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NUMERICAL TOOLS FOR MODELING NONLINEAR INTEGRABLE OPTICS IN IOTA WITH INTENSE SPACE CHARGE USING THE CODE IMPACT-Z

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Abstract

The Integrable Optics Test Accelerator (IOTA) is a novel storage ring under commissioning at Fermi National Accelerator Laboratory designed to investigate the dynamics of beams with large transverse tune spread in the presence of strongly nonlinear integrable optics. Several new numerical tools have been implemented in the code IMPACT-Z to allow for high-fidelity modeling of the IOTA ring during Phase II operation with intense proton beams. A primary goal is to ensure symplectic treatment of both single-particle and collective dynamics. We describe these tools and demonstrate their application to modeling nonlinear integrable dynamics with space charge in IOTA.

INTRODUCTION

The Integrable Optics Test Accelerator lattice design [1–3] makes use of a 1.8 m-long nonlinear magnetic insert with an s-dependent transverse magnetic field that is shaped to generate bounded, regular (integrable) motion in the transverse plane for on-momentum particles. The strong dependence of particle tunes on amplitude leads to a decoherence of transverse oscillation modes that may help to suppress the particle-core resonances primarily responsible for beam halo development and particle loss in intense beams [4].

Several challenges are associated with modeling accurately the beam dynamics in IOTA, including the strong intrinsic nonlinearity of the system, the complex structure of the fields within the nonlinear magnetic insert, the need for robust long-term tracking with space charge over >1K turns (for investigations of beam stability and low-level particle losses), and the sensitivity of the integrability of the system to a variety of perturbative effects.

A primary goal of this study was to implement tools within the code IMPACT-Z [5] for modeling nonlinear integrable optics (with space charge) in IOTA while avoiding sources of non-symplectic numerical artifacts, preserving as far as possible the structure of the ring as an integrable or near-integrable Hamiltonian system.

STRUCTURE OF THE NONLINEAR INTEGRABLE POTENTIAL

The ideal 2D magnetic field within the nonlinear insert is given by $\vec{B} = \nabla \times \vec{A} = -\nabla\psi$, where the magnetic vector potential $\vec{A}$ and the magnetic scalar potential ψ at a longitudinal position s are most easily expressed in terms of the dimensionless quantities [6]:

$$F = \frac{A_s + i\psi}{B\rho}, \qquad z = \frac{x+iy}{c\sqrt{\beta(s)}}, \qquad \tilde{t} = \frac{\tau c^2}{\beta(s)}, \qquad (1)$$

using the complex function:

$$F(z) = \left(\frac{\tilde{t}z}{\sqrt{1-z^2}}\right)\arcsin(z).$$

Here $\beta = \beta_x = \beta_y$ is the betatron amplitude across the drift space that will contain the magnet, $B\rho$ is the magnetic rigidity, τ is a dimensionless parameter characterizing the strength of the magnet, and $c \neq 0$ [$\mathrm{m}^{1/2}$] characterizes the length scale of the potentials in the transverse plane. Fig. 1 illustrates the function F, together with the associated magnetic field lines.

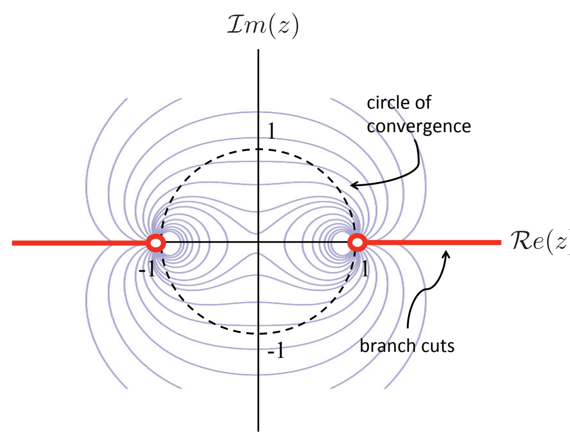

Figure 1: Domain of analyticity of the complex function F, which defines the vector potential of the nonlinear insert in the transverse plane. The curves in blue denote magnetic field lines. The dashed circle denotes the circle of convergence of the multipole series. Singularities occur at the points $z = \pm 1$.

Since $\vec{A}_\perp = 0$ in this model, the single-particle Hamiltonian within the nonlinear magnetic insert takes the following form, using the longitudinal coordinate s as the independent variable:

$$H = -\sqrt{1 - \frac{2P_t}{\beta_0} + P_t^2 - |\vec{P}|^2} - \mathcal{A}_s - \frac{1}{\beta_0}P_t, \qquad (2)$$

where the transverse momenta $\vec{P}$ are normalized by the design momentum $p^0 = mc\beta_0\gamma_0$, the longitudinal variables are $T = c\Delta t$ and $P_t = -\Delta\gamma/(\beta_0\gamma_0)$, and $\mathcal{A}_s = A_s/B\rho$.

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In the paraxial approximation $P_x, P_y << 1$, the Hamiltonian for an on-energy particle ($P_t = 0$) within the nonlinear magnetic insert takes the form:

$$H_\perp(X, P_x, Y, P_y; s) = \frac{1}{2}(P_x^2 + P_y^2) - \mathcal{A}_s(X, Y, s). \quad (3)$$

It can be shown [1,6] that the Hamiltonian (3) yields integrable transverse motion with the two invariants

$$H_N = \frac{1}{2}(P_{xN}^2 + P_{yN}^2 + X_N^2 + Y_N^2) - \tau U(X_N, Y_N), \quad (4)$$
$$I_N = (X_N P_{yN} - Y_N P_{xN})^2 + P_{xN}^2 + X_N^2 - \tau W(X_N, Y_N),$$

where

$$U = \mathcal{R}e\left(\frac{z}{\sqrt{1-z^2}}\arcsin(z)\right), W = \mathcal{R}e\left(\frac{z+z^*}{\sqrt{1-z^2}}\arcsin(z)\right).$$

Here the star * denotes complex conjugation, and

$$\begin{pmatrix} X_N \\ P_{xN} \end{pmatrix} = \begin{pmatrix} 1/c\sqrt{\beta} & 0 \\ \alpha/c\sqrt{\beta} & \sqrt{\beta}/c \end{pmatrix} \begin{pmatrix} X \\ P_x \end{pmatrix}, \quad (5)$$

with a corresponding expression for (Y_N, P_{yN}), where $\alpha(s) = -\beta'(s)/2$. In IOTA, the transfer map $\mathcal{R}$ from the exit of the nonlinear insert to its entrance is assumed linear with a phase advance $n\pi$ for integer n (in both planes). It follows that H_N and I_N are each invariant under $\mathcal{R}$, and are therefore invariant under the one-turn map for the ring.

SYMPLECTIC INTEGRATOR

Within the nonlinear magnetic insert, tracking is performed using a second-order symplectic integrator [5, 7] via the Hamiltonian splitting:

$$H = H_{drift} + H_{NLL}, \quad (6)$$

where H_{drift} is the Hamiltonian for a drift and $H_{NLL} = -\mathcal{A}_s$. The map for a step of length h is evaluated as:

$$\mathcal{M}(s \to s+h) = \quad (7)$$
$$\mathcal{M}_{drift}\left(\frac{h}{2}\right)\mathcal{M}_{NLL}\left(h, s+\frac{h}{2}\right)\mathcal{M}_{drift}\left(\frac{h}{2}\right) + O(h^3),$$
$$\mathcal{M}_{drift}(h) = e^{-h:H_{drift}:}, \quad \mathcal{M}_{NLL}(h,s) = e^{-h:H_{NLL}(s):}.$$

The maps $\mathcal{M}_{drift}$ and $\mathcal{M}_{NLL}$ are exactly known. In particular, if we define the quantity $\mathcal{P} = P_x + iP_y$, the map $\mathcal{M}_{NLL}(h,s)$ acts only on momenta, taking $\mathcal{P} \to \mathcal{P}_f$ where:

$$\mathcal{P}_f = \mathcal{P} + \frac{h}{c\sqrt{\beta(s)}}\left(\frac{dF(z)}{dz}\right)^*. \quad (8)$$

Here the derivative in (8) is known, and the map is evaluated numerically using Fortran complex arithmetic. This symplectic integration procedure avoids numerical errors associated with the presence of small denominators that are present in the equations of motion derived from the nonlinear potential in its original form [1].

TREATMENT OF SPACE CHARGE

An explicitly symplectic space charge tracking algorithm [8,9] was implemented within the code IMPACT-Z for modeling 2D space charge in an unbunched, coasting beam using a spectral method. For a system of N_p macroparticles with phase space coordinates $(\mathbf{r}_j, \mathbf{p}_j)$, $(j = 1, \ldots, N_p)$, the collective Hamiltonian of the system is expressed in the form [9]:

$$H = \sum_{j=1}^{N_p} H_{ext}(\mathbf{r}_j, \mathbf{p}_j) + \frac{K}{2}\sum_{i=1}^{N_p}\sum_{j=1}^{N_p} G(\mathbf{r}_i, \mathbf{r}_j), \quad (9)$$

where H_{ext} denotes the single-particle Hamiltonian including external fields, K is the generalized perveance of the beam, and G is a Green function for the 2D Poisson equation in a rectangular conducting pipe. An approximation for G is obtained by using a finite number of Fourier modes in x and y as:

$$G(\mathbf{r}_i, \mathbf{r}_j) = 4\pi\frac{4}{ab}\frac{1}{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\sin(\alpha_l x_j)\sin(\beta_m y_j)$$
$$\times \sin(\alpha_l x_i)\sin(\beta_m y_i), \quad (10)$$

where

$$\alpha_l = \frac{l\pi}{a}, \quad \beta_m = \frac{m\pi}{b}, \quad \gamma_{lm}^2 = \alpha_l^2 + \beta_m^2. \quad (11)$$

Here N_l, N_m are the number of Fourier modes and a, b are the aperture sizes in x and y, respectively.

At each longitudinal step, the splitting (9) may be applied to evaluate the map for a second-order symplectic integrator on the collective N_p−particle phase space. By grouping terms appropriately, the computational complexity of this algorithm scales as $O(N_{\text{mode}} \times N_p)$, where $N_{\text{mode}} = N_l N_m$ is the total number of modes. The algorithm is easily parallelized by distributing particles uniformly among computational cores [9].

ADDITIONAL CAPABILITIES

Additional capabilities were implemented that include: numerical diagnostics for statistical characterization of the two invariants (4), a diagnostic for characterizing beam mismatch to the nonlinear integrable lattice, a 2D particle-in-cell solver with free space boundary conditions, and improvements in the quadrupole and dipole models relevant for modeling proton rings at low energy. Dipoles can currently be modeled using symplectic tracking through 3rd order.

APPLICATION TO IOTA MODELING

We studied the effect of space charge on the preservation of the invariants H_N and I_N using a version of the IOTA lattice designed for 2.5 MeV protons with a current of 0.411 mA (space charge tune depression $\Delta Q = -0.03$). For propagation in the external fields we use linear tracking for all elements external to the nonlinear magnetic insert (the "arc") to isolate the effect of space charge on the integrability of

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motion. The lattice quadrupole settings were tuned [10] to restore nearly integer tune advance across the arc in the presence of the linearized space charge fields. A particle distribution with 1.024M particles is initialized according to the waterbag-like distribution function:

$$f \propto \Theta(H_N - \epsilon_0), \qquad \Theta(x) = \begin{cases} 1, & x \leq 0 \\ 0, & x > 0 \end{cases}, \tag{12}$$

where ϵ_0 is chosen so that $\langle H_N \rangle c^2 = 4$ mm-mrad. Fig. 2 illustrates the evolution of the moments of the two invariants of motion, taken among all particles in the beam. The results suggests that space charge may induce slow stochastic diffusion due to the breakup of invariant tori [11]. To illustrate the contribution of macroparticle noise, Fig. 3 shows the dependence of the observed diffusion rate on the number of simulation particles in a tracking study with identical beam and lattice parameters. (Note that a slight variation of the distribution type (12) was used in the latter study.)

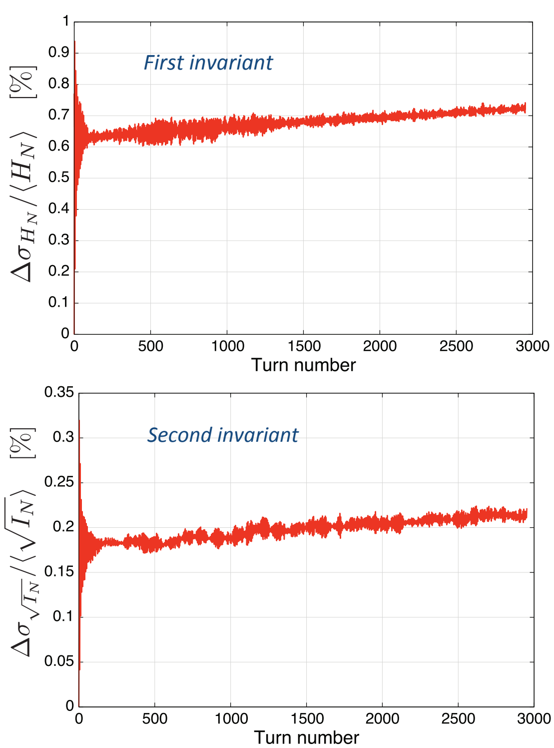

Figure 2: Evolution of the standard deviation within the beam of the two invariants of ideal single-particle motion (4). Here, we use $\sqrt{I_N}$ rather than I_N for practical reasons related to numerical benchmarking. An initial period of rapid phase mixing occurs, followed by slow linear growth.

CONCLUSIONS

A variety of new numerical tools have been implemented in the code IMPACT-Z to facilitate the modeling of nonlinear integrable optics in IOTA with space charge. A treatment of the nonlinear integrable potential of the IOTA magnetic

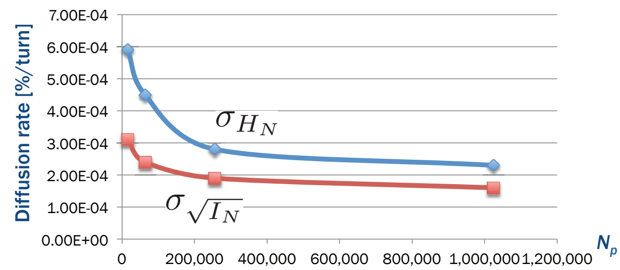

Figure 3: Convergence of the diffusion rate, evaluated as $\Delta\sigma_G/\langle G\rangle$ per turn, for the invariants $G = H_N$ and $G = \sqrt{I_N}$ with the number of simulation particles for a fixed number of spectral modes (64×64). The scaling is not a simple power law, but behaves approximately as $\sim N_p^{-\nu}$, with $1 \leq \nu \leq 2$.

insert in the complex plane is used as an alternative to [1] for numerical tracking, avoiding a previously problematic numerical instability. This is performed using a second-order symplectic integrator based on Yoshida splitting. Space charge can be treated using either a traditional grid-based Poisson solve (with a variety of possible boundary conditions) or using a new spectral solver that is symplectic (by design) on the N-particle phase space of the macroparticle system [9]. Simulations indicate slow diffusion of the invariants of motion in the presence of space charge, which is well-captured using $\sim$ 1 M particles and 64x64 spectral modes. Studies further exploring the interplay between space charge, numerical noise, and integrability in IOTA are ongoing.

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