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the linear analysis and in subsequent sections, nonlinear analysis, results, and conclusions are presented.

LINEAR ERROR ANALYSIS

Detailed studies of the sensitivity of the chosen lattice are carried out against the linear errors in dipole and quadrupole field as well as alignment error. These studies include the effect on closed orbit and optics due to dipole field error in the dipole magnets, alignment error in quadrupole magnets and quadrupole gradient errors in the quadrupole magnets. At this compact synchrotron, all dipole magnets are driven by the same power supply and thus correction in power supply will bring the field at the same baseline level for all the magnets of the family. Thus we emphasized the study under the random errors.

Effect On Orbit

The expression of the closed orbit distortion (COD) due to any perturbation, which is dipole in nature, is given by

$$u_{s \rightarrow 0} = \frac{\theta \sqrt{\beta_s \beta_0}}{2 \sin(\pi\nu)} \cos(\pi\nu - |\varphi_s - \varphi_0|). \quad (1)$$

where (β_s, φ_s) and (β_0, φ_0) are the beta function and phase advance at the place of measure and perturbation, ν is the tune, θ is the equivalent kick strengths of the perturbation. The major contribution to COD comes from two sources, namely the alignment error of dipoles and quadrupoles and random field error in the dipole magnets, and then individually the effects of these errors on orbit are studied. The results show that all kinds of errors have a significant effect on the closed orbit in one direction, either horizontal or vertical. For dipole magnets, the field errors are particular keys, which generate horizontal COD. In addition, by the significant effect on the edge angle focusing the displacement of vertical and the rotation around longitudinal direction generate COD at vertical. For quadrupole magnets, the displacement of the horizontal and vertical introduce corresponding dipole errors, which result in COD at orthogonal directions. We assigned gaussian random errors to these dipole and quadrupole magnets, which are shown in Table 2.

Table 2: Various Errors in the Field and Alignment

Type	Random error
Field error in the dipoles	$<\pm 5E-4(3\sigma)$
Gradient error in quadrupoles	$<\pm 1.5E-3(3\sigma)$
Displacement error in all magnets	0.2mm(1 σ)
Rotation error in all magnets	0.2mrad(1 σ)

Different 500 random seeds are chosen for studies. The max COD with and without correction plot in the Fig. 3. There are a few mm COD at either horizontal or vertical, and thus the correction is needed. After correction, COD reduce below 1mm at horizontal and vertical, which is less than 1/10 beam size at injection energy. The max kick strength of the correctors is shown in Fig. 3. All the kicks at either horizontal or vertical less than the 1 mrad limited

corresponding to the max current of the corrector power supply, which can be acceptable.

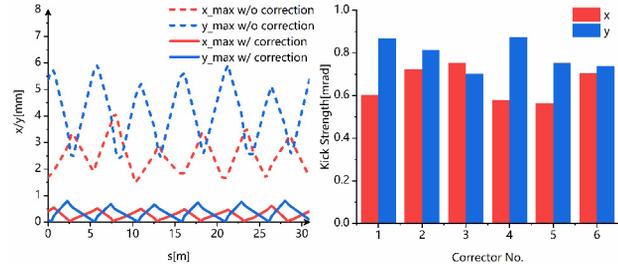


Figure 3: COD (left) and max kick strength (right).

Effect On Optics

The gradient errors in the quadrupole generate tune shift, beta beat and also breaks the symmetry of the lattice. The effect on the working point and beta function is presented in this section. Symmetry breaking causes shrinkage in the dynamic aperture, which is presented in the next section. The change in beta i.e. beta beat due to gradient errors is given by

$$\Delta\beta_{u \rightarrow 0} = \frac{\Delta K \beta_s \beta_0}{2 \sin(2\pi\nu)} \cos(2(\pi\nu - |\varphi_s - \varphi_0|)). \quad (2)$$

where (β_s, φ_s) and (β_0, φ_0) are the beta function and phase advance at the place of measure and errors, ν is the tune, ΔK is the equivalent kick strengths of the gradient errors. The tune shift due to these errors is given by

$$\Delta Q_u = -\frac{1}{4\pi} \beta \Delta K \Delta s. \quad (3)$$

Where ΔK is the equivalent kick strengths of the gradient errors, β is the average beta function in the quadrupole, Δs is the length of the quadrupole. A 500 random gaussian distribution of gradient errors, which also can be see in Table 2, are generated for the simulation, the results shown in Fig. 4.

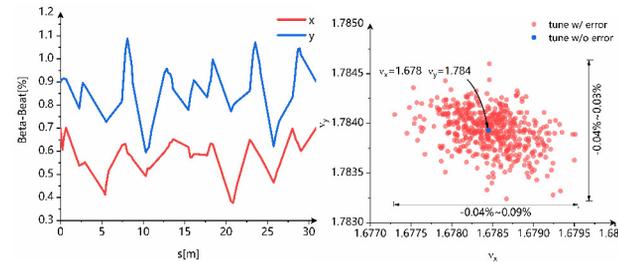


Figure 4: Beta-beat and tune shift on gradient errors.

From Fig. 4, the gradient errors generate a very small change in the tune shift and meanwhile the beta beat is less than 1.2%.

NONLINEAR ERROR ANALYSIS

Above linear analysis provides the required physical aperture, allowed dipole errors in the dipole magnets and allowed alignment errors in the dipole and quadrupole magnets. The analysis does not give a clear picture of required tolerances in the gradient errors, which breaks the symmetry of the lattice. It also does not give an indication of the effect of the main dominating multipole

error in dipole and quadrupole magnets. The relative multipole errors for a combined dipole and quadrupole magnets respectively are [3]:

$$\frac{\Delta B}{B_0} = \frac{b_n x^n}{B_0} \quad (n=0,1,2,\dots) \quad (4)$$

$$\frac{\Delta B}{B_1} = \frac{b_n x^{n-1}}{B_1} \quad (n=1,2,3,\dots) \quad (5)$$

where

$$b_n = \frac{1}{n!} \left(\frac{\partial^n B_y}{\partial x^n} \right)$$

$n=0,1$ correspond to dipole, quadrupole respectively. So for obtaining the allowed multipole errors in the dipole magnets, a preliminary study of magnetic field nonlinear is carried out by Opera3D [4] code in magnet design report from Sigmaphi [5], and thus the systematic errors at multipole components see in Table 3 from the polynomial fitting of simulation data.

Table 3: Tolerance on Multipole Component for Magnets

Multipole errors	Dipole $n=0$		Quadrupole $n=1$	
	Sys	Ran	Sys	Ran
B_1/B_n	-1.50E-4	$<\pm 3E-4$		$<\pm 1.5E-3$
B_2/B_n	2.55E-4	$<\pm 3E-4$		$<\pm 1.5E-3$
B_3/B_n	3.45E-4	$<\pm 3E-4$	0.24E-4	$<\pm 1.5E-3$
B_4/B_n	3.15E-4	$<\pm 3E-4$		$<\pm 1.5E-3$
B_5/B_n			2.52E-4	$<\pm 1.5E-3$

*The reference radii at dipole and quadrupole magnet are 50mm and 45mm respectively.

But the variation in construction from magnet to magnet leads to multipole components that are unique for each magnet. At the present time, it is not known precisely how large the random errors are likely to be, so we assign random errors by the experiences of magnet construction also in Table 3.

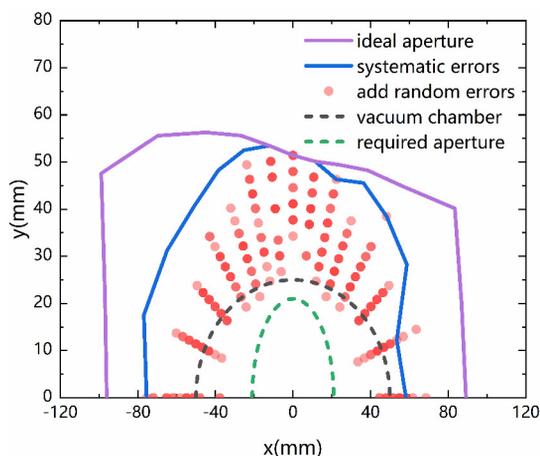


Figure 5: Dynamic aperture on multipole components.

The results on dynamic aperture with no error, with systematic errors only, with systematic and different 20 random errors at injection energy, are shown in Fig. 5. Max beam size at injection energy is plot in Fig. 5 as

required aperture. The dynamic aperture with systematic error is bigger than vacuum chamber and significantly shrink into the vacuum chamber by adding random errors. But it is safe that the dynamic aperture is also bigger than required aperture.

CONCLUSION

Linear and nonlinear studies have been done for the compact proton synchrotron for the tolerances of dipole and quadrupole magnets. Significant reduction in dynamic aperture observed here gives an indication that the dynamic aperture is sensitivity against to the multipole components at the dipole magnets. The difference in the manufacture of magnets is inevitable, and further works need to be carried out to fix the multipole errors to arrive at the tolerances by means of collaboration among magnet construction, measurement, and physical design.

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