

INVERSE FREE ELECTRON LASER SEPARATRIX CROSSING FOR ENERGY GAIN AND STABILITY

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Abstract

The laser wakefield accelerator (LWFA) has been proposed as a driver for next generation compact light sources. However, the beams produced by LWFA's typically exhibit correlated energy spread and energy jitter too large for many applications, in particular the Free Electron Laser. We present here a novel scheme whereby using a strongly tapered undulator interaction directly after the LWFA we are able to trap and accelerate a large fraction of charge initially outside of a growing Inverse Free Electron Laser ponderomotive potential. The final output energy is determined by the stagnant undulator parameters, resulting in significant reduction of the output energy jitter. This interaction is treated numerically.

INTRODUCTION

The laser wakefield accelerator is an all optical, compact source of high brightness electron beams potentially capable of driving the next generation of light sources. However, electron beams produced by these accelerators exhibit fluctuations in the output energy proportional to fluctuations in the drive laser intensity and plasma density, typically on the order of 10%. Furthermore, the typical output correlated energy spread (>1%) is too large for many applications [1].

The inverse free electron laser (IFEL) is a unique advanced accelerator. By coupling a laser to an electron beam in a strongly tapered undulator magnet, up to GeV/m gradients can be achieved and sustained over long distances [2, 3]. This interaction does not require any medium and occurs in vacuum far from any boundaries and is thus not subject to many instabilities applicable to other advanced accelerators. The IFEL dynamics are dictated by the undulator and laser normalized vector potentials, $K = \frac{eB}{k_w mc}$ and $K_1 = \frac{eE}{k mc^2}$, where k_w and k are the undulator and laser wave numbers respectively. Typically $K \gg K_1$ and the output energy is stable, depending only on the static undulator parameters. Recent experiments have shown the IFEL accelerator to be a mature technology capable of achieving high gradient acceleration with stable output energy and charge [4, 5].

Due to available permanent magnet and laser technology, the IFEL in general requires the input electron beam to be highly relativistic. A scheme using the beam produced by an LWFA as the input beam for an IFEL could result in GeV level, stable electron beams without significant increase in the accelerator footprint while taking advantage of the already existing high intensity laser system. However, the phase space acceptance of the IFEL, determined by the initial ponderomotive potential, can be smaller than the en-

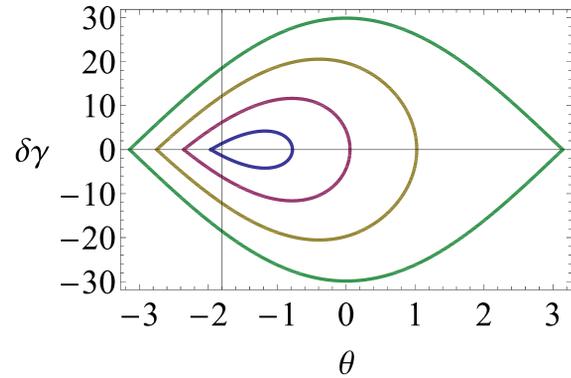


Figure 1: The ponderomotive bucket varying the resonant phase with $\theta_r = 0, -\frac{\pi}{8}, -\frac{\pi}{4}, -\frac{3\pi}{8}$.

ergy fluctuations from the LWFA resulting in a significant decrease in the output charge stability.

We present here a novel scheme where the initial loading of the IFEL ponderomotive potential is achieved by trapping initially non-resonant particles. By sweeping a growing IFEL potential through the range of energy fluctuations produced by the LWFA, particles will experience a non-zero probability of crossing the separatrix into the region of trapped orbits. Choosing undulator parameters that keep this trapping probability approximately constant will result in a trapping fraction independent of the initial beam energy. Continuing the IFEL interaction after this "trapping section" will produce an output beam stable in both energy and charge. We first investigate the phenomenon of separatrix crossing in the IFEL potential followed by a numerical example case of the proposed scheme.

IFEL DYNAMICS

Phase synchronicity between an electron and an electromagnetic wave copropagating in an undulator field can be achieved by choosing the undulator parameters to satisfy the FEL resonance condition for a given resonant electron energy, $\gamma_r^2 = \frac{k(1+K^2)}{2k_w}$. Energy exchange between the electrons and the field is then determined by the ponderomotive gradient, $\frac{d\gamma^2}{dz} = -2kK_1K \sin(\theta)$, where θ is the ponderomotive phase. The IFEL mechanism introduces tapering of the undulator parameters such that a particle at a particular accelerating "resonant phase", $\theta_r < 0$, will continue to satisfy the FEL resonance as it gains energy. This is accomplished by matching the change in resonant energy to the resonant

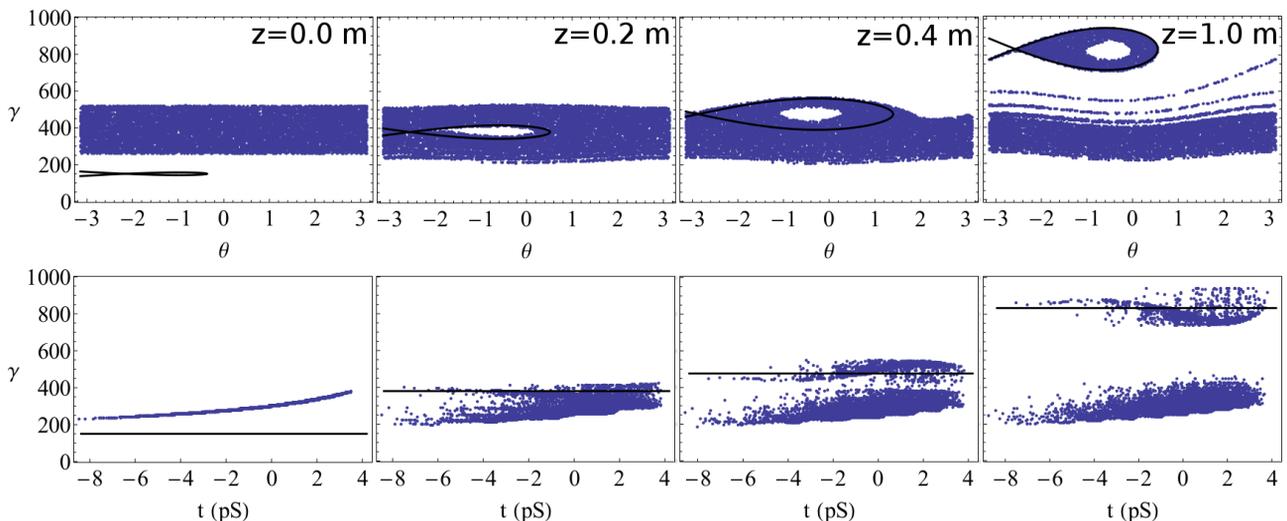


Figure 2: (Top) Growing IFEL ponderomotive potential (black line) trapping particles through separatrix crossing. The IFEL undulator tapering is shown in Figure 3. (Bottom) IFEL trapping using a hypothetical LWFA longitudinal phase space. The black line shows the resonant energy.

ponderomotive gradient:

$$\frac{d}{dz} \left(\frac{k(1 + K(z)^2)}{2k_w(z)} \right) = -2kK_1(z)K(z) \sin(\theta_r) \quad (1)$$

The electron beam longitudinal phase space evolution in the IFEL can then be described by the approximate hamiltonian:

$$H(\delta\gamma, \theta; z) = \frac{k_w \delta\gamma^2}{\gamma_r} - \frac{kK_1K}{\gamma_r} \cos(\theta) + \frac{d\gamma_r}{dz} \theta \quad (2)$$

where $\delta\gamma = \gamma - \gamma_r$ and $\frac{d\gamma_r}{dz} = \frac{-kK_1K \sin(\theta_r)}{\gamma_r}$ [6].

The ponderomotive potential, or bucket, derived by the hamiltonian defines a region of stable orbits. Particles initially injected within the separatrix of the potential will undergo synchrotron oscillations about γ_r and θ_r , remaining trapped for the duration of the interaction. The area of the potential is given by:

$$S \approx 8 \sqrt{\frac{2kK_1K}{k_w} \frac{1 + \sin(\theta_r)}{1 - \sin(\theta_r)}} \quad (3)$$

The area of the ponderomotive potential decreases rapidly as the magnitude of θ_r increases, Fig. 1. The amplitude of the ponderomotive potential can also be controlled by choice of undulator parameters. In the case of constant bucket area, particles injected outside of the separatrix will remain detrapped. However, by either decreasing the magnitude of the resonant phase and/or increasing the bucket amplitude, particles will cross into the region of stability and remain trapped due to the incompressibility of phase space, Fig. 2.

PROBABILISTIC APPROACH

The IFEL hamiltonian is identical to that of a forced non-linear pendulum. Separatrix crossing and particle trapping in this system has been studied in the context of a variety

of phenomena. This has been treated in the literature as a probabilistic phenomenon, with the probability of separatrix crossing given by the increase in the bucket area divided by the total area of phase space swept out by the bucket as the resonant energy increases [7–10]. The instantaneous probability is thus given by:

$$p_r[K, k_w, \theta_r; z] = \frac{dS/dz}{d\gamma_r/dz} \quad (4)$$

The total probability is then given by:

$$P_r = \frac{\int_{z_1}^{z_2} p_r[K, k_w, \theta_r; z] dz}{\Delta z} \quad (5)$$

The K and k_w tapering and θ_r variation can be chosen to maximize this functional for given laser parameters.

Table 1: Numerical Model Parameters

Parameter	Value
Peak K_1	0.07
λ	800 nm
Rayleigh length	0.4 m
Laser waist position	1.2 m
Laser power	34 TW
Trapping section	0-0.4 m
Acceleration section	0.4-2 m
Initial average energy	200 MeV
Initial energy variation	+/- 60 MeV
Final average energy	1000 MeV

NUMERICAL MODEL

We present here an example case using an initial longitudinal phase space distribution produced by an LWFA, Fig. 2.

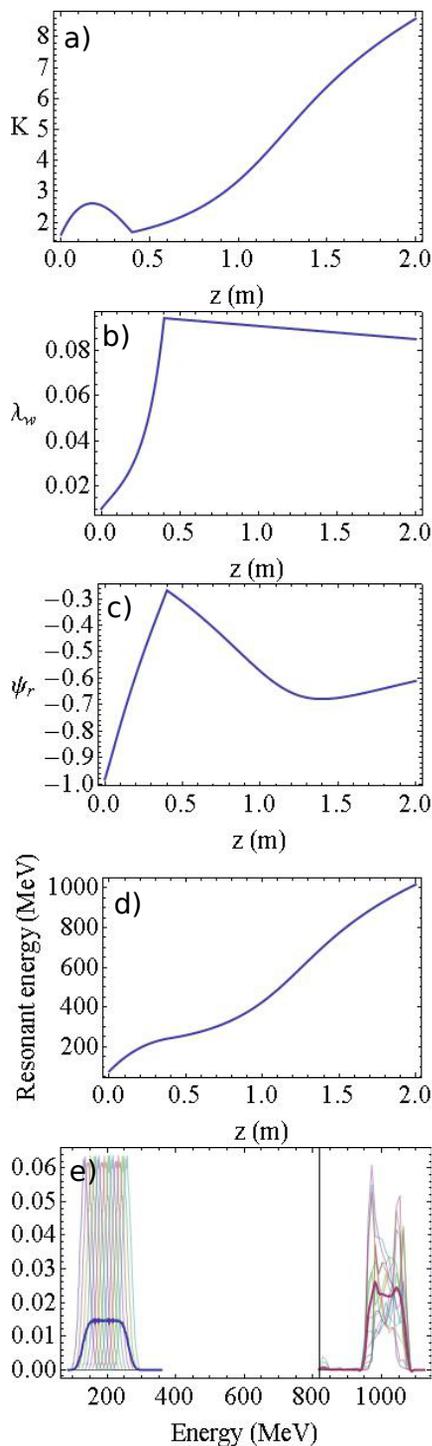


Figure 3: a-d) Undulator tapering parameters used in numerical model. e) Input energy spectra varying the initial energy from 140-260 MeV in steps of 20 MeV with the average spectrum shown (blue). Also shown are the output energy spectra for each input energy and average output spectrum (red). The initial energy distribution bin width is $b_i = 0.01(200 \text{ MeV})$ and the final distribution bin width is $b_f = 0.01(1000 \text{ MeV})$ in order to reflect the enhancement in normalized energy spread.

The IFEL dynamics are solved numerically considering the use of a separate laser pulse to drive the interaction. Electron beam and laser parameters are listed in Table 1 and undulator parameters are shown in Fig. 3 a-d. Undulator tapering in the trapping section utilizes the approach described in the previous section, while tapering in the acceleration section is chosen to maximize the resonant gradient while keeping the bucket area constant in order to maintain trapping.

Figure 3e shows the initial, final and average energy spectra from the numerical model. Considering the normalized energy spread to be the figure of merit for many applications, the initial and final energy spectra are normalized to their respective average energies. With this in mind, we find that the final average energy is stable over the range of initial energy fluctuations, the average charge per percent energy spread has increased by a factor of two, and the charge at the average energy is effectively constant. Up to 40% of the initial charge is trapped and accelerated to the final energy. Although this numerical model did not consider fluctuations in K_1 , as previously stated, the final energy is a function of the static undulator parameters with laser intensity fluctuations contributing primarily to the size of the final bucket which scales as $\sqrt{K_1}$.

CONCLUSION

We have demonstrated that separatrix crossing in the IFEL interaction can be used to trap and accelerate an electron beam exhibiting considerable energy fluctuations. The model discussed above considered only longitudinal dynamics using an approximate hamiltonian. Full 3-D simulations of the scheme are still necessary. Furthermore, although the output energy is stable and the charge per percent energy spread has increased, the overall normalized energy spread is still far too large for most applications. Further investigation into controlled de-trapping or further manipulation of the ponderomotive potential could lead to significant decrease in the final energy spread.

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