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RELIABILITY IMPROVEMENT ON WIGGLER PERIOD AVERAGING APPROXIMATION.*

Kilean Hwang†, Ji Qiang, Lawrence Berkeley National Laboratory, Berkeley, USA

Abstract

As the wiggler period averaging is subject to reliability issue, many efforts on FEL codes without such approximations are made at the cost of heavier computation loads. However, efforts toward increasing the reliability of such approximation are few. In this report, we present a new capability of IMPACT code suite based on such approximation with the addition of perturbative corrections to wiggler period averaging error.

INTRODUCTION

Wiggler Period Averaging (WPA) has been widely used in FEL simulation codes to reduce numerical load significantly. Once the slowly varying envelope approximation (SVEA) is applied, the remaining length dimensions of important physical meaning includes wiggler period, Rayleigh length, gain length, betatron oscillation period and etc. Among them, in general, the wiggler period is significantly short compared to others. Rayleigh length can also be as short as the wiggler period for the high order Gaussian modes [1]. However, in presence of the electron beam with FEL interaction, the Gaussian modes are no longer eigenmodes as the diffraction is compensated by the gain. For this reason, WPA is a quite efficient method unless gain length, betatron period or any other length scales of importance are close to the wiggler period. The reliability of WPA can be further enhanced if perturbative corrections are added to it. In this report, we calculate the next order terms in equations of motion and field equation of WPA and add them as correctional terms. We especially focus on the non-resonant condition, i.e. large energy deviation from the resonant energy γ_R defined by the user input wavenumber k_r.

PARTICLE PUSHER

An effective Hamiltonian integrated over a wiggler period can be obtained using Lie map method.

Bare Hamiltonian

We start from the following Hamiltonian

$$H\left(\boldsymbol{x}_\perp, \boldsymbol{p}_\perp, ct, -\gamma; z\right) = -\sqrt{\gamma^2 - 1 - (p_x - a_x)^2 - \left(p_y - a_y\right)^2}$$

where we ignored space-charge potential and $a_{x,y}$ is the normalized vector potential assumed to be

$$a_x = -K_{\text{eff}} \cos\left(k_u z\right) + a_r \quad (1)$$
$$a_y = K k_x^2 x y \cos\left(k_u z\right) \quad (2)$$

for a planar wiggler with

$$K_{\text{eff}} = K\left(1 + k_x^2 \frac{x^2}{2} + k_y^2 \frac{y^2}{2}\right) \quad (3)$$

where K is the normalized peak wiggler strength, k_u is the wiggler wave number, $k_{x,y}$ are the natural focusing strength, and a_r is the vector potential of the radiation field which we write by

$$a_r = \Re \sum_{h\geq 1} K_h\left(\boldsymbol{x}, t\right) e^{ihk_r(z-ct)} \quad (4)$$

where h is the harmonic number, k_r is the fundamental mode wave number and K_h is the h-th harmonic radiation envelope.

We further transform the Hamiltonian into

$$H = \left(k_u + k_s\right)\eta - \sqrt{k_r\eta^2 - 1 - (p_x - a_x)^2 - \left(p_y - a_y\right)^2}$$

whose longitudinal canonical pairs are

$$\theta \equiv k_r\left(z - ct\right) + k_u z, \qquad \eta \equiv \gamma/k_r \quad (5)$$

Then, the phase of Eq. (4) becomes $ih\left(\theta - k_u z\right)$. From now on, we assume the following order of magnitudes: $k_x x$, $k_y y$, p_x, $p_y \lesssim O\left(10^{-2}\right)$, and $1/\gamma \lesssim O\left(10^{-3}\right)$. We also consider large energy deviation $\delta = \gamma/\gamma_R - 1 \lesssim O\left(10^{-2}\right)$ from the resonant energy encompass future research like the use of laser-plasma accelerator [2]. And keep terms to the next leading order during the derivation of the Effective Hamiltonian.

Lie Map Perturbation

We decompose the Hamiltonian by $H = S + F + V$ where $S = \int_0^{\lambda_u} H\, dz/\lambda_u$ is wiggler period averaged Hamiltonian, V is the potential of the 1st order of radiation field and $F = H - S - V$ the fast oscillating (of wiggler frequency) part. Accordingly, we factorize the map by [3],

$$\mathcal{H} = \mathcal{V}\cdot\mathcal{F}\cdot\mathcal{S} \quad (6)$$
$$\mathcal{F} = e^{\mathcal{G}_F} \quad (7)$$
$$\mathcal{V} = e^{\mathcal{G}_V} \quad (8)$$

where $\mathcal{S}$ is the unperturbed exact map of S, and

$$\mathcal{G}_F = -\int_0^z dz :F^{\text{int}}: + \frac{1}{2}\int_0^z dz_1 \int_0^{z_1} dz_2 :: F_2^{\text{int}} : F_1^{\text{int}} : + \ldots \quad (9)$$
$$\mathcal{G}_V = -\int_0^z dz :V^{\text{int}}: + O\left(K_h^2\right) \quad (10)$$

are the generators. The colons represent Poisson bracket [3], $z = 0$ is the starting location of integration, and

$$F_i^{\text{int}} \equiv \mathcal{S}(z_i) F(z_i) \quad (11)$$
$$V_i^{\text{int}} \equiv \mathcal{F}(z_i)\mathcal{S}(z_i)V(z_i) \quad (12)$$

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† kilean@lbl.gov

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are the interaction picture potentials propagated by $\mathcal{S}$ and $\mathcal{F}\cdot\mathcal{S}$. It can be shown that, to the leading order, $\mathcal{G}_F$ is [4]

$$\mathcal{G}_F = -\lambda_u \frac{K^4 k_x^2}{16 k_u^2 \gamma^3} \tag{13}$$

which is negligible in general. Therefore, the effective Hamiltonian is

$$\mathcal{H}_{\rm eff} = S - \mathcal{G}_V/\lambda_u \tag{14}$$

On the other hand, the interaction picture field envelope $K_h^{\rm int} = \mathcal{F}\mathcal{S}K_h$ can be written as

$$K_h^{\rm int}(z) = K_h(z) + \frac{K_{eff}}{k_u\gamma}\sin(k_u z)\frac{\partial}{\partial x}K_h(z). \tag{15}$$

The 2nd term is the coupling between the wiggling motion and the transverse field variation which can serve as a correction to WPA. This term was also included in TDA-H [5]. However, it can be shown that, when integrated over a wiggler period, it only appear at even harmonics to the leading order. It can be significant correction term to even harmonics for small transverse beam size and low energy. Here, we consider corrections to WPA only for $h = 1$ mode. Therefore, we drop this term but include longitudinal field variation as a corrections to WPA by,

$$K_h^{\rm int}(z) = \mathbb{K}_h + z\frac{\partial}{\partial z}\mathbb{K}_h \tag{16}$$

where $\mathbb{K}_h \equiv \int_0^{\lambda_u} K_h\, dz/\lambda_u$.

In the same way, FEL phase propagates as

$$\theta^{\rm int} = \theta + \dot\theta z - \xi \sin 2(k_u z) - \zeta\sin(k_u z) \tag{17}$$

where

$$\dot\theta \equiv k_u - \frac{k_r}{2\gamma^2}\left(1 + p_x^2 + p_y^2 + \frac{K_{eff}^2}{2}\right) \tag{18}$$

$$\xi \equiv \frac{k_r K_{\rm eff}^2}{8 k_u \gamma^2} \tag{19}$$

$$\zeta \equiv \frac{k_r K}{k_u \gamma^2} p_x \tag{20}$$

Then, the generator of the field potential become

$$\mathcal{G}_V = \lambda_u \Re \sum_{h\ge1} \frac{e^{ih\theta}}{\gamma}\left[K_{eff}\int_C^h + p_x\int_1^h + K_{eff}\int_{zC}^h \partial_z\right]\mathbb{K}_h \tag{21}$$

where integration parameters, to the leading order, are

$$\begin{aligned} \int_C^{h\in{\rm odd}} &= \frac{1}{2}\left(J^{h\xi_R}_{-\frac{h-1}{2}} + J^{h\xi_R}_{-\frac{h+1}{2}}\right) \\ \int_C^{h\in{\rm even}} &= \frac{1}{2}\left(J^{h\xi_R}_{-\frac{h+2}{2}} - J^{h\xi_R}_{-\frac{h-2}{2}}\right)\frac{h\zeta}{2} \\ \int_1^{h\in{\rm even}} &= J^{h\xi_R}_{-\frac{h}{2}} \\ \int_{SC}^{h\in{\rm even}} &= \frac{1}{4i}\left(J^{h\xi_R}_{-\frac{h-2}{2}} - J^{h\xi_R}_{-\frac{h+2}{2}}\right) \end{aligned} \tag{22}$$

where J_i^f is the Bessel function of order i of argument f, and $\xi_R \equiv \frac{k_r K^2}{8k_u\gamma_R^2}$ is the resonant amplitude of wiggling motion which is defined to reduce numerical burden of Bessel function evaluation. As for the fundamental mode $h = 1$, we include terms to the next leading order.

$$\begin{aligned} \int_C^1 &= \frac{1}{2}\left(J_0^{\xi_R} - J_1^{\xi_R}\right)\left(1 + \frac{ih\dot\theta\lambda_u}{2}\right) - \frac{1}{4}\frac{\dot\theta}{k_u}\Xi \\ &\quad -\frac{\Delta\xi}{2}\left[\left(1 - \frac{1}{\xi_R}\right)J_1^{\xi_R} + J_0^{\xi_R}\right] \end{aligned} \tag{23}$$

$$\int_{zC}^1 = \frac{1}{4}\left(J_0^{\xi_R} - J_1^{\xi_R}\right) + \frac{i}{8\pi}\Xi \tag{24}$$

where $\Delta\xi \equiv \xi - \xi_R$, and

$$\Xi \equiv \sum_{l\neq-\frac{h-1}{2}} \frac{J_l^{\xi_R}}{l} + \sum_{l\neq-\frac{h+1}{2}} \frac{J_l^{\xi_R}}{l+1} \tag{25}$$

$\dot\theta$ and $\Delta\xi$ are important only when $\gamma/\gamma_R - 1 \gtrsim O\left(10^{-2}\right)$.

FIELD SOLVER

Using SVEA, the Maxwell's equation reads

$$\begin{aligned} &\left[\nabla_\perp^2 + 2ihk_r\left(\frac{\partial}{\partial z} + k_u\frac{\partial}{\partial\theta}\right)\right]K_h \\ &= -2\frac{eZ_0}{mc^2}\int_0^{\lambda_u} J_x(z)e^{-ih(\theta - k_u z)}dz \end{aligned} \tag{26}$$

The integration on right hand side is difficult as particle positions are function of z. We take only the monopole out of the multipole expansion method [6] on the source term. Higher order multipoles represent coupling between wiggling motion and field variation and thus can also serve as a correction to WPA. TDA-H [5] included the dipole terms. However, it can be shown that the dipole terms are significant only for even harmonics. Instead, we include correction to WPA through the integration parameters Eqs. (23, 24) of the monopole source,

$$\begin{aligned} S_h^i &= -2\frac{eZ_0}{mc}(k_r + k_u)\sum_j \delta\left(x - \bar x_j\right)\delta\left(y - \bar y_j\right) \\ &\quad \times q_j w_{i,j}\frac{e^{-ih\theta_j}}{\gamma}\left(K_{eff}\int_C^h + p_x\int_1^h\right)^\dagger \end{aligned} \tag{27}$$

where $w_{i,j}$ is the weight function of j-th particle governing source deposition on the grid point $\theta = \theta_i$, S_h^i is the source at the grid point θ_i, Z_0 is the vacuum impedance, and $\bar x_j$, and $\bar y_j$ are the monopole location defined by averaging $x(z)$, and $y(z)$ over a wiggler period respectively.

SIMULATION

Here, we use a example of SASE simulation to see how big can be the effects of the WPA correction terms. The corrections to the WPA and the WPA are implemented in IMPACT

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code suite [7] including particle migration across numerical mesh [8]. Here, we suppress the particle migration effect by running in steady-state mode. The parameters we used are $\gamma = 1.0 \times 10^3$, $I_{\text{peak}} = 3kA$, $l_u = 3cm$, and $l_r = 32nm$. Figures 1 and 2 show the case of $\sigma_\delta = 0.005$. Although we have large energy spread, the effect of correction term is shown to be negligible.

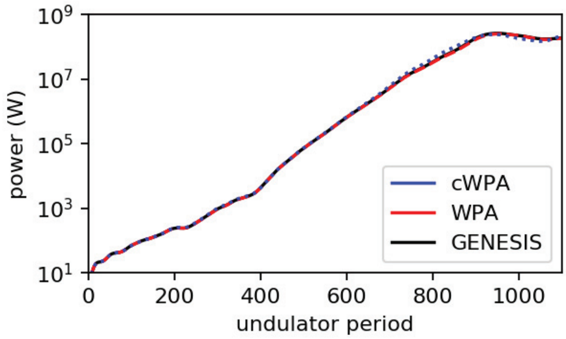

Figure 1: Comparison between WPA and addition of correction terms(cWPA) for a energy spread $\sigma_\delta = 0.005$. Exactly same particle data, mesh number and sizes are used.

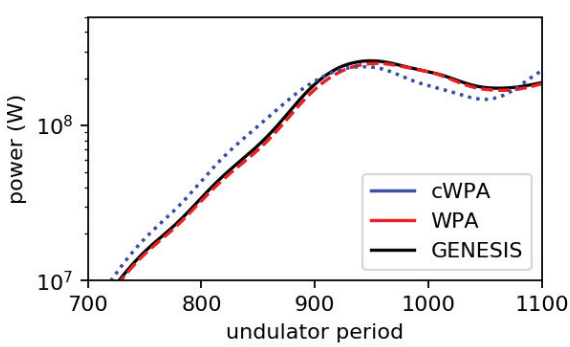

Figure 2: Closer look of Fig.1. There also is a subtle differences between GENESIS [9] and our WPA implementation.

CONCLUSION

The next leading order terms of WPA, in consideration of large energy spread, is derived and implemented in IMPACT code suit [7]. Simulation shows little difference between WPA and the next leading order terms added to WPA, even when large energy spread $\sigma_\delta = 0.005$ is assumed. This result supports the reliability of WPA for the future laser-plasma accelerator beam.

REFERENCE

[1] W.M. Fawley, "Some Issues and Subtleties in Numerical Simulation of X-ray FEL's", *eScholarship*, 2002.

[2] J. Faure *et al.*, "A laser–plasma accelerator producing monoenergetic electron beams", *Nature*, volume 431, pages 541–544, 2004.

[3] Alex J. Dragt, "Lie Methods for Nonlinear Dynamics". `http://www.physics.umd.edu/dsat/dsatliemethods.html`

[4] Kilean Hwang, Ji Qiang, "Lie Map Formalism for FEL Simulation", in *Proc. FEL2017*, Santa Fe, NM, USA, 2017, paper WEP073.

[5] Juhao Wu, "Harmonic contents of and detailed study on a High-Gain Harmonic Generation Free Electron Laser", in *Proc. PAC2003*, Portland, OR, USA, 2003, pp.1035-1037.

[6] M.J. Schmitt, C.J. Elliott, *Phys. Rev. A* , 41, 3853, 1990.

[7] J. Qiang, R. Ryne, S. Habib, V. Decyk, *J. Comp. Phys.* vol. 163, 434, 2000.

[8] Kilean Hwang, Ji Qiang, "Numerical shot noise modeling and particle migration scheme", presented at IPAC18, Vancouver, Canada, 2018, paper THPMF084, this conference.

[9] S. Reiche, *Nucl. Instrum. Methods Phys. Res., Sect. A*, vol. 429, p.243, 1999.