

# ANALYTICAL ESTIMATION OF THE BEAM ION INSTABILITY IN HEPS

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## Abstract

The High Energy Photon Source (HEPS) is a new designed photon source at beam energy of 6 GeV, with natural beam emittance less than 100pm. Due to the small transverse beam size and high beam intensity, beam ion instability is one of the potential issues for HEPS. The growth time of the instability is estimated analytically for different operation scenarios. The results show considerably good agreement with the wake strong simulations.

## INTRODUCTION

In an electron ring, when a charged particle beam pass through a beam pipe, the ions generated from the residual gas molecules can be trapped by the beam. If an electron bunch with a small transverse offset passes through the ion cloud, the cloud can be disturbed and its centre of mass will start to oscillate. Then the subsequent bunches can be deflected by the perturbed ion cloud, and finally cause beam-ion coherent instability, tune shift, emittance blow-up or beam losses. The fast beam ion instability (FBII) is induced by the ions generated during a single passage of the bunch train. Several theories have been developed to evaluate and suppress the instability [1-6].

In this paper, we first review the existing theories on evaluating the growth rate of the instability with different approximations. Then numerical results of HEPS calculated with these theories will be discussed. Finally, the analytical results will be compared with the wake-strong simulations.

## REVIEW ON THEORIES OF BEAM ION INSTABILITY

For a single bunch train consisting of  $n_b$  bunches, with number of particles per bunch  $N_e$  and bunch spacing of  $L_{sep}$ , followed by a long gap, the growth rate at the end of the bunch train can be described with the following three approximations:

### Simple Linear Treatment for Small Amplitude

With a simple linear treatment, the force between the beam and the ions is assumed to be linear. This is valid only when the coherent oscillation amplitude of the beam is smaller than the transverse beam size, i.e.  $y \ll \sigma_y$ . With this assumption, an initial perturbation  $\hat{y}$  was found to increase quasi-exponentially as [1, 2]

$$y = \hat{y} \frac{1}{2\sqrt{2\pi}(t/\tau_c)^{1/4}} \exp(\sqrt{t/\tau_c}), \quad (1)$$

with a characteristic time of

$$\frac{1}{\tau_c} = \frac{4d_{gas}\sigma_{ion}\beta N_e^{3/2}n_b^2r_e^{1/2}L_{sep}^{1/2}c}{3\sqrt{3}\gamma\sigma_y^{3/2}(\sigma_x + \sigma_y)^{3/2}A^{1/2}}, \quad (2)$$

where  $d_{gas}=p/kT$  denotes the residual gas density with  $p$  is the gas pressure,  $k$  is the Boltzmann's constant, and  $T$  is the absolute temperature,  $\sigma_{ion}$  is the ionization cross section,  $\beta$  is the average vertical betatron function,  $c$  is the velocity of light,  $\sigma_{x,y}$  is the horizontal and vertical beam size,  $A$  is the atomic mass number of the ions,  $\gamma$  is the relativistic beam energy,  $r_e$  and  $r_p$  are the classical electron and proton radii.

### With Ion Decoherence and Ion Frequency Spread

When considering the nonlinearity of the ion oscillations in the beam and variation of the transverse beam size along the lattice, the ion oscillation frequency can have a certain spread. This frequency spread can induce further landau damping based on the simple linear theory. With these considerations, the beam oscillation amplitude increases exponentially along the bunch train as [3]

$$y \sim \exp(t/\tau_e), \quad (3)$$

with an e-folding time of

$$\frac{1}{\tau_e} \approx \frac{1}{\tau_c} \frac{c}{4\sqrt{2\pi}L_{sep}n_b a_{bt} f_i}, \quad (4)$$

and coherent ion oscillation frequency of

$$f_i = \frac{c}{\pi} \sqrt{\frac{QN_e r_p}{3AL_{sep}\sigma_y(\sigma_x + \sigma_y)}}, \quad (5)$$

where  $Q$  denotes the charge of the ions, and  $2a_{bt}$  is the peak-to-peak variation of the ion oscillation frequency. For multiband lattice,  $a_{bt}$  is normally smaller than 1. The expressions are still only valid for small beam oscillation amplitude compared with the beam size, i.e.  $y \ll \sigma_y$ .

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### For Large Oscillation Amplitude

According to the simulation and experimental measurements, the instability growth rate at large beam oscillation amplitude will reduce rapidly, and the instability growth rate finally get saturate when it is comparable with the synchrotron radiation damping or transverse feedback damping [4, 7-8].

In this case, for large beam oscillation amplitude, i.e.  $y \gg \sigma_y$ , the interaction between the ions and the beam become nonlinear, and the oscillation will increase almost linearly as [2, 4]

$$y \sim \sigma_y \frac{t}{\tau_H}, \quad (6)$$

with a time constant of

$$\frac{1}{\tau_H} \approx \frac{1}{\tau_c} \frac{c}{2\pi f_i L_{sep} n_b^{3/2}}, \quad (7)$$

For the theoretical estimations, it is always assumed that the ions in the bunch train will not be over focused. This condition can be written as  $4L_{sep}f_i/c \leq 1$ .

### ANALYTICAL ESTIMATION ON HEPS

HEPS adopts hybrid 7BA lattice with super-bends and anti-bends to achieve low natural emittance. The design beam current is around 200 mA. Two operational modes with different filling patterns are considered. One is the high brightness mode with one bunch train of 680 bunches and bunch spacing of 1.8 meters. The other one is the high charge mode with 63 bunches uniformly filled in the ring. The main beam parameters of HEPS relevant to this study are listed in Table 1.

First, we estimated the ion density along the bunch train. We assume that the ion line density increases by  $\lambda_i$  when one bunch passes, and decreases exponentially with characteristic time of ion oscillation during the bunch gap. Here,  $\lambda_i$  can be expressed as

$$\lambda_i = \sigma_{ion} \frac{P}{kT} n_b, \quad (8)$$

and the effective ion density is

$$\rho_{ion} = \frac{2\lambda_i}{3\sigma_y(\sigma_y + \sigma_x)}, \quad (9)$$

Considering an average vacuum pressure of 1 nTorr and ion species of CO, the ion densities along the bunch train are calculated for the two operation modes. The results are shown in Fig. 1 and Fig. 2. From the results, we get the equilibrium ion density for the high brightness mode is around  $7.3 \times 10^{11} \text{ m}^{-3}$  and for the high charge mode is around

$5.0 \times 10^{11} \text{ m}^{-3}$ . The equilibrium ion density is lower for the high bunch charge mode due to the over-focus of the ions between the large bunch spacing.

With the ion density, the equilibrium transverse dimensions of the ions are recalculated according to Eq. (9), while keep the ratio of the horizontal and vertical size to be equal to  $\sigma_x / \sigma_y$  [9]. Then the ion oscillation frequency and the instability growth time with different assumptions are calculated based on the equilibrium transverse dimensions. The results are presented in Table 1.

Table 1: Main Beam Parameters of HEPS

Parameters, Unit	High Brightness Mode	High Charge Mode
Beam energy, GeV	6	
Circumference, m	1360.4	
RF frequency, MHz	166.6	
Beta function $\beta_x/\beta_y$ , m	4.5/8.1	
Bunch number	680	63
Bunch spacing, m	1.8	21.6
$\rho_{i, eff}$ , $10^{11} \text{ m}^{-3}$	7.3	5.0
$f_i$ , MHz	4.8	3.8
$4L_{sep}f_i/c$	0.1	1.1
$\tau_c$ , $\mu\text{s}$	19.3	31.9
$\tau_e$ , ms	1.9	2.8
$\tau_H$ , ms	61.7	27.2

The results show that the instability can develop in tens of micro seconds from the simple linear theory. When considering the damping effect due to the ion oscillation frequency spread, the growth time of the instability can be increased to several milliseconds. While at large beam oscillation amplitude, the instability growth time increases to tens of milliseconds.

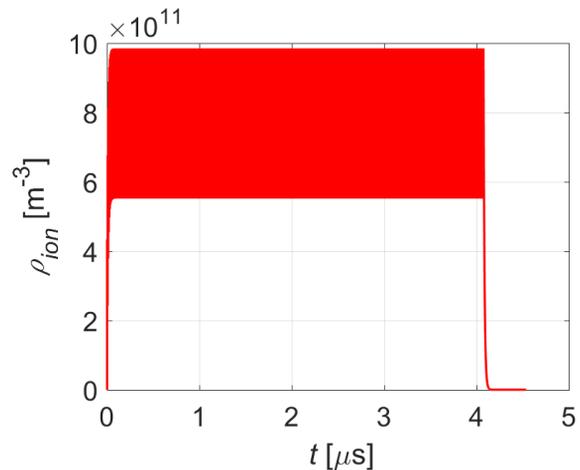


Figure 1: Ion density along the bunch train for the high brightness mode ( $n_b=680$ ).

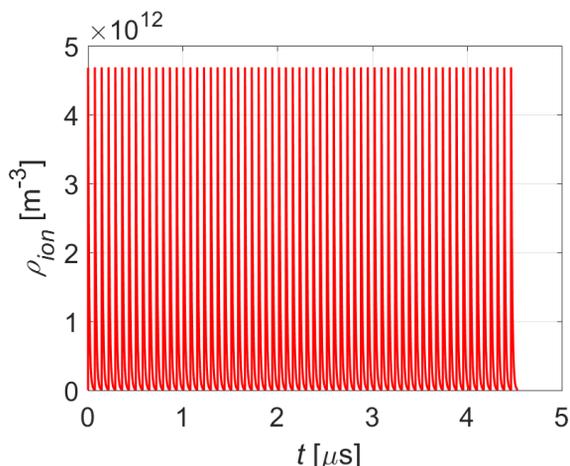


Figure 2: Ion density along the bunch train for the high charge mode ( $n_b=63$ ).

The analytical results are also benchmarked with the wake strong simulations [10]. Figures 3 and 4 show the development of the vertical oscillation amplitude of the beam with number of turns for the two operation modes. For the high brightness mode, the instability develops in around 4 ms. For the high charge mode, the instability develops in around 2 ms. The simulations show reasonably good agreements with the analytical estimations.

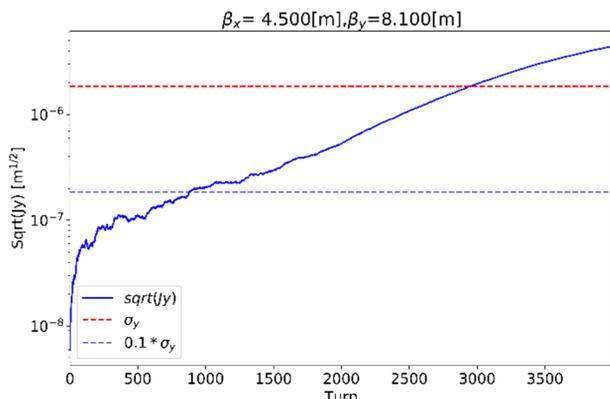


Figure 3: The evolution of the vertical oscillation amplitude of the beam with number of turns ( $n_b=680$ ).

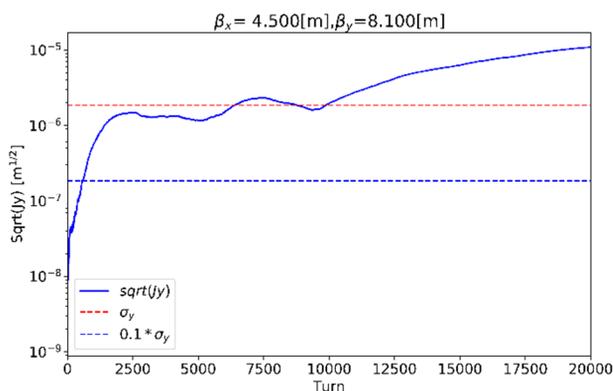


Figure 4: The evolution of the vertical oscillation amplitude of the beam with number of turns ( $n_b=63$ ).

## CONCLUSION

The existing theories on evaluating the fast beam ion instability with different approximations are reviewed. The ion densities along the bunch train in HEPS are calculated numerically for different operation scenarios. With the equivalent ion density, the growth time of the fast beam ion instability is estimated analytically. The estimated growth time at small oscillation amplitude, which is less than the transverse beam size, is around 1.9ms for the high brightness mode, and 2.8 ms for the high charge mode. While at large oscillation amplitude, the growth times are increased to tens of milliseconds. The theoretical estimations have also been benchmarked with the wake-strong simulations and reasonable good agreements have been reached. Since the growth rates are faster than the synchrotron radiation damping, efficient transverse feedback system is required to control the instability.

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