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NUMERICAL MULTIPARTICLE TRACKING STUDIES ON COUPLED-BUNCH INSTABILITIES IN THE PRESENCE OF RF PHASE MODULATION*

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Abstract

In this paper, a numerical simulation code is presented which is capable of tracking particles through a circular accelerator under the influence of RF phase modulation and with narrow band wake fields giving rise to coupled-bunch instabilities. RF phase modulation enhances the stability of the beam by adding additional Landau damping suppressing the excitation of coupled-bunch effects. The code combines the usual concept of using rigid macroparticles representing bunches to calculate coupled-bunch effects with a multi-particle approach to calculate phase modulation effects. The basic concept and the mathematical basis are shown together with a study that determines the dependency of coupled-bunch growth rates on the RF phase modulation amplitude. The results show a quadratic correlation, which is in good agreement with experimental results obtained at DELTA.

INTRODUCTION

Studies of RF phase modulation suppressing coupled-bunch instabilities in circular accelerators date back to the 1990s [1, 2]. At DELTA, a 1.5 GeV electron storage ring operated by the TU Dortmund University (see Table 1), this effect is used during user operation since 2008 [3]. With this system, the voltage in the accelerating cavity is described by

$$U(t) = U_0 \sin \omega_{\mathrm{RF}} t + A_m \sin \omega_{\mathrm{mod}} t$$

with the RF amplitude U_0, the RF frequency ω_{RF}, the modulation amplitude A_m and the modulation frequency ω_{mod}.

Besides of its capability to increase the beam lifetime by about 20 % at the cost of an increased bunch length, RF phase modulation is also able to suppress the excitation of coupled-bunch instabilities successfully by introducing additional Landau damping to the electron beam [3–5].

In 2011, a digital bunch-by-bunch feedback system [6] was installed at DELTA, offering the possibility to look into the additional damping effect in more detail. For this purpose, a measurement method was developed at DELTA to analyze the growth rates of coupled-bunch instabilities in the presence of RF phase modulation even with high modulation amplitudes A_m [7].

Previous Results

This paper is based on beam measurement data obtained at DELTA [7, 8]. With the bunch-by-bunch feedback system, the optimum modulation frequency was obtained to be slightly below twice the synchrotron frequency as expected and shown before at several other machines (e.g. [1, 2]). In addition, the dependency of the growth rate on the modulation amplitude was achieved.

Looking at the solution of the equation of motion

$$z(t) = k e^{-t/\tau} \sin \omega t + \phi \quad (1)$$

with the factor $\tau^{-1} = \tau_D^{-1} - \tau_G^{-1}$ being a combination of the damping rate τ_D^{-1} and the growth rate τ_G^{-1}. If $\tau_D^{-1} > \tau_G^{-1}$, the oscillation amplitude decays while it grows exponentially if $\tau_D^{-1} < \tau_G^{-1}$ [9]. At DELTA, the damping rate is mainly based on synchrotron radiation damping while the growth rate characterizes the coupled-bunch mode excited by the impedance of the ring. To obtain the dependency of the growth rate τ_G^{-1} on the modulation amplitude A_m, we measure τ^{-1} as a function of A_m with a constant offset due to natural damping.

The results, as shown in Fig. 2, show a quadratic behavior. With no analytic description available, this behavior should be confirmed by numerical calculations. For this purpose, a tracking code has been developed which is capable of tracking multiple bunches with thousands of particles per bunch through a circular accelerator with an impedance being able to give rise to coupled-bunch instabilities, which is presented in the next section.

NUMERICAL TRACKING CODE

Numerical codes for either tracking rigid macroparticle bunches to analyze coupled-bunch effects (e.g. [10]) or Hamiltonian-based multiparticle calculations for studying the effects of RF phase modulation on the longitudinal phase space (e.g. [1]) are well known today. In this paper, a code is presented which combines both effects. However, we are not interested in the phase space, but in the dynamics of the center of mass of the electron bunches sufficiently characterized by the growth rate.

Table 1: Storage Ring Parameters

Parameter	Value
Revolution frequency	2.6 MHz
RF frequency	500 MHz
Beam power loss	25 kW
Maximum beam current	130 mA
Bunch length rms	100 ps / 18° at RF
Synchrotron frequency	15.2 - 16.4 kHz
Instability threshold	50 ± 6 mA

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Concept and Demands

Our goal was to develop a numerical tracking code with a minimum amount of assumptions to validate the measurement data. Additionally, the code has to run on a standard desktop PC restricting its complexity. The code is able to calculate phase focusing effects in an accelerating cavity with RF phase modulation together with wake field effects allowing the beam to act back on itself giving rise to coupled-bunch instabilities.

Due to these demands, the observed circular accelerator consists only of an accelerating cavity loaded with a voltage described by equation (1) and a momentum compaction factor α representing the rest of the ring. Further, we add a passive cavity-like structure at the position of the cavity, called *impedance structure* in the following, which represents a higher order mode of the cavity serving as the source for wake field effects.

To be able to take phase focusing and RF phase modulation into account, a great number of particles per bunch is needed. For wake field effects on the other hand, intra bunch effects can be neglected. The basic concept to be able to simulate both effects with a decent performance is to calculate phase focusing in the accelerating cavity for every single particle while using the center of mass of the bunch in the passive impedance structure limiting the calculation effort to one matrix operation per bunch.

At startup, the beam is generated by h equidistant bunches with n_{ppb} particles each. All particles are created with a random time deviation δt and energy deviation δE with respect to the reference particle of the bucket with $\delta t = \delta E = 0$. For both, energy and time, a Gaussian distribution with standard deviations σ_t and σ_E is used.

Interactions

A particle with charge q_1 traveling through our virtual circular accelerator experiences three interactions during each turn as presented below.

First, it passes the accelerating cavity which has a length of $l = 0$. For simplicity, we neglect synchrotron radiation completely setting the synchronous phase to $\phi = \pi$. This way, the particle is phase focused due to

$$\frac{\Delta E_{\mathrm{RF}}}{Q} = U_0 \sin \omega_{\mathrm{RF}} \delta t + A_m \sin \omega_{\mathrm{mod}} \delta t,$$

with the energy deviation of the particle applied by the cavity ΔE and the quality factor of the cavity Q.

Afterwards, the particle passes the rest of the ring represented by the momentum compaction factor

$$\alpha = \frac{\delta T / T}{\delta E / E}$$

changing its time deviation based on its δE coordinate.

The last interaction happens inside the narrow-band impedance structure characterized by its quality factor Q_r and its resonance frequency ω_r and is mandatory for coupled-bunch effects. To describe this structure, we simplify it to

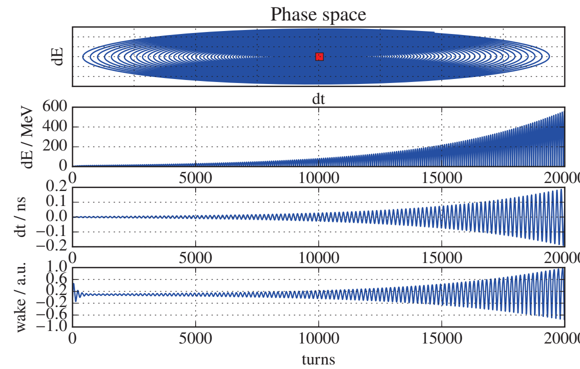

Figure 1: Simulation results of one particle under the effects of RF phase modulation and wake fields. After a short transient behavior, the relative coordinates δt and δE of the particle rise exponentially as well as the wake field itself.

an RLC resonator circuit and solve the equations of motion for the voltage $U(t)$ and the current in the inductance $I_L(t)$. This way, the structure can be described by [11]

$$\begin{pmatrix} U \\ I_L \end{pmatrix}_k = \mathbf{M(t)} \begin{pmatrix} U \\ I_L \end{pmatrix}_{k-1}$$

with the iteration variable k, the wake matrix

$$\mathbf{M(t)} = e^{-\chi t} \begin{pmatrix} M_{11} & M_{21} \\ M_{21} & M_{22} \end{pmatrix}$$

and the matrix elements $M_{11} = \cos \omega_n t - \frac{\omega_r}{2Q\omega_n} \sin \omega_n t$, $M_{12} = -\frac{R\omega_r}{Q\omega_n} \sin \omega_n t$, $M_{21} = \frac{\omega_r Q}{R\omega_n} \sin \omega_n t$ and $M_{22} = \cos \omega_n t + \frac{\omega_r}{2Q\omega_n} \sin \omega_n t$. With this formalism, the wake field effect can be taken into account by calculating the charge vector

$$\mathbf{q_{n+1}} = \frac{R_s \omega_r}{Q_r} \left[\mathbf{M}(t - t_p)\mathbf{q_n} + \begin{pmatrix} q_1 \\ 0 \end{pmatrix} \right]$$

in every time step with t_p being the time elapsed since the last particle passed the impedance structure. In addition, the energy change of a second particle with charge q_2 based on the wake field

$$\Delta E_{\mathrm{WF}} = -e \frac{R_s \omega_r}{Q_r} \left(\frac{q}{2} \left[\mathbf{M}(t_2 - t_1)\mathbf{q_0} \right]_1 \right)$$

generated by the first particle can be determined, where the notation $[\mathbf{M}(t)\mathbf{q}]_1$ means the first term of the charge matrix [11].

Due to the fact that coupled-bunch interactions in first order do not depend on intrabunch motions, this interaction is calculated only once per turn even if the bunches consist of multiple particles since the synchrotron frequency is low. In this case, the used charge is replaced by the bunch charge $q_{\mathrm{bunch}} = n_{\mathrm{ppb}} \cdot q_0$ and the energy change is applied to all particles inside the bunch respectively.

In conclusion, the coordinates δE_1 and δt_1 for turn $n = 1$ are calculated depending on the current wake field vector $\mathbf{q}$

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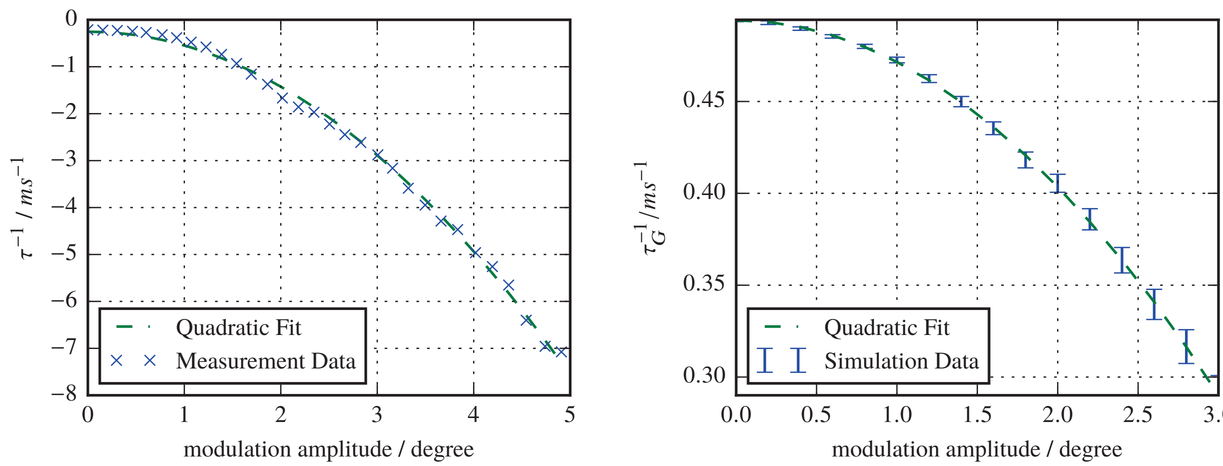

Figure 2: Value τ^{-1} gathered by experimental studies (left) and growth rate τ_G^{-1} obtained by numerical calculations (right) as a function of the modulation amplitude. As shown by a quadratic least-squares fit in the given amplitude interval, the growth-rate dependency can be modeled as quadratic in good approximation.

and δE_0 and δt_0 of the previous turn ($n = 0$) leading to

$$\delta E_{1,p} = \delta E_{0,p} + U_0 \sin \omega_{\mathrm{RF}} \delta t_{0,p} + A_m \sin \omega_{\mathrm{mod}} \delta t_{0,p}$$
$$\delta t_{1,p} = \delta t_{0,p} + \alpha \delta E_{1,p} \frac{Lc}{E_0}$$
$$\delta E_{1,p} = \delta E_{1,p} - \Delta E_{\mathrm{WF}}$$

for every particle p. Since the particles can be considered independently for the first two interactions, all these calculations are done in one matrix operation.

Results

In order to reproduce the measurement data (shown in Fig. 2), we swept the modulation amplitude A_m from 0° up to 3° in steps of 0.2° and used the tracking code to determine the coupled-bunch growth rate in every iteration. Due to performance reasons, only one singlebunch out of $h = 192$ buckets is tracked with $n_{ppb} = 10\,000$ particles over up to $n = 50\,000$ turns. The high number of turns and particles per bunch are necessary to see the effect of RF phase modulation leading us to use only one bunch for not exceeding the memory and to reduce the runtime. The particle charge is increased to be a multiple of e to set the beam current to $I = 110$ mA without increasing the number of particles. We chose a high beam current to get high, distinguishable growth rates. However, this leads to huge phase spaces resulting in the loss of particles. To prevent this, we shrunk the bunch length to $\sigma_t = 1$ ps to be able to track more turns.

With the coupled-bunch mode frequencies of a bunched beam [12]

$$\omega_\mu = p \cdot \omega_{\mathrm{RF}} + \mu\omega_0 + \omega_s,$$

the impedance structure is tuned to the fundamental mode with $\mu = 0$, has a quality factor $Q_I = 1 \times 10^5$ and a shunt impedance of $R_s = 5\,\mathrm{M\Omega}$ to give a reasonable order of magnitude. The results of one particle, excited by the wake field, are shown in Fig. 1. To obtain the growth rate of the instability, an exponential curve is fitted to the peaks of the oscillation of either one of the coordinates (δE or δt) of the center of the bunch (sum of all particles) or the wake field inside the impedance structure. The growth rates gathered by the three fits are identical and we chose to fit the wake field.

The results of the parameter sweep are shown in the right-hand side of Fig. 2. After testing, a second order polynomial function proved to be the best match to the data. Comparing the data obtained by the measurement and the ones from the numerical calculations, we see that both show a quadratic behavior. The data values from the measurement are negative, since τ^-1 was measured at a beam current of $I_{\mathrm{beam}} = 30$ mA, which is well below the instability threshold. The data points from the numerical calculations, on the other hand, are the growth rates τ_G^{-1}, since we neglected synchrotron radiation damping which are always positive. Taking damping effects into account would add a constant negative offset τ_D^{-1} leading to negative results in the simulation as well.

CONCLUSION

A numerical tracking code combining intrabunch and coupled-bunch effects has been developed to calculate the phase focusing effect of the accelerating cavity together with the longitudinal position change described by the momentum compaction factor for every particle in every turn. In addition, the wake field effects in a passive cavity-like impedance structure are calculated for the center of mass of the bunch and applied to all particles. The results are in good agreement with experimental data obtained at DELTA verifying that the dependency of coupled-bunch growth rates on the modulation amplitude is quadratic.

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