$\frac{\partial \tilde{E}_\nu}{\partial z} \propto e^{-\frac{\alpha^2}{2\phi_c^2} - \frac{1}{2}\left(k^2 z \epsilon \phi_c \left(\phi_y - \alpha_b\right)\right)^2 + i\left(k_u \Delta\nu + k\frac{(\alpha - \phi_y)^2}{2}\right) z}, \qquad (18)$$

where α_b and ϕ_c are Eqs. (10) and (13) subject to Eq. (11).

Eq. (18) may be integrated with respect to z, but the result involves error functions, and cannot be written compactly. In lieu of additional approximations we discuss the maximization of $|\tilde{E}_\nu|^2$ through the minimization of the factors in the exponent of $\partial\tilde{E}_\nu/\partial z$.

There is an unavoidable field degradation when α approaches ϕ_c. This is a consequence of microbunch smearing. The $(\phi_y - \alpha_b)^2$ dependence indicates a tendency to radiate perpendicular to the microbunches, while the $(\alpha - \phi_y)^2$ dependence indicates a tendency to emit along the new travel direction.

Fortunately the microbunches can rotate toward the new travel direction, so the field growth is strongest when $\phi_y = \alpha_b$ and

$$\Delta\nu = -k(\alpha - \alpha_b)^2/(2k_u).$$

This can be converted into a detune in undulator K by noting that the beam will radiate at a wavenumber k set by the pre-microbunched beam,

$$\frac{\Delta K}{K_0} \approx -\frac{k(\alpha - \alpha_b)^2}{4k_u},$$

where K_0 is the rms undulator parameter resonant to the pre-microbunched beam with zero kick. This detune is a moving target, since the microbunch angle α_b changes as the beam propagates. The exact optimal detune must be calculated numerically by integrating Eq. (18), but the rotation of the microbunches serves to move the optimal detune closer to zero when $f > 0$. For $f < 0$, the rotating microbunches move away from the kick direction, thus moving the optimal detune farther away from 0.

This can be contrasted with a formalism developed without microbunch rotation, $\alpha_b = 0$ [3,4]. A detailed comparison is made in a forthcoming publication.

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