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# SIMULATION OF PERTURBATIVE EFFECTS IN IOTA

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*Abstract*

The Integrable Optics Test Accelerator (IOTA) is being commissioned at Fermi National Laboratory for study of the concept of nonlinear integrable optics. The use of a special nonlinear magnetic element introduces large tune spread with amplitude while constraining the idealized dynamics by two integrals of motion. The nonlinear element should provide suppression of instabilities through nonlinear decoherence. We examine the case of a bunch injected off-axis and the resulting damping of centroid oscillations from decoherence. A simple model of the damping is described and compared to simulation.

## INTRODUCTION

The Integrable Optics Test Accelerator (IOTA), currently under construction at Fermi National Laboratory (Fermilab), is intended to be a test facility for novel concepts in beam dynamics [1]. As part of this work IOTA will test the concept of a nonlinear integrable magnetic element, first proposed by Danilov and Nagaitsev [2]. The use of this nonlinear element in a lattice with carefully specified properties to retain time invariance for bunch transport outside the nonlinear element should make it possible to achieve large tune-spread-with-amplitude while still maintaining a large dynamic aperture.In order for this nonlinear element to function is is necessary that the lattice between nonlinear elements act as a thin lens with equal focusing in both planes. While easy to achieve in a linear lattice this assumption will be disrupted by effects such as chromaticity, nonlinearities, and space charge.

To test the strength of nonlinear decoherence provided by the nonlinear element the centroid oscillations of a beam injected off-axis may be studied. The interference of space charge with the decoherence may be observed in simulation. For the case of single particle motion the decoherence time may be estimated from an analytic model. We first present the general model for describing the decoherence and then look at the analytically tractable case of an octupole term in the potential. The inclusion of higher-order terms is briefly discussed along with some of the difficulties that arise from their inclusion.

## NONLINEAR POTENTIAL

The nonlinear magnet constructed for IOTA uses the 'elliptic' potential described in [2]. This potential may be characterized in terms of a unitless strength parameter $t$ and geometric scale factor $c$, with units of $m^{\frac{1}{2}}$, that describes the location of two singularities in the x-plane. The first few terms of the multipole expansion for the elliptic potential in normalized coordinates $\hat{x}, \hat{y} = x/\sqrt{\beta}, y/\sqrt{\beta}$ are given by [3]

$$U(\hat{x},\hat{y}) = \frac{-t}{c^2}\, Im \left\{ (\hat{x}+i\hat{y})^2 + \frac{2}{3c^2}(\hat{x}+i\hat{y})^4 + \frac{8}{15c^4}(\hat{x}+i\hat{y})^6 + \ldots \right\}. \quad (1)$$

Though this expansion is only valid in the region $\sqrt{\hat{x}^2+\hat{y}^2} < c$ it does allow for approximation of the lowest order amplitude dependence in tune. This will be shown to yield reasonable agreement to simulations using the exact potential, as long as the emittance is sufficiently small.

## DECOHERENCE MODEL

Based on the model of Meller et al. [4], the tune of a particle with an amplitude-dependent tune can be written as

$$\nu = \nu_0 - \sum_{i=1} \mu_i a^{2i}, \quad (2)$$

where $\nu_0$ is the linear tune and $\mu_i$ are coefficients based upon the octupole, duodecapole, etc. multipole components in the nonlinear element. For a given beam distribution $\rho(a,\varphi)$, with normalized betatron amplitude $a = \sqrt{\beta\varepsilon}/\sigma$ and phase $\varphi$, a bunch with an initial centroid offset $\Delta x$, will experience nonlinear decoherence and the centroid position as a function of turn number, $N$, will be

$$\bar{x}(N) = \sigma_x \int_0^\infty da \int_0^{2\pi} d\varphi\, a\cos(\varphi)\rho(a,\varphi - 2\pi N\nu). \quad (3)$$

For a Gaussian distribution and only second-order amplitude dependent tune the integral in Eq. (3) may be performed exactly. For other distributions and higher-order amplitude terms the integration generally must be performed numerically. This can prove difficult as the integrand will tend to become highly oscillatory at large $N$.

To evaluate decoherence from previous work [5] we need to be able to integrate Eq. (3) for an offset waterbag distribution, given by

$$\rho(a,\varphi) = \begin{cases} \frac{1+Z^2-2Z\cos(\varphi)}{\pi} & a \le 1 \\ 0 & a > 1 \end{cases} \quad (4)$$

where the normalized offset is given by $Z = \Delta x/\sigma_x$. Inserting the distribution from Eq. (4) into Eq. (3) the angular integral can be performed and, with a change of variables the general result is

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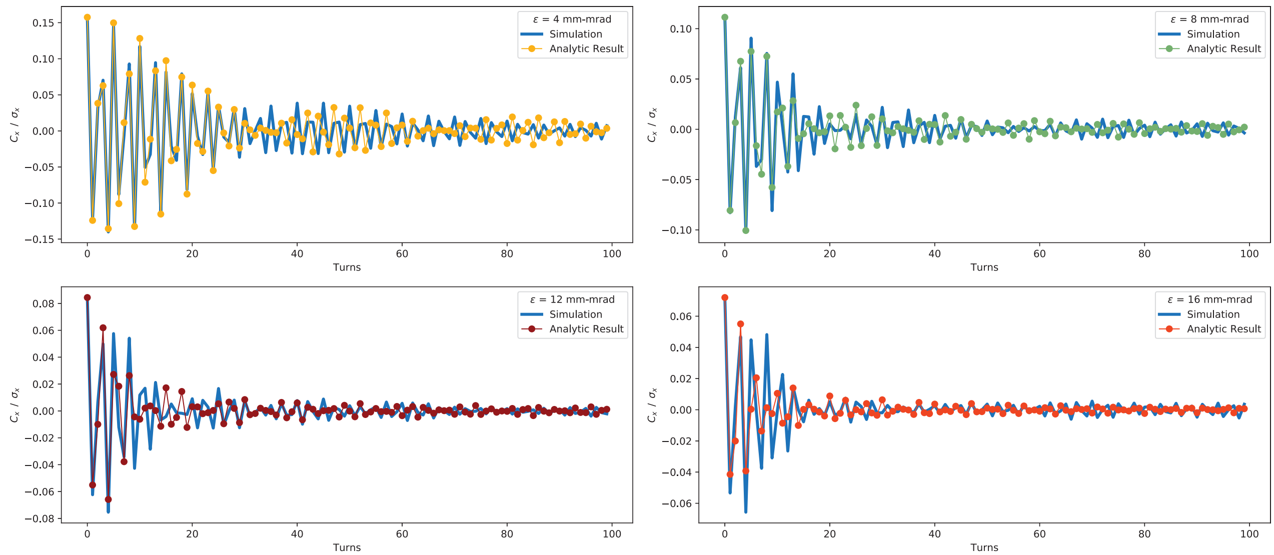


Figure 2: Comparison of the analytic decoherence model to Synergia simulations of IOTA. The comparison is performed for bunches with four different emittances. From left to right and top to bottom $\varepsilon = 4, 8, 12, 16$ mm-mrad are shown.

$$\bar{x}(N) = \frac{Z}{2N} \int_0^{2\pi N} d\hat{u} \, \cos(2\pi \nu_0 N) \cos\left(\sum_{j=1} \frac{\mu_j \hat{u}^j}{2\pi N}\right) + \sin(2\pi \nu_0 N) \sin\left(\sum_{j=1} \frac{\mu_j \hat{u}^j}{2\pi N}\right). \quad (5)$$

If only the lowest order in $\hat{u}$ is used this integral may be evaluated analytically. A second special case when going up to second order in $\hat{u}$ may be evaluated semi-analytically in terms of Airy integrals. Beyond this in the general case this equation must be evaluated numerically.

### *Analytic Terms*

For the case where only the quadratic amplitude term is used the expected coupling strength may be estimated from [6]

$$\mu_1 = \frac{3\varepsilon}{16} \sum_{i=1}^{n} k_{3,i} \beta_{x,i}^2, \quad (6)$$

where the octupole strength of a segmented representation of the nonlinear element of length $l$ is

$$k_{3,i} = \frac{16}{n} \frac{t\ell}{c^2 \beta_{x,i}^3}. \quad (7)$$

In the ideal case the field in the magnet should continuously and smoothly vary along the length of the element. In the actual nonlinear element the the magnet is composed of 20 segments to approximate this variation. In our simulation the magnet is split into 60 segments for slightly better convergence. The matched betatron function calculated for the element and the resulting variation in $k_3$ are shown in Fig. 1.

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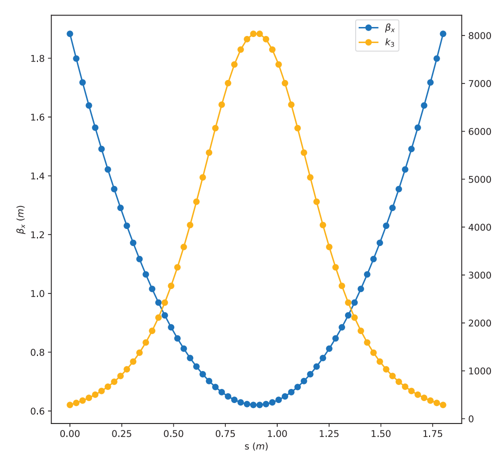


Figure 1: The betatron amplitude $\beta_x$ in blue and the octupole strength $k_3$ in yellow for the nonlinear lens.

For comparison to this model bunches with several different emittances were simulated in IOTA using the code Synergia [7]. In each cases the bunch was started with an offset of 100 µm in the x centroid. The nonlinear element used a strength parameter $t = 0.4$ and scale factor $c = 0.01\ m^{\frac{1}{2}}$. A model of the nonlinear element with the full elliptic potential (that is, no truncated multipole expansion) was used in the simulations.

The results of the simulations are compared against the exact analytic model, calculated only using the octupole coefficients from Eq. (6), is shown in Fig. 2. The truncated analytic model appears to give good agreement with the envelope of the centroid oscillations, especially for smaller emittances where the decoherence is weaker. For very rapid decay of oscillation amplitude in the 16 mm-mrad case the analytic result seems to over-estimate the decoherence

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strength. The disagreement is likely due to the area of validity of Eq. (1) and to the increasing importance of higher order terms at larger amplitudes.

## NUMERICAL MODEL

For $j_{max} > 1$ in Eq. (5) the coupling coefficients $\mu_j$ may need to be found numerically. Using simulation data as a baseline a fitting routine may be used to fit Eq. (3) with the $\mu_j$ terms, up to a given order, and the base tune $\nu_0$ as the free parameters. This can prove challenging from a numerical standpoint, however, due to the previously discussed, highly-oscillatory nature of the integrand and the observation that most fitting algorithms appear to struggle with falling into local minima. An example for $\varepsilon = 4$ mm-mrad is shown in Fig. 3 with fittings up to 8th order in amplitude ($j = 4$).

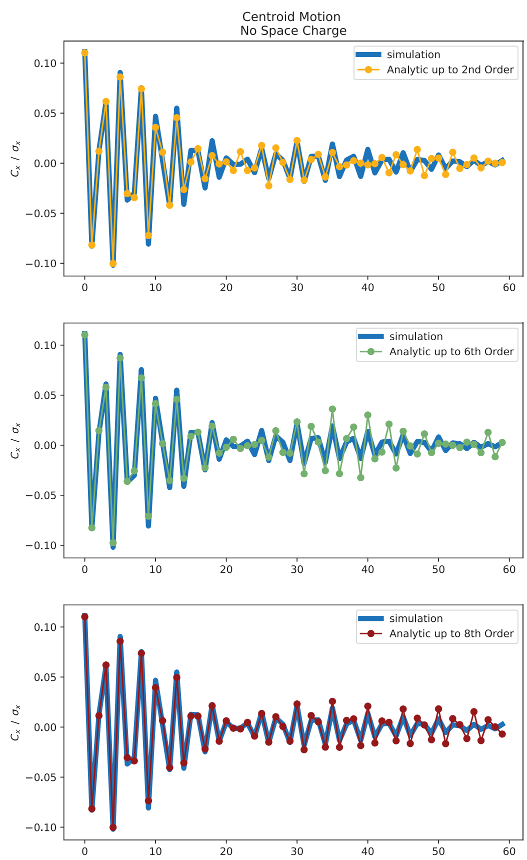


Figure 3: Comparison of simulations in Synergia for a $\varepsilon = 4$ mm-mrad bunch against the semi-analytic model based on Eq. (5). The top plot shows terms up to $j = 1$, the middle plot up to $j = 3$, and the bottom plot includes up to $j = 4$.

For these cases the comparison is truncated after 60 turns, as due to the difficulty of performing the fit only this limited region of data was used. The comparison is significantly improved with inclusion of the higher order terms, and appears to converge to the simulation data in the region of the fit. The value of $\mu_1$ calculated from the analytic calculation was 0.0296 while the fit gave a result of 0.0513. Of note, the fit determined that the natural tune should be 5.409 which matches the expected tune of the IOTA lattice with the shift from the nonlinear element included.

## CONCLUSION

The nonlinear integrable optics concept being tested in IOTA offers a path to stable transport of very-high intensity beams by suppressing instabilities while maintaining reasonable dynamic aperture. In this work we have studied the nonlinear decoherence induced by the special elliptic potential of the nonlinear magnet. An analytic model is shown that provides reasonable estimates of the damping time for centroid oscillations of a bunch injected off-axis. While extending to this model to include higher-order effects from the nonlinear element is possible, it is numerically challenging. Future work will explore adding in higher-order terms and comparing against decoherence in simulations that include strong space charge effects.

## ACKNOWLEDGMENTS

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